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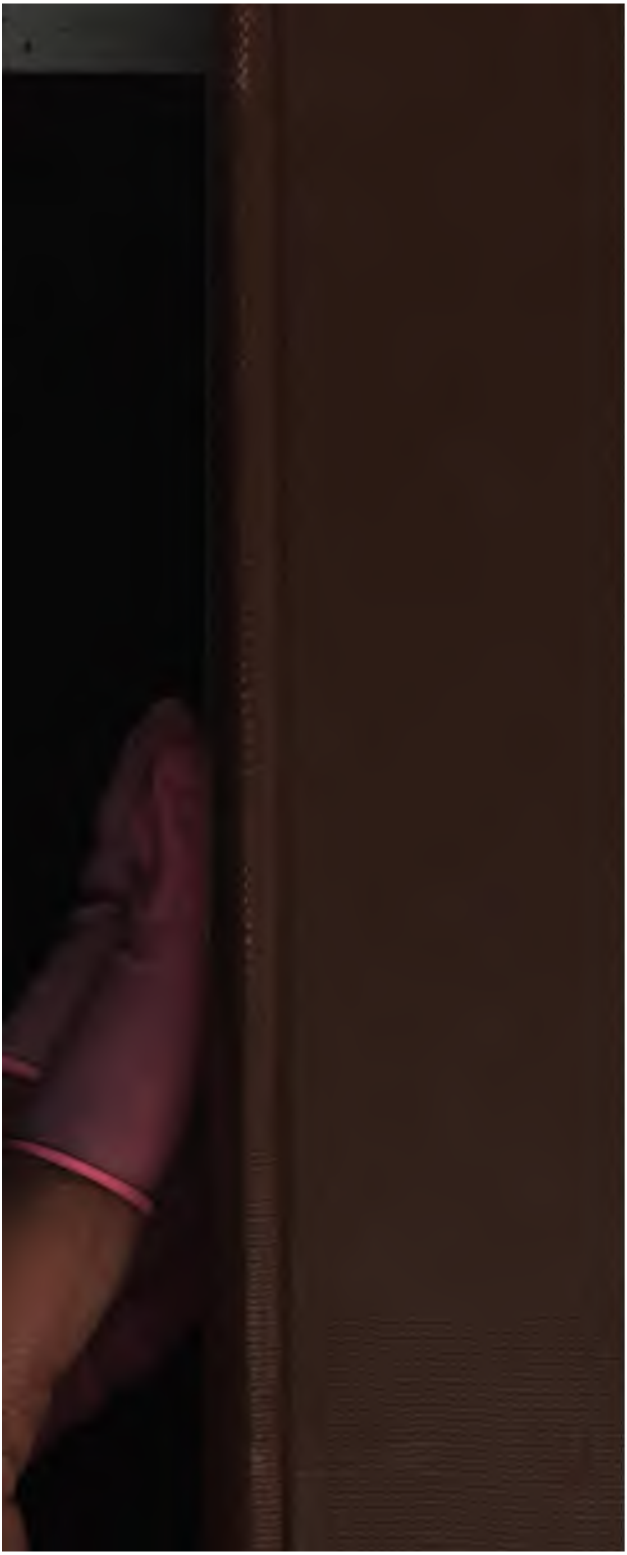

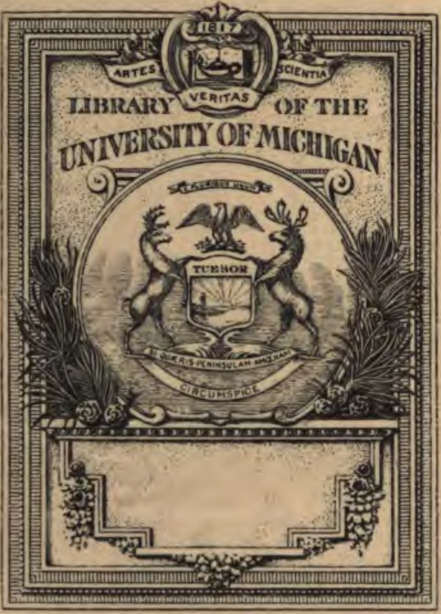

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THE
LEMENTS
OF
ſtronomy,
ICAL and GEOMETRICAL.

VID GREGORY M. D. *Savilian*
feſſor of ASTRONOMY at *Ox-*
, and Fellow of the Royal-
ety.

to *Engliſh*, with Additions and Corrections.

To which is annex'd,
HALLEY's Synopſis of the
Aſtronomy of Comets.

In Two Volumes.

Vol. I.

LONDON:
for J. NICHOLSON at the *King's-Arms*
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THE
AUTHOR's
PREFACE.

MY deſign in publiſhing this Book, was,
that the Celeſtial Phyſics, which the
moſt ſagacious *Kepler* had got the
of, but the Prince of Geometers Sir *Iſaac*
on, brought to ſuch a pitch as ſurpriſes all
World, might, by my care and pains in il-
ting, become eaſier to ſuch as are deſi-
of being acquainted with Philoſophy and
nomy. The Title informs you ſuffici-
that the Arithmetical or Calculatory part
ſtronomy is here omitted, tho' that, per-
, may be publiſhed hereafter in its proper
. As for the Phyſics, it is all taken out of
bovemention'd Authors; but is here inter-
d with Aſtronomy, in ſuch places as ſeem'd
er and convenient; the Geometry to be met
in it, I have either borrow'd elſewhere,
quoted the place where 'tis to be found,

or deliver'd it Lemmatrically. Whatever is done in each Section, you have it expreſs'd either in the Title or Preface thereof, in ſuch a manner, as that thoſe who are leſs vers'd in the more abſtruſe parts of Geometry, or leſs concern'd about the Phyſical parts, may paſs over, and only read the Aſtronomy ſeparately and diſtinct from them.

The Celeſtial Phyſics, or Phyſical Aſtronomy, is not only the firſt in dignity of all inquiries into Nature whatever, but the firſt in order, becauſe it is the eaſieſt. For the Sun and Planets are ſeparated from one another by ſo immenſe a diſtance, as renders them incapable of exerting moſt of thoſe forces whereby all Bodies act upon one another; ſo that they have no other force left them whereby they can affect one another, but the ſingle force of univerſal Gravity: Whereas in the production of ſeveral Phænomena, that are obſerv'd upon our Earth, innumerable other forces are exerted, ſuch as are very hard to be diſtinguiſh'd from one another; which notwithſtanding, if not accurately done, in vain do we attempt Nature, and make any inquiry into it. Upon this account it is, that every Problem in the Terreſtrial Phyſics is very operoſe and perplex'd, on the contrary, in the Celeſtial Phyſics, much more eaſy and ſimple; tho' even the latter has its difficulties, ariſing from the different diſtances and magnitudes of the Celeſtial Bodies. For the Fix'd Stars are ſo vaſtly diſtant aſunder, that they have no mutual action upon

ther, obſervable by us who are the In-
nts of the Earth. The Primary Pla-
re remov'd ſo far from each other, that,
hey have ſome ſmall power and effect
one another, yet we cannot be ſenſible
till after many Years obſervation. The
dary Planets are not at ſo great a di-
from their Primaries, or from the Sun,
at they may be conſiderably affected by
wers of both, (if regard be had to the
ity of Matter that is in theſe latter,) and
the ſpring of thoſe manifold inequali-
und in them, ſuch as, for inſtance, mani-
ſhews itſelf in our Moon; which yet is
g at all, if compar'd with the inequali-
und among Terreſtrial Bodies, which
ed upon by an innumerable variety of
forces, preſſing every way upon them.
at thoſe perſons ſeem to apply their
ts but to a very indifferent purpoſe in
dy of Nature, that overlook this part
onomy, from whence the principal and
imple Laws of Nature are to be learn'd.
t none may think the Phyſics deliver'd in
lowing Work intirely new and unknown
ronomy, I ſhall take the liberty to ſhew
was both known and diligently culti-
by the moſt ancient Philoſophers. And
dwell a little longer upon this argument,
e there is no need of ſpending a Preface,
upon the order of the parts of this
which may be ſeen in the Index, or
he uſefulneſs, dignity, hiſtory and pro-

greſs of Aſtronomy, or even of the true Syſtem of the World, approv'd of by *Pythagoras*, and others among the Ancients; theſe things being all of them treated of at large by the common Writers of Aſtronomy. What I ſhall now therefore make out is, that we do ſtill tread in the ſteps of the Ancients in this Phyſical Aſtronomy; inaſmuch as they knew that the Celeſtial Bodies gravitated towards each other, and were retain'd in their Orbits by the force of Gravity; and were alſo apprized of the Law of this Gravity.

For if we look back to the firſt Riſe of Aſtronomy, and take a view of it in its Infancy, as it were, we ſhall find nothing better approv'd of, nothing more univerſally entertained among the ſeveral Sects of Philoſophers, than this notion of the Gravity of the Celeſtial Bodies. That ſaying is well known, ſo often uſed by [a] *Anaxagoras*, and his Scholars, [b] *Achelaus* and [c] *Euripides*. Namely, "That "the Sun and Stars were *fiery or red-hot Stones* "and *Golden Clods*." Of the ſame mind alſo were [d] *Democritus*, *Metrodorus*, and [e] *Diogenes*,

By

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[a] *He affirm'd the Sun to be a Maſs of Red-hot Iron.* Diog. Laert. in Anaxag. *That the ſubſtance of the Sun was Stone, that of the Moon, Earth.* Plat. in Apol. Socr.

[b] *That the Stars were Plates of Red hot Iron.* Stob. Ecl. Phyſ. cap 25.

[c] *They ſay that* Euripides *uſed to call the Sun a Clod of Gold, or Golden Maſs.* Diog. Laer. in Anaxag.

[d] Anaxagoras, Democritus *and* Metrodorus *affirm'd, that the Sun was a Maſs of Iron or Stone red-hot.* Plut. de Placit. Phil. lib. 2. c. 20.

[e] Diogenes *thought that the ſubſtance of the Stars was not unlike that of a Pumice Stone.* Ibid, c. 13.

ſe expreſſions they meant no more, than
ey were heavy, denſe and fix'd Bodies,
s Stones are) ſo as to bear a conſidera-
gree of heat: And that this was really
neaning will evidently appear, if we do
quire more narrowly into the firſt Au-
of this Opinion. For, as we are told
emocritus, theſe notions about the Sun
oon are not to be aſcrib'd to *Anaxago-*
their original, for he had really bor-
them of the Ancients. Nor is it a diffi-
atter to find out, who they were that he
v'd them of, or from whom they were
d down to him. He had them from his
r [g] *Anaximenes*, whoſe Opinion we know
hat the Stars were of a fiery nature and
nce, that there were alſo mingled with
certain Earthly Bodies, which were car-
ound about them, tho' not viſible to us:
ich words he plainly means, Planets of
eſtrial nature, performing their revolu-
in the Syſtem of every Fix'd Star. Theſe
ns *Anaximenes* received from *Anaximan-*
Anaximander from [h] *Thales* himſelf, who
he Head and Founder of the *Jonic* Philo-

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vorinus, *in his various Hiſtory, relates, that* Democritus
ſay of Anaxagoras, *that the Opinions which he taught*
ing the Sun and Moon, were not his own, but far more anci-
n Anaxagoras'*s time; and that he had ſtollen them.* Laert.
nocrit.

naximenes *ſaid, the nature of the Stars were fiery, and*
here were certain terreſtrial Bodies that are inviſible, carried
er about them. Stob. Ecl. Phyſ. c. 25.

Thales *was of Opinion that the Stars conſiſted of an Earthly*
nce, which was continually red-hot. Stob. Ecl. Phyſ. c. 25.
ſo Plutarch. De Plac. Phyloſ. lib. 2. c. 13.

ſophy; and ſpread this opinion of the Gravity of the Fix'd Stars among his Sect. Nor did this Doctrine concerning the Stars ſtop here, but afterwards it diffus'd it ſelf thro' the *Italic* Philoſophy, the [i] followers of which taught, that each Star was a World in the infinite Æthereal Space, containing Earth, Air and Æther; and that the [k] Moon, not only was like our Earth, but inhabited by Animals of a larger ſize, and furniſh'd with Plants of a more beautiful appearance.

Nor were they ſo abſurd in their conceptions about Gravity, as to think that it was done by the virtue of any point within the Earth, or of a Center, to which all heavy Bodies placed any where tended; but they thought it was done by the [l] power of the whole Matter in the Terreſtrial Globe attracting all things to it ſelf: And as the power of the Loadſtone is compos'd of the powers of the ſeveral

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[i] *The* Pythagoreans *affirm'd, that every Star is a World in the infinite Æthereal Space, wherein are contain'd Earth, Air, and Æther,* Plut. de Plac. Philoſ. Lib. 2. c. 13.

[k] *The* Pythagoreans *aſſerted, that the Moon ſeem'd to be of a ſimilar nature with the Earth, it is inhabited as our Earth is, by Animals, tho' of a larger ſize than ours, and fill'd with the ſame Plants, tho' much more beautiful than ours.* Plut. de Plac. Phil. Lib. 2. c. 30.

[l] *And yet if every heavy Body inclines towards the ſame place, and does with every one of its parts tend to its middle or center, the Earth certainly will not appropriate to itſelf theſe heavy Bodies, which are its parts, becauſe it is the Center of the Univerſe, but rather becauſe it is the whole, of which they are the parts.* Plut. de facie in Orbe Lunæ.

[m] *As for that which is incorporeal, 'tis not probable, nor will they themſelves allow it to be poſſeſs'd with ſo great a power as to draw all things towards it, and retain them about it.* Ibid.

l parts combin'd together, ſo they be-
that the Gravity towards the whole
, reſulted from the Gravity towards
ingle part of it. Beſides, they believ'd
was a [m] Gravity towards the Moon and
acting in the ſame manner as it does to-
the Earth; and that each [n] Planet, like
e, whirl'd in a ſling, was kept in its
by the ſame principle, and for the ſame
revolving always about us. From
hings mention'd by [o] *Diogenes Laertius*,
ning *Plato*, which alſo are obſcurely
at in his [p] *Timæus*, I am apt to believe
[q] *Galileo*, that the divine Philoſopher
'd the Mundane Bodies, when they were
rmed, were moved with a Rectilinear mo-
by the means of Gravity,) but after that
they

---

d this meeting together of Bodies here, and their coalition
with the Earth's Body, ſhew us the manner how it is pro-
t the parts, which are aſſembled at the Moon's Body, con-
there.

the Moon is help'd, and preſerv'd from falling down, by
Motion and that impetuoſity of her Revolution; as Stones
weighty Bodies put in Slings and ſwung round, are kept
ping out by the ſwiftneſs of their Motion, and their being
rcularly. ——— *Wherefore the Moon does not move*
ds, as her own weight wou'd naturally carry her, her ten-
at way being ſtopt by the violence of her circular revolu-
bid.

ſe at firſt were mov'd in a confus'd and irregular manner,
n they were duely adjuſted and rightly ſettled, then the
as eſtabliſh'd by God in juſt order and proportion. Diog.
Plat.

gave it a Motion altogether agreeable to its nature as a
at is a direct one,) and a little further. *Therefore he*
ds made it continue its courſe in a Circle. Plat. in Timeus.
his Coſmical Syſtem,

they had arrived to ſome determined places, they began to revolve by degrees in a Curve, the Rectilinear Motion being chang'd into a Curvilinear one. 'Tis from this Doctrine of Gravity, that all Bodies gravitate mutually to one another, 'tis by this that [r] *Lucretius*, taught by *Epicurus* and *Democritus*, labours to prove, that the Univerſe has no Center or loweſt Place, but that there is an infinity of Worlds like ours in the immenſe Space. His Argument runs thus; If the nature of things were bounded any where, then the outmoſt Bodies, ſince they have no other beyond them, towards which they may be made to tend by the force of Gravity, wou'd not ſtand in an Equilibrio, but make towards the inner and lower Bodies, being neceſſarily inclin'd that way by their Gravity, and therefore having made towards one another, during an infinite ſpace of time, would have long ago met, and lye in the middle of the whole, as in the loweſt place. 'Tis evident therefore from hence that *Lucretius*, and thoſe whom he followed, believ'd that all Bodies did Gravitate towards the Matter placed around them, and that every ſingle Body was carried by the more prevailing Gravity, towards that region where there was moſt Matter.

As

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[r] *Suppoſe they all had Bounds, ſuppoſe an End;*
Then Bodies which by Nature muſt deſcend,
And from Eternity purſu'd the Race,
Had long e'er this time reach'd the Loweſt Place. Lucr. l. 1. v. 986.

t is manifeſt that the Ancients were
d of, and had diſcover'd the Gravity of
lies towards one another, ſo alſo they
ot unacquainted with the Law and Pro-
n which the action of Gravity obſerv'd
ling to the different Maſſes and Diſtan-
For that Gravity is proportional to the
ity of Matter in the heavy Body, [s] *Lu-*
does ſufficiently declare, [t] as alſo that
we call light Bodies, don't aſcend of
own accord, but by the action of a force
neath them, impelling them upwards,
s a piece of Wood is in Water; [u] and
er, that all Bodies, as well the heavy as
ght, do deſcend *in vacuo*, with an equal
ty. It will be plain likewiſe, from what I
preſently obſerve, that the famous Theorem
t the proportion whereby Gravity decrea-
n receding from the Sun, was not un-
vn at leaſt to *Pythagoras*. This indeed
s to be that which he and his followers
ld ſignify to us by the Harmony of the
res: That is, they feign'd *Apollo* playing
n an Harp of ſeven Strings, by which
bol, as it is abundantly evident from
ny, *Macrobius* and *Cenſorinus*, they meant
Sun in Conjunction with the ſeven Planets,
for

Beſides; why have not Bodies equal Weight
thoſe, whoſe Figure is but juſt as great? Lucr. l. 1. v. 415.
And this I think a proper place to prove,
nothing of it ſelf can upwards move. Lucr. l. 2. v. 178.
Therefore thro' Void, unequal Weights muſt be,
Swift in Motion, all of like degree. Lucr. l. 2. v. 228.
Plin. lib. 2. c. 22. Macrob. lib. 1. c. 19. Cenſorin. c. 11.

for they made him the leader of that Septenary Chorus, and Moderator of Nature; and thought that by his Attractive force he acted upon the Planets (and called it Jupiter's Priſon, becauſe it is by this Force that he retains and keeps them in their Orbits, from flying off in Right Lines) in the Harmonical ratio of their Diſtances. For the forces, whereby equal Tenſions act upon Strings of different lengths (being equal in other reſpects) are reciprocally as the Squares of the lengths of the Strings.

[y] For *Pythagoras* as he was paſſing by a Smith's Shop, took occaſion to obſerve, that the Sounds the Hammers made, were more accute or grave in proportion to the weights of the Hammers; afterwards ſtretching Sheeps Guts, and faſtning various Weights to them, he learn'd that here likewiſe the Sounds were proportional to the Weights. Having ſatisfy'd himſelf of this, he inveſtigated the Numbers, according to which Conſonant Sounds were generated. Whether the whole of this Story be true, or but a Fable, 'tis certain *Pythagoras* found out the true ratio between the ſound of Strings and the Weights faſten'd to them. The ſame Tenſion acts upon a String as ſhort again, four times more powerfully: For it produces an Octave, and an Octave is ſounded by a force that is four times greater; for if a String, put upon the ſtretch by a given Weight,

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[y] Macrobius lib. 2. inſomn. Scip. cap. 1,

, generates a given Tone, the ſame
tretch'd by a Weight four times greater,
und an Octave. Thus likewiſe the
enſion upon a ſubſeſquialteran Chord,
a double ſeſquiquartan ratio: For it
es a Fifth or Diapente, and a String
nds a given Note, with a given Weight
to be ſtretch'd by a Weight that is a
ſeſquiquartan to ſound a Fifth. And
ally, the Weights which generate all
in Strings, are reciprocally as the
s of the lengths of Strings of equal
n, producing the ſame ſound in any
l Inſtrument. *Pythagoras* afterwards
the proportion he had thus found
eriments, to the Heavens, and from
learn'd the Harmony of the Spheres.
by comparing theſe Weights with
ights of the Planets, and the inter-
the Tones, produced by the Weights,
e interval of the Spheres; and laſtly,
gths of Strings with the Diſtances of
nets from the Center of the Orbs; he
ood, as it were by the Harmony of the
s, that the Gravity of the Planets to-
the Sun (according to whoſe meaſures
anets move) were reciprocally as the
s of their Diſtances from the Sun.

have thus far been ſhewing what was
inion of the Ancients concerning Gra-
and it is evident they were perſwaded
ravity was not an affection of Terre-
Bodies only, but of the Celeſtial alſo,

that all Bodies gravitate towards one another, and that the Planets are retain'd in their Orbits by the force of Gravity, and laſtly, that the Gravity of the Planets towards the Sun are reciprocally as the Squares of their Diſtances from it. What the induſtry and ſkill of the Moderns have added to theſe inventions of the Ancients, the following Pages do declare at large.

THE
UBLISHER
TO THE
EADER.

HE *Science of* ASTRONOMY,
which is as much eſteem'd and ad-
for its great and manifold uſes for
rvice of Mankind, as it is delightful
tertaining to the more curious and
plative, has in all ages been cul-
d and improv'd, by Men the moſt
nt for their parts and learning; and
brought, as it were, to the ut-
legree of perfection, and that chiefly
ſuperior Genius and Induſtry of
of our own Nation. But ſince no-
conſiderable therein, has been as yet
in our own Language, I thought I
not oblige my Country-Men more
in publiſhing an Engliſh *Edition of*
moſt valuable and finiſh'd piece of
ONOMY *now extant. It is generally*

reckon'd to be a Book that contains not only all the Diſcoveries and Philoſophical Sentiments of the great Kepler, *and the various Hypotheſes of the moſt noted Aſtronomers before and ſince his Time; but is chiefly valued by the beſt Judges, for the large and inſtructive Comments deliver'd in it, on the Writings of the illuſtrious Sir* Iſaac Newton, *as well as on the ſeveral Aſtronomical Diſſertations of the ſagacious Dr.* Halley, *which the Reader will find here every where interſpers'd.*

And in order to render this work as compleat as poſſible, I ſhall, in a very little time, preſent you with another Volume, containing correct Aſtronomical Tables, for the ready computing of the Planets Places, Eclipſes, &c. *all done by a Perſon of known ability, from the true Theory of Gravity, deliver'd in this Book: For it was by no means judged proper that I ſhould annex to ſo intire a piece as this is, any imperfect Tables, drawn from a different Principle from what is here eſtabliſhed, ſuch it ſeems all thoſe as yet publiſhed are.*

THE
LEMENTS
OF
ſtronomy,
ical and Geometrical.

The FIRST BOOK.
he Syſtem of the World.

SECTION I.

ning the Order, Diſtances, and Peri-
f the Primary Planets revolving about
un, and the principal Phænomena
ce ariſing.

PROPOSITION I.

O give a general account of the Order, and
Periods of the Primary Planets revolving
about the Sun, and their Diſtances from it;
hat we are to think of the Comets and Fixt

Sun is to be look'd upon as immovable,
ced in the midſt of that immenſe Space,

in which the Planets perform their Revolutions. And there are ſix opake Spherical Bodies that revolve about it, as their Center, from Weſt to Eaſt, [*Fig.* 1.] from *A*, along *B*, *C*, and *D*, in the following Order: Mercury neareſt the Sun, compleating its Revolution in about three Months; next to Mercury, Venus in about ſeven Months and an half; then the Earth in a Year; Mars in about two Years; Jupiter in twelve; and, laſt of all, Saturn, which is outermoſt, in thirty. Their Diſtances from the Sun are nearly the ſame as they are repreſented in the Scheme: namely, ſuppoſing the diſtance of the Earth from the Sun to be divided into ten equal Parts, of theſe the diſtance of Mercury will be about four, of Venus ſeven, of Mars fifteen, of Jupiter fifty two, and that of Saturn ninety five.

'Tis to be obſerv'd, that all their Orbits are not in the ſame Plane, but variouſly inclin'd to one another; ſo that ſuppoſing the Plane of the Earth's Orbit to coincide with the Plane of this Scheme, one half of the Plane of any other Planet's Orbit will be above, and the other half below it; ſo that the Planes interſect in a Line that paſſes thro' the Sun.

Beſides the abovemention'd Planets moving round the Sun, there are likewiſe Comets; and theſe, if they are laſting in their Nature and Motions, move in very excentric Orbits, and conſequently ſhew themſelves only, when they are in thoſe Parts of their Orbits that are neareſt to us and the Sun: Some of them move, like the Planets, from Weſt to Eaſt, others from Eaſt to Weſt; ſome again from North to South, and laſtly, others from South to North. Their Orbits are various and different in Magnitude, Situation, and Inclination both to one another, and to the

its of the Planets. Their Periods are
et known from Obſervation, nor in-
t fully certain, that they move in Lines
rn into themſelves. But all theſe things,
, will be determined by proper Obſer-
made in future Ages.
reſt of the Mundane Space is to be con-
s divided into Spaces juſt like that we
en deſcribing, each having one of thoſe
its Center, which are called *Fixt Stars*,
ing the office of a Sun to it, and having,
be, Planets and Comets of its own re-
about it.
nough at firſt to have a general con-
of theſe things as they are here deſcribed;
the Periods of the Planets, and their
s from the Sun, as here laid down, are
ctly true, yet they are neareſt the Truth
l Numbers. Again, tho' the Paths they
are not perfect Circles, concentric to
and the Motion of the ſame Planet not
equable; yet the difference is ſo ſmall,
y need not, at preſent, be taken other-
ll by Obſervations and Methods of uſing
ereafter to be ſhewn in their proper
ll theſe things ſhall be preciſely and
ettled, and theſe niceties examined; or,
till the ſame are purpoſely handled, in
inquire into their Phyſical Cauſes.

PROPOSITION II.

deſcribe the Phænomena that ariſe from the
uation of the Sun and motion of the Earth,
l above.

If the Obſerver be ſuppos'd to be plac'd
un, 'tis evident the Earth will ſeem to
move from Weſt to Eaſt perpetually, as

really it does. Then ſince, beſides the Earth, our Obſerver ſees the Fixt Stars plac'd round him, as it were (the nature of the Eye requiring it) in a concave Sphere, that has the Eye for its Center; 'tis likewiſe evident, that he will obſerve the Earth moving, as it were, among the Fixt Stars, and approaching nearer and nearer the more Eaſtern ones, till, in a Year's ſpace, having compleated its Revolution, it returns to the ſame place among them again. And becauſe the Earth always goes the ſame track over again, the Obſerver will take eſpecial notice of the Stars the Earth paſſes over; alſo the Plane of the Earth's Orbit, and the Circle in the Sphere of the Fixt Stars, called the *Ecliptic*, made by that Plane; and this will be a great Circle, becauſe it paſſes thro' the Sun, or Eye, which is the Center of that Concave Sphere that terminates the Sight. But if the Obſerver, for the advantage of making Obſervations, imagines this Ecliptic divided into twelve equal Parts, or *Signs*, calling them by the name of any neighbouring Conſtellation, or Figure thoſe Stars ſeem to make; in this caſe, I ſay, [*Fig.* 2.] the Earth will ſeem to move from ♈ to ♉, and from thence to ♊, and ſo on; from Weſt to Eaſt, thro' all the Signs, till it return to ♈ again.

Secondly, If you imagine the Obſerver to be removed from the Sun to our Earth, if the Earth be at *A*, where it is ſeen from the Sun among the Fixt Stars, at ♈; the Sun will appear, when ſeen from the Earth, in the oppoſite Sign ♎, among the Fixt Stars; the Earth being then the Center of the Sphere of the Fixt Stars: For the place of the Eye is the Centre of a Sphere, on whoſe ſurface the Stars are conceiv'd

o be plac'd. If the Earth be moved
thro' *B*, to *C*, in its Orbit, or if look'd
rom the Sun, from ♈ thro' ♉, to ♊,
uentia, or according to the order of the
the Sun, to an Obſerver on our Earth,
nks the place he ſtands upon immovable,
ear to move among the Fixt Stars, ac-
to the order of the Signs alſo, from ♎,
, to ♐, *&c.* in the ſame Plane, during
e time, and towards the ſame Region of
avens, as the Earth ſeen from the Sun
t in the oppoſite Points of the Ecliptic.

SCHOLIUM.

like Phænomena happen in reſpect of
and any other Planet; nay indeed, the
ne, excepting that the time of that Pla-
olution about the Sun, or the Sun's ap-
evolution about the Planet, when view'd
at Planet, is various, according to the
t Period of each, mention'd in the fore-
'ropoſition; and that the Plane of the
f that Planet produced, will cut other
n thoſe which the Plane of the Earth's
es, when produced; and conſequently,
Path of the Sun among the Fixt Stars,
n any other Planet, is different from its
hen ſeen from the Earth, that is, from
ptic.

PROPOSITION III.

*deſcribe the Phænomena of the Planets ſeen
om the Sun, ariſing from their Motion in
hoſe Planes are inclin'd to the Planes of the*

the Orbits of the Earth and Planets are
ed, as that their Planes are inclin'd to
her, and interſect each other, (as was

ſhown in general in *Prop.* 1.) in right Lines paſſing thro' the Sun; the Inclination of the Plane of the Orbit of each, to the Plane of the Ecliptic, or Earth's Orbit, is to be taken into conſideration, in explaining the Phænomena of the Planets viewed from the Sun. For the Plane of the Ecliptic is taken by Aſtronomers as the Standard to which the Planes of the other Orbits are judged to incline; and that with very good reaſon, ſince it is that in which the Earth (the habitation of the Aſtronomer) moves round the Sun, or in which the Sun ſeems to move round the Earth: And an Obſerver plac'd in any other Planet, would make the Plane of that Planet's Orbit, the Standard of all the reſt, and conſider them as inclin'd to it.

The right Line which paſſes thro' the Sun, and is the common Section of the Plane of the Orbit, with the Plane of the Ecliptic, is call'd the *Line of the Nodes* of that Planet, and the Points themſelves, wherein the Orbit of the Planet cuts the Ecliptic, are call'd the *Nodes*. Thus, [*Fig.* 3.] let ⊙*T* ♁ *t* be the Plane of the Orbit of the Earth produced indefinitely, *NPn* the Orbit of any Planet, interſecting the Plane of the former Orbit or Ecliptic in *N* and *n*, which are the Nodes of that Planet; ſo as that one part *NPn* of that Orbit be ſuppos'd above the Plane of this Scheme, and the other, *npN*, below it, (which makes it look like an Ellipſe:) The right Line *Nn* joining the Nodes, being the common Section of the Plane of the Orbit of the Planet with the Plane of the Ecliptic, is the Line of the Nodes.

'Tis evident then, that if a Planet be ſeen from the Sun, when it is in one of the Nodes, as *N*, it will appear to be in the Plane of the Ecliptic. But when gone forwards as far as P, it will

deviate from the Ecliptic: and (by *Def-*
. XI.) the Inclination of the right Line
the Ecliptic, and conſequently of the
at *P* ſeen from the Sun, call'd the Planets
ntric Latitude, is meaſur'd by the Angle
where *PE* is ſuppos'd to be a perpendicular
from *P* to the Plane of the Ecliptic. This
entric Latitude is continually upon the
e, till the Planet is got as far as *L*, its
where it is equal to the Inclination of
ne of the Planet's Orbit, to the Plane of
iptic. But during the Planet's paſſage
hence to the other Node *n*, it is decrea-
ill at laſt it vaniſhes at the Node. Having
the Node *n*, it begins again, chang-
name, becauſe 'tis towards the con-
parts of the Ecliptic, and grows bigger
igger, till the Planet has arriv'd at the
Limit *l*, from whence again it grows leſs
ſs, till it vaniſhes at the other Node *N*.

Orbs of the Planets (that is, the Planes of
Orbits) are inclin'd to the Ecliptic in the
ing manner: The Orb of Saturn makes
gle of 2 $\frac{1}{2}$ Degrees; Jupiter, 1 $\frac{1}{3}$ Degree;
a little leſs than 2 Degrees; Venus, ſome-
above 3 $\frac{1}{3}$ Deg. Mercury almoſt 7 Degrees.
ccount of the Poſition of the Line of the
s of each Planet ſhall be given in a
proper place. We have, in theſe two laſt
oſitions, conſider'd the Planets as they ap-
to one plac'd in the Sun: becauſe it
neceſſary to the underſtanding their Motion
view'd from the Earth.

PROPOSITION IV.

TO *deſcribe the Phænomena ariſing from the Motion of the Earth, and inferior Planets, Venus and Mercury, when view'd from the Earth.*

Since Venus and Mercury revolve about the Sun, in leſſer Orbits than the Earth, as you ſee in the Scheme, [*Fig.* 4.] where *T* repreſents the Earth, carried in its Orbit *T* ♁ from Weſt to Eaſt; and *ACEG*, the Orbit deſcrib'd by Venus in a leſs ſpace of Time the ſame way; 'tis evident, that when Venus is in *D E F*, that part of its Orbit that is fartheſt off the Earth, it will appear to us on the Earth to move *in conſequentia*, or according to the order of the Signs, and is then ſaid to be *Direct*. When it is in *G*, moving from thence to *H*, it will appear to move as ſwift as the Sun, becauſe then its Motion tends directly towards the Earth, and it does not ſeem to move at all, but as its Orbit is carried along by the Sun, whoſe Motion is towards the Eaſt: Venus moves now therefore ſlower than before, but is ſtill Direct. When it is got beyond *H* in its Motion, thro' *A* to *B*, it paſſes between the Earth and Sun, becauſe it is nearer to us than the Sun, and moves ſwifter than the Earth, (the cauſe of which we ſhall hereafter aſſign,) conſequently will ſeem to us to change its Place among the Fixt Stars, and move *in antecedentia*, or contrary to the order of the Signs, and then it is ſaid to be *Retrograde*, tho' really Direct ſtill, if view'd from the Sun. Between Direct and Retrograde, for inſtance, about *H*, it will appear *Stationary*, the right Lines that join the Earth and Venus, continuing for ſome ſenſible time parallel. Thus likewiſe, after its Retrogradation, before it becomes Direct again, it will appear Stationary

ſecond time, and is Stationary in order
ing Direct, as it was before in order
ng Retrograde at *H.* In all this affair,
s to be had to the Motion of the Earth,
us is Direct, Stationary or Retrograde,
ng as it is poſited in ſuch parts of its
have the ſame relation to the Earth in
on as the Points aforementioned have.
at has been ſaid, 'tis evident that Venus,
etrograde, as at *A*, is nearer to the Earth,
ſequently appears bigger than at other
r the contrary when Direct, as at *E*, it
off, and conſequently appears leſs.
ecauſe Venus moves round the Sun at a
nce than the Earth; 'tis evident that
eem always to attend the Sun, ſome-
go to the Weſt of it, ſometimes to the
the Heavenly Bodies ſeeming to be at
diſtance from the Eye of the Spectator.
reſſion to the Eaſtward or Weſtward of
is call'd the *Elongation*, and is meaſured
gle contain'd under a right Line drawn
Eye to the Sun and Venus, which is
reater than the Angle ☉*TC* or ☉*TG*,
nes *TC* or *TG*, when drawn, are Tan-
the Orbit *A D F.* Conſequently the
ion of Venus will never be above half
ant from the Sun, as is evident from
idiameters laid down in *Prop.* i. And
has arrived to its fartheſt Elongation
eturn to the Sun, and paſs as far be-
n the other ſide as if its Motion were
ory.
ury has all the ſame Phænomena; but its
ns, Stations, and Retrogradations hap-
ner, becauſe it finiſhes its Courſe in a
time, and conſequently overtakes the
Earth,

Earth oftner than Venus. And ſince Mercury's Orbit is leſs than that of Venus, its greateſt Elongations muſt be alſo leſs, and it muſt be a nearer and more conſtant attendant of the Sun, being, by *Prop.* i. never a whole Sign diſtant from it, and conſequently ſeldom to be ſeen by us.

PROPOSITION V.

TO *deſcribe the Phænomena of the Latitude of the inferior Planets ſeen from the Earth.*

Let T♁t be the Earth's Orbit, whoſe Plane is the ſame with that of the Ecliptic; [*Fig.* 5.] and let N♀n be the Orbit of an inferior Planet, for inſtance Venus, whoſe Plane is inclin'd to that of the Ecliptic, and therefore will look like an Ellipſe, whoſe greater Axe is the interſection of the Planes, or Line of the Nodes Nn. And while Venus is in ♀, let the Earth be in T; in which ſuppoſition Venus will be neareſt the Earth, and Retrograde, by *Prop.* iv.

'Tis evident from *Def.* 5. *Elem.* xi. that the Inclination of the right Line ♀T to the Plane of the Ecliptic, or the Latitude of Venus in ♀, ſeen from the Earth (which is hereupon called the *Geocentric Latitude*) is meaſured by the Angle ♀TE, the right Line ♀E, being made perpendicular to the Plane of the Ecliptic. If Venus be ſuppos'd to continue in ♀, and the Earth to be at t, in which ſuppoſition Venus is Direct, and furtheſt off the Earth; the Geocentric Latitude of Venus will be the Angle ♀tE, leſs than ♀TE, almoſt in the ratio of T♀ to t♀, tho' the Heliocentric Latitude of Venus be in both caſes the ſame.

And all that has been now ſaid is true of Mercury as well as of Venus; conſequently all other

rcumſtances being alike, the Latitude
ferior Planets is greater, when they are
ade and neareſt the Earth; leſs, when
nd fartheſt off.

ver if any inferior Planet be moſt Retro-
d neareſt the Earth, and at the ſame time
ar a Node, it will be found directly be-
he Obſerver and the Sun: If it be at a
able diſtance from a Node, it will paſs
to the Northward or Southward of him.
manner, if it be moſt Direct and fartheſt
Earth, and be alſo near, or in a Node, it
cover'd by the Sun; if it be otherwiſe
, it will paſs on one ſide of the Sun.

PROPOSITION VI.

explain the Phænomena of the Inferior Pla-
ets, ariſing from their being opake Spherical
nlighten'd by the Sun.

e all the Planets, as well as the Earth,
ake Spherical Bodies, reflecting the Rays
ht that fall upon them every way, 'tis evi-
at that half of each Planet which is ex-
or turn'd to the Sun, is enlighten'd by
ile the other half, which is turn'd from
n, is in darkneſs. And ſince that half
which is towards the Earth is ſeen by the
ver, if we conſider which Face of Venus is
ten'd in this or that ſituation in reſpect of
n, together with the Face that is viſible at
arth, [*Fig.* 6.] in the part *T* of its Orbit
it will be manifeſt, that when Venus is in *A*,
is, moſt Retrograde and neareſt to us, it
t viſible at all, having its dark Face turn'd
: And if Venus were at the ſame time in
lane of the Ecliptic, that is, in one of its
es, it would appear like a Spot in the Sun,

being ſituated directly between the Earth and Sun, by the foregoing Propoſition.

When Venus is got into the ſituation *B*, that is, begins to be Retrograde, if view'd from the Earth, then ſome part of the enlighten'd half is turn'd towards the Earth, tho' the greater remaining part belongs to the obſcure half. And becauſe Venus is of a ſpherical Figure, and appears like a Plane, its enlighten'd part will appear Horned, and thoſe Horns turned towards the Weſt, or from the Sun.

When Venus is at *C*, half the enlighten'd part will be ſeen by us, and then it is ſaid to be half Full; at *D* it is ſaid to be Gibbous, above half the enlighten'd part being viſible; and at *E*, where it is fartheſt off the Earth, and moſt Direct in its Motion, it appears Full, all the enlighten'd Hemiſphere being turned towards us.

Venus will have the ſame variety of Phaſes in its paſſage thro' *F*, *G*, and *H*, that is, it will be Gibbous at *F*, half Full at *G*, and Horned at *H*, with its Horns turned from the Sun, that is, in the preſent caſe, looking towards the Eaſt, exactly contrary to what they were when it was at *B*.

For the better underſtanding of this matter, we have delineated the ſeveral Phaſes of Venus, as they appear to us, in the Scheme [*Fig.* 7.]; where the ſame Letters are uſed that were in the Diagram [*Fig.* 6.] repreſenting its Phaſis when it was in that part of its Orbit, marked by the ſame Letter. Thus, in this Scheme, [*Fig.* 7.] at the Point *A*, it is drawn all obſcure, becauſe that is its condition in reſpect of the Earth at *T*, when Venus is at *A*, in the former Diagram. At *B* it is repreſented Horned, be-

it appears ſo, when it is at *B*, in the
Scheme, and look'd upon from the Earth
nd ſo on in the other caſes of *C*, *D*, *&c.*
ike Phænomena will appear in Mercury,
being had to its Orbit and Revolution.

SCHOLIUM.

ie Phænomena deſcribed in the three laſt
tions, manifeſtly follow upon the Situation
otion of the Earth, Venus and Mercury,
wn in *Prop.* 1. So it follows converſely,
e Obſervation of theſe Phænomena in
eſtabliſhes and confirms that Order and
on, namely, that Venus and Mercury re-
bout the Sun, in Orbits, that are included
the Earth's Orbit.

PROPOSITION VII.

describe the Phænomena ariſing from the
Motion of the Earth, and of the ſuperior Pla-
Mars, Jupiter, and Saturn. [*Fig.* 8.]

M ♂ be the Orbit of any one of the ſupe-
anets; for inſtance Mars, *A C* ♄ *G* the Or-
the Earth, nearer to the Sun. 'Tis evident,
that this Planet will not always attend the
but ſometimes be diametrically oppoſite
For the Earth finiſhing its Revolution
r than any of the ſuperior Planets, will
imes be exactly between the Sun and
Planet; thus, when Mars is in *M*, the
n may be at *H*; and, to ſpeak univerſally,
ngle at the Earth, made by lines drawn
ce to the Sun and that Planet, may be equal
ny given one.

t us ſuppoſe Mars to be in *M*, and the Earth
e ſame time in *A*; Mars in this caſe will
ar Stationary, in order to its being Direct,
uſe the right Lines that join the Earth and
Planet

Planet at that moment, will continue parallel for ſome ſenſible time; during the whole of which, notwithſtanding, Mars will ſeem to go forwards, as uſual, if viewed from the Sun.

While the Earth moves along thro' *B*, *C*, *D*, *E*, *F* and *G*, Mars likewiſe will ſeem to go forwards among the Fixed Stars, upon a double account; firſt, becauſe it really does move about the Sun, *in conſequentia*; and then again, becauſe the Earth in the oppoſite Semicircle is carried the ſame way, and about the ſame Center: And, conſequently, Mars in this caſe being moſt remote from the Earth, and in conjunction with the Sun, view'd from the Earth, will ſeem to move faſter than ordinary, *in conſequentia*, and become Direct. But when the Earth is arriv'd to the Point *G*, in reſpect of Mars at *M* (which ſome time or other will happen, tho' Mars be carried in the mean while about the Sun, namely, when the Earth has almoſt overtaken Mars) Mars will again become Stationary, in order to its being Retrograde, as it will be ſoon after. For when the Earth in its Motion from *G*, thro' *H* to *A*, has paſs'd Mars, and that Planet is ſeen in oppoſition to the Sun, and biggeſt, becauſe neareſt the Earth, which is lower and ſwifter, will make Mars appear to move *in antecedentia* from *S* thro' *K* to *P*; whereas in the mean while view'd from the Sun, it ſeem'd to move as always before, *in conſequentia*.

The like Phænomena will happen to Jupiter and Saturn, excepting that Saturn's Retrogradations are more frequent than Jupiter's, and Jupiter's than Mars's: becauſe the Earth oftner overtakes Saturn than Jupiter, and Jupiter oftner than Mars, and paſſes between them and the Sun.

PROPOSITION VIII.

eſcribe the Phænomena of the Latitude of the
perior Planets ſeen from the Earth.

e Earth's Orbit be *T t* ♁ ; [*Fig.* 9.] that
iperior Planet as Mars ♂ *M*, whoſe Plane
ed to that of the Ecliptic, and cuts it in
of the Nodes *N* ⊙ *n* : Let the ſituation
and the Earth, to the Sun, be ſuch, as that
ing in ♂ , the Earth may be in *T*, almoſt
Mars and the Sun; in which caſe Mars
eareſt the Earth, and conſequently big-
moſt Retrograde, as was ſhown in the
g *Propoſ.* Its Geocentric Latitude will
ured by the Angle ♂ *TE*, ♂ *E* being ſup-
rpendicular to the Plane of the Ecliptic.
lars continuing in the ſame ſituation in
Orbit, and conſequently having the ſame
ntric Latitude, the Earth being ſuppoſed
the Sun being between it and Mars;
h Caſe, by the foregoing, Mars will be
off, and conſequently leaſt, and moſt
in its Motion; Its Geocentric Latitude
d by the Angle ♂ *t E*, is always leſs than
gle ♂ *TE*, in the ratio of the Diſtances of
th from Mars, that is, of the right Lines
. Thus in whatever ſituation Mars and
th be placed, in reſpect of the Sun, its
tric Latitude will be changed, ſo as, *ca-*
ibus, it will be leſs as Mars is nearer to
unction with the Sun, and its ſwifteſt di-
otion; and greater, as it is nearer its Re-
lation and Oppoſition to the Sun.

evident from what has been ſaid, that
f the Superior Planets can ever be ſeen
ne Earth to cover the Sun; tho' any of
nay be covered by the Sun, when it is
and pretty near a Node.

PRO-

PROPOSITION IX.

TO *deſcribe the Phænomena of the Superior Planets, ariſing from their being Opake Bodies, and enlightned by the Sun.*

Saturn and Jupiter being Opake Bodies, and illuminated by the Sun, that half of each Planet which is turn'd towards the Sun, (that is, the illuminated half) is likewiſe turn'd towards the Earth, which is never far off from the Sun, or Center of Saturn and Jupiter's Orbit: For, by *Prop.* i. Jupiter's diſtance from the Sun is above five times, and Saturn's almoſt ten times greater than the Earth's diſtance from it.

In Mars indeed it is ſomething different: For the diſtance of Mars from the Sun being but half as much more as the Earth's Diſtance from it; its inlighten'd Hemiſphere, towards the Sun, is not always, as to Senſe, turn'd towards the Earth. [*Fig.* 10.] Let *T* be the Earth's place in its Orbit *T*♁, 'tis evident that Mars being at *A* or *B*, in Conjunction or Oppoſition to the Sun, has the ſame Face towards the Earth, as it has towards the Sun, that is, its enlightened one, and conſequently appears Full; but in the ſituation of the Points *D* or *C* (when the Angle ⊙*CT*, or ⊙*DT* is greateſt, or when ⊙*TC*, or ⊙*TD* is almoſt a right Angle) neither is the whole enlighten'd Face ſeen, nor is that Face that is ſeen entirely illuminated, but it appears Gibbous, the light being deficient a little towards thoſe parts that are turn'd from the Sun.

SCHOLIUM.

As the Phænomena deſcribed in theſe three laſt Propoſitions follow from the Order and Motion of the Earth, Mars, Jupiter, and Saturn, laid

n in *Prop.* 1. So by the converſe of
ie Obſervation of theſe Phænomena
d eſtabliſhes that Order.

PROPOSITION X.

ſcribe the Phænomena of the Motion of Co-
ts, ſeen from the Earth.

e different Comets have different Or-
ch can't be determin'd till ſome Obſer-
ave been made about them: 'Tis evi-
during their deſcent, near their Peri-
r while they are in that part of their
hich is within the Region of the Pla-
y have the ſame Phænomena as the
xt them has, regard being had to the
and Inclination of the Orbit of the

uently a Comet's apparent Path a-
Fixt Stars, will be nearly a Great Cir-
pting that its deviation from it, on this
e of it, will be like that of a Planet,
as the Motion of the Earth is, which
ng with it the Spectator. For were the
ew'd from the Sun, it would deſcribe
ircle among the Fixt Stars accurately.
viation from one, on the account of
n of the Earth round the Sun, will
ry ſenſible in a ſmall ſpace of time.
Earth happen to be between the Sun
met, that moves *in conſequentia*, the
ill be Retrograde; for the Earth
much the ſwifter, as the right lines that
he Earth and Comet converge towards
on beyond the Comet: But if the
ves ſlower, then the Comet's Motion,
away the Earth's Motion, likewiſe be-
wer: But if the Sun be between the

Earth and the Comet, the Comet's Motion will appear ſwifter than really it is. On the contrary, the Comets that move *in antecedentia* are ſwifter than they ſhould be, when the Earth is between them and the Sun; and ſlower, or perhaps moving according to the natural Order of the Signs in appearance, when the Earth is ſituated on the contrary ſide. All this happens from the Motion of the Earth, and its various Poſition, as it does in the Planets; that are, according as the Motion of the Earth falls in with, or is contrary to their Motion, ſometimes Retrograde, ſometimes Slower, ſometimes Swifter than they ſhould be, as has been ſhewn in *Prop.* iv and vii. Thoſe Phænomena will be moſt ſenſible, and eaſily taken notice of, a little before the diſappearance of the Comet; its apparent Motion being at that time ſlower, and the Earth's Motion proportionably greater, with which the Spectator is carried.

The Latitude of the Comet likewiſe, *cæteris paribus*, is varied by the different Situation of the Earth, it being greater in Oppoſition; but in Conjunction greater or leſs, according as the Comet is between the Earth and Sun, or the Sun between the Comet and the Earth, as was demonſtrated of the Planets in *Prop.* v and viii.

SECTION II.

Direction of the Forces, which re-
he Primary Planets in their Orbits.

PROPOSITION XI.

Body be moved according to the Direction of
iven right line as A Z, [Fig. 11.] *and at*
time be urged by a Centripetal Force tend-
ds a certain given immoveable Point as S,
without the aforeſaid right line; the Line
y the Body will be a Curve, and Concave to-
, lying all of it in the ſame immoveable
ing thro' the right line A Z *and the Point* S:
Areas contain'd under any portions of the
d Right lines drawn to the Center S, *are to*
r, as the Times wherein thoſe portions of the
re deſcribed.

e Time be imagin'd to be divided into
ts; in the firſt of which let the Body
the line *A B*, a part of *A Z*, by its *Vis*
ie, by which it tends to move accord-
he Direction of the Line *A Z*; in the
art of Time it will deſcribe the Line
l to the former *A B*, if nothing hinder
the firſt and chief Law of Motion, all
ice put into Motion, and not meeting
Impediment, will continue to move
y on, in the ſame right line they were
ed in. But when the Body is arrived at
t *B*, let us ſuppoſe a Centripetal Force
owards the Point *S*, to act upon it by
mpulſe, ſo as that, had it been impell'd
hat Impulſe, it would in the ſecond
ime deſcribe the line *B G*. 'Tis evident

that

that drawing the Line *c C* parallel to *BG*, thro' the Point *c*, and *GC* to *Bc*, thro' *G*, the Body acted upon by both Impulſes at once, will arrive, in the ſecond part of time, at the Point *C*, deſcribing the right line *B C*. For as it is well known in Mechanics, a Body deſcribes the Diagonal of a Parallelogram, both the Forces combin'd, in the ſame time as it would do the Sides, with the Forces ſeparated. 'Tis certain likewiſe that the right line *B C* is in the Plane of the Parallelogram *BGCc*, both of whoſe Sides *B G* and *B c* is in the Plane of the Triangle *A S B*, that paſſes thro' the Center *S*, of the Centripetal Forces, and the immoveable right line *A Z*. Beſides, the Triangles *S C B*, *S c B* are equal, becauſe they are upon the ſame Baſe *B S*, and between the Parallels *S B*, *C c*; but *S c B*, *S B A* are equal, becauſe their Baſes are equal, and height the ſame: Conſequently *S B A*, *S C B* alſo are equal. By the ſame method of reaſoning, if in the third particle of Time, a Body deſcribes any other right line as *C D*; it may be proved that the Triangle *S C D* is equal to the Triangle *S B C*, and that the right line *C D* is in the ſame Plane with the right lines *S B*, *B C*; that is, in the ſame with that which is drawn through the right line *A B*, and the Point *S*. And ſo we may go on, as long as the Motion is continued, and in equal times the Area deſcribed by Radii drawn to the immoveable Center of the Forces, will be equally increaſed; and by compounding Ratios, any ſums of Areas, are to one another, as the Times wherein they are deſcrib'd. The Line deſcrib'd by the Body will be in the immoveable Plane, and it will be concave towards *S*, becauſe every part of it that is a right line, as *B C*, declines from the foregoing

wards the Center. If you ſuppoſe the
of the Triangles, *S A B*, *S B C*, *S C D*
gmented, and their breadth to be di-
l *in infinitum*; their Baſes, *A B*, *B C*, *C D*,
m a Curve Line, concave towards the
rts, and lying in the ſame Plane; and
tripetal Force, which before acted, as
y ſtarts, and at equal intervals of Time,
the Body is continually drawn off from
gent of that Curve (which the baſes of
igles meaſure) now acts conſtantly:
Areas thus deſcrib'd, *S A B C S*,
D E S, will be, as before, proporti-
e Times of deſcription. *Q. E. D.*

PROPOSITION XII.

y moved in a Curve Line A B C D,
g. 12.] *deſcrib'd upon a Plane, and con-*
ds the ſame parts, and by a Radius drawn
mmoveable Point ſituated in the ſame Plane
Concavity of the Curve, deſcribing Areas
to the Times, is urged by a Centripetal
ng to the Point S.

h a Curve be imagined to be divided
rts *A B*, *BC*, *C D*, *&c.* differing as little
from right lines, ſo as to be deſcribed
Particles of Time: Let the Centripe-
likewiſe be conceiv'd to act only in
B,*C*,*D*, *&c.* by ſtarts, as in the forego-
Let *A B* be produc'd to *c*, ſo as that
ual to *A B*; in like manner *B C* to *d*,
equal to *B C*; and ſo on. The Tri-
B, will be equal to *S B C*, becauſe, by
s, the Areas deſcribed are proportio-
Times; and *S A B* is equal to *S B c*,
B is equal to *B c*: Wherefore *S B C*
o *S B c*, and conſequently (by 39

Elem.

Elem. 1.) *Cc* is parallel to *S B*. Moreover, the Body moved along *A B*, in the firſt particle of Time, by its *Vis inſita* alone would deſcribe *B c*, in the ſecond; but it really does deſcribe *B C*, in that ſecond Particle of Time: Wherefore the Force acting in the Point *B*, in conjunction with the *Vis inſita* to carry the Body along *B C*, is directed according to a Right line parallel to *Cc*; that is, according to the Right line *B S*. After the ſame manner the Force acting in the point *C*, and in conjunction with the *Vis inſita*, (by which alone the Body would deſcribe the Line *C d* in the third Particle of Time) carrying it in the ſame time along *C D*, is directed according to a Right line parallel to *d D*, that is, according to *C S*: But the Right lines *B S*, *C S*, *&c.* tend towards the Point *S*. Therefore the Centripetal Force, that draws the Body off from the Tangent of the Curve, acts along right lines tending to the immoveable Point *S*. *Q. E. D.*

PROPOSITION XIII.

THE Forces whereby the Primary Planets, Mercury, Venus, Mars, Jupiter, and Saturn, are drawn off from their Rectilineal Motions, and kept in their Orbits, do not tend towards the Earth, but towards the Sun.

For every Body that is moved in a Curve Line, is turn'd off from its Rectilineal courſe, that it naturally affects, by ſome Force or other. And it is evident that the Planets move in Curve Lines, becauſe their Orbits return into themſelves. Now this Force exerciſed upon them, does not tend towards the Earth, becauſe the Orbits of two of them, namely, Mercury and Venus, do not ſurround the Earth, as is evi-

dent

m *Prop.* vi; and conſequently they are
ave in all their parts towards the Earth:
re (by *Prop.* xi.) the Forces by which
kept in their Orbits, do not tend to it.
the Primary Planets, *viz.* Mercury,
Mars, Jupiter and Saturn, in reſpect of
h, do ſometimes go forwards from the
wards the Eaſt, ſometimes backwards
Eaſt towards the Weſt, and ſometimes
l: But the time in which theſe Moti-
erform'd, always flows uniformly; and
e the Areas deſcrib'd by a Radius drawn
arth from any Planet, are not propor-
the Times of deſcription. Conſe-
by the foregoing Propoſition, each
s urged and retained in its own Orbit,
ce that does not tend towards the Earth.
rom the *Scholium* to *Prop.* vi.) 'tis certain,
Orbits of Mercury and Venus ſurround
; which is evident likewiſe concerning
its of Mars, Jupiter and Saturn, from
lium to *Prop.* ix. And all the Planets, in
f the Sun, are always Direct in their
as is evident from *Prop.* 1; and that al-
iformly, juſt as the Time flows. 'Tis
eed, the Planets move a little ſwifter in
ints of their Orbits, even in reſpect of
than in others, as ſhall be ſhewn hereaf-
the difference is ſo ſmall, as it may
ry well be neglected. But even at that
Planets are nearer to the Sun (for their
are not Circles concentric to the Sun)
ir Motion is ſo temper'd, as that the A-
ſcribed by Radii drawn to the Sun, are
augmented, as will appear in the ſequel
Subject. And conſequently the Forces,
y the Planets are drawn off from a Recti-

lineal Motion, and retained in their Orbits, tend towards the Sun.

PROPOSITION XIV.

THE Forces, by which the Comets are kept in their Trajectories (if they be Curvilineal) do not tend towards the Earth, but towards the Sun.

If the Trajectories of the Comets were Right lines, they would not be urged by any Forces at all tending to a point ſituated without thoſe Right lines; for if they were, they would be drawn off from a Rectilineal Motion, and made to deſcribe Curvilineal Orbits, by *Prop.* xi. And if the Trajectories of Comets were Curve lines, yet the Force by which any ſuch Comet is retained in that Curve, is not directed towards the Earth, becauſe the Earth is generally found to be without the Plane of that Trajectory; beſides, in reſpect of the Earth, the Comet is ſometimes Direct and ſometimes Retrograde, and conſequently does not deſcribe Areas proportional to the Times: Yet in reſpect to the Sun, which is placed in the Planes of all their Trajectories, a Comet always moves the ſame way, and the nearer the Sun, the ſwifter; ſo as to encreaſe its Area, which is equally deſcribed by a Radius drawn from the Sun: Wherefore what was aſſerted, is evident from *Prop.* xii.

SECTION III.

)rder, Diſtances, and Periods of the dary Planets revolving about their ry ones, and their Phænomena; to- with the Direction of the Forces, by they are kept in their Orbits.

PROPOSITION XV.

ribe the Order, and Periods of the Secon- Planets, or Satellites, about their Primary Diſtances from them.

Six Primary Planets, that revolve a- un, there are but three, as we are ſure ſervation, that have Satellites, or o- lving about them, hence called, Se- Planets. The Earth has one, *viz.* the compleating its Revolution in $27\frac{1}{3}$ d diſtant about 60 Semidiameters of from it.

has four; the inmoſt of which revolves ay, at the diſtance of $5\frac{2}{3}$ Semidiame- piter from his Center; the ſecond re- $3\frac{3}{5}$ Days, at the diſtance of 9 Semidi- the third in $7\frac{1}{6}$ Days, at the diſtance nidiameters; the fourth and outermoſt bout Jupiter in the ſpace of $16\frac{3}{4}$ Days, tant from his Center $25\frac{1}{3}$ Semidiame- piter.

has five; of which the innermoſt re- $1\frac{7}{8}$ of a Day, at the diſtance of $4\frac{1}{8}$ Se- ers of Saturn, from the Center of Sa- ſecond, in the ſpace of $2\frac{3}{4}$ Days, at the diſtance of $5\frac{2}{5}$ Semidiameters of Saturn; the third, in $4\frac{2}{5}$ Days almoſt, at the diſtance of 8 Semidiameters; the fourth, in almoſt 16 Days, at the diſtance of 18 Semidiameters; the fifth and outermoſt of all, that have been diſcovered, revolves about Saturn in $79\frac{1}{3}$ Days, at the diſtance of 54 Semidiameters from the Center of Saturn. And the Solar Syſtem, as far as it has been hitherto diſcover'd is of a Figure much like that in the Scheme [*Fig* 13.] But between the two laſt Satellites of Saturn, *Hugenius* ſuſpects there revolves a Sixth, the ſpace being larger than it ſhould be, in proportion to the diſtances of the others, or that there are others beyond the fifth revolving about Saturn, but not ſeen as yet, by reaſon of their Obſcurity. The Planes of the Orbits of the Satellites of the ſame primary Planet, do not coincide, but are variouſly inclin'd to one another, and to the Plane of the Orbit of the Primary one.

Saturn likewiſe is encompaſs'd with a thin plane *Ring*, that does not touch the Body of Saturn at all, but is like an Orbicular Arch, built round about it. The Plane of this Ring is at this time nearly parallel to the Plane of Earth's Equator. And from the various Poſition of it, in reſpect of the Sun illuminating it, it being Opake like a Planet, and in reſpect of the Eye of the Spectator, the various Phaſes of the *Anſæ* of Saturn ariſe, which have ſo long baffled the attempts of Aſtronomers, with their various Shapes. This was firſt diſcover'd by *Hugens*, in his *Syſtema Saturnium*, Printed 1659. The Diameter of the Ring is to the Diameter of Saturn, as 9 to 4; and the breadth of the ſpace between the Ring and the Body of Saturn is equal to the breadth of the Ring it ſelf.

PRO-

PROPOSITION XVI.

ribe the Phænomena of the Moon view'd
the Earth, as it is a Spherical Body illu-
the Sun.

on, being a ſecondary Planet, accom-
Earth *T*, [*Fig.* 14] during its Revo-
ut the Sun, in its Orbit *R T*, and
ts Orbit the ſame way as the Earth
A thro' *B*, *C*, *&c.* The Plane of the
bit does not coincide with the Plane
ptic, but is inclin'd to it by an Angle
e Degrees; and the common inter-
theſe Planes is the Right Line ☊ ☋;
☊ and ☋, where the Orbit of the
rſects the Plane of the Ecliptic are
Nodes of the Moon, as was ſaid be-
we treated of the primary Planets.
Orbit of the Moon is to be conceiv'd
lf above and half below the Plane of
c, the former ☊ *A* ☋ to the North,
☊ *K* ☋ towards the South, and on
nt is repreſented in the Diagram,
gure of an Ellipſe. One of the
. that where the Moon, having paſs'd
c, is aſcending into the Northern
ll'd the *Dragon's Head*, and is mark'd
e other, the *Dragon's Tail*, mark'd thus
Line of the Nodes does not always
ame poſition, but is moved with an
otion, ſo as that the Nodes go back-
. ☊ thro' *G*, *F*, *&c.* and ☋ thro'
contrary to the Order of the Signs,
ng a Revolution in about 19 Years.
ence 'tis certain, that the Moon can
ound in the Ecliptic above twice in
ly Courſe, namely, when it is in the

Node ☊ or ☋; all the reſt of the time it is wide of it; and the Latitude of the Moon's deviation is meaſur'd by the Angle in which the Right line connecting the center of the Moon and Eye is inclin'd to the Plane of the Ecliptic, (by *Def.* 5. *Elem.* xi.) agreeable to what was ſaid above concerning the primary Planets, *Prop.* v. and viii.

Beſides, the Moon, like the other primary Planets, being an Opake, Rough, Spherical Body, reflecting the Sun's Rays falling upon it; 'tis evident, that half of it being turn'd towards the Sun is illuminated and bright, while the other half, that is turn'd from the Sun, continues obſcure and dark. Now only that Hemiſphere of the Moon which is towards the Earth, can be ſeen by an Inhabitant of the Earth viewing it: Conſequently the Phaſes of the Moon will be various, according to the various habitude of the inlighten'd Hemiſphere to that which is turn'd towards the Earth; as was ſhewn in the like caſe concerning the inferior Planets, in *Prop.*vi.

If the Moon be at *A* in the Scheme [*Fig.* 14.] the point juſt oppoſite to the Sun, which we will ſuppoſe to be at ☉, and the Earth in *T*; 'tis evident that the whole enlighten'd face of the Moon is turn'd towards the Earth; this Phaſis or appearance of the Moon is call'd the *Full Moon.* If the Moon be remov'd to *B*, the Sun and Moon continuing as they were, part of the bright or illuminated half will be turn'd from the Earth, and part of the obſcure or darken'd half turn'd towards it; hereupon the Moon will not appear full, but Gibbous, or a little deficient or obſcure in oppoſite to the Sun. The Moon arriving to the ſituation *C*, where the Angle *CTS* is Right, and the Moon in one

of

dratures, half the Hemiſphere which
the Earth is illuminated, and half
which account it will appear a half
d in the decreaſe. The Moon having
, for inſtance, to *D*, but a little of its
is turn'd to the Earth is inlighten'd,
greater part remaining dark: and
ly the inlighten'd part will appear
y reaſon of the Moon's really ſpheri-
parently plane Figure; and the Horns
turn'd away from the Sun, as ſhall
rated in the next Propoſition, or in
caſe, turned towards the Weſt. At
n the Moon is come to the point *K*,
junction with the Sun, all the in-
lf is turned from the Sun; and con-
he Moon will be intirely dark, and
New Moon; becauſe it will ſoon ap-
nd ſhew its Horns ſtill turn'd from
nd conſequently now towards the
rwards being upon the increaſe, it
a half Moon at *G*, at the end of the
r, as it did in *C* at the beginning of
arter; then Gibbous at *H*, till it be-
gain at *A*. For greater evidence
ſs of the whole, we have drawn the
of the Moon ſeen by us, corre-
every diſtinct ſituation, and plac'd
d it.

time of the Moon's Revolution a-
rth be but 27 Days, 7¾ Hours, which
Periodic Month, yet by a Month, or
ion is commonly underſtood all that
e that is ſpent from one New Moon
t following, which is greater than
ention'd Periodic Month; becauſe
Periodic Month, in which the

Moon departing from the point *K*, where the New Moon now happens, returns to it again; the Earth itſelf, together with the Moon its Attendant, is carried an intire Sign *in conſequentia*; ſo that the ſaid point of the Orbit of the Moon *K*, is more towards the weſt of the Sun; and conſequently the Moon is not yet arrived to a conjunction with the Sun; but ſtill lacks 2 days and 5 hours to it, or to be become New; that is, to finiſh an intire Lunation, and all the ſeveral Changes of its Appearances; which ſpace of time is call'd a *Synodic Month*, and conſiſts of 29 days, 12¾ hours.

A little before and after a New Moon, *viz.* when the Moon is in *D* or *F*, the Rays of the Sun reflected from the Earth, meeting with the Moon, are the cauſe of that faint Light by which the reſt of Moons Diſc beſides its Horns, is then render'd viſible. But when the Moon is got out of the way of the reflexion of the Earth's light, that faint light vaniſhes, being falſly ſuppos'd native.

PROPOSITION XVII.

TO *draw the Phaſis or Appearance of the Moon at any given Time.*

Let a Plane, the ſame with that of the Scheme, for inſtance, paſſing thro' the Centers of the Sun, Earth and Moon, cut the Globe of the Moon, and let its Section be the Circle *ADBC*, [*Fig.* 15.] Let *CD* be a Diameter of the Circle, and *SL*, a right line connecting the Centers of the Sun and Moon, perpendicular to it; and *AB* another Diameter to which the Right line *TL*, connecting the Centers of the Earth and Moon, is perpendicular. From the Point *D*, let *DE* fall a perpendicular to *AB*, meeting it in *E*.

Since

lane perpendicular to that of this
ected upon the Diameter *C D*, ſepa-
nlighten'd Hemiſphere from the ob-
d a Plane erected after the ſame man-
A B, ſeparates the viſible from the in-
niſphere: That part of the Globe of
which is common to both Hemiſpheres,
vhich lies between the Planes erected
D L, will be the enlightened part of
Hemiſphere; and that which lies be-
Planes erected upon *B L*, *D L*, will
ure or dark Portion. Both theſe Por-
e viſible Hemiſphere, as well the in-
as the obſcure, are terminated in the
oints of the Globe of the Moon;
oſe in which a Right line raiſed from
dicular to the Plane of the Circle
nd produced both ways, meets the
rface; that line being the common
the two Planes erected upon *A B*, and
greateſt breadth of theſe parts is in
ſference *A D B*; the breadth of ob-
B D, of the enlightened *A D*. The
the meaſure of the Angle *B L D*, which
the Angle *S L T*, contained under the
drawn from the Center of the Moon
ters of the Sun and Earth. For if to
Angles *S L D*, *T L B*, (becauſe right
add the common Angle *D L T*, the
ention'd will be formed, which are
tly equal. But at any given time, the
the Moon's Diſtance from the Sun,
is known; conſequently *S L T*, its
ent to two right: For the Diſtance of
from the Earth is ſo ſmall, in com-
th the diſtance of the Earth from the
the Right lines *T S*, *L S*, drawn from

the Earth and Sun, may be taken for parallels. Make therefore, that as twice the Radius is to the verſed line of the Angle *TLS*, ſo is *BA* to *BE*. And ſince the Hemiſphere of the Moon ſeen from the Earth, appears like a Circle, as it happens to any Globe ſeen at a great Diſtance, according to the Principles of Optic: Let *AMBN*[*Fig.*16.]be a Circle repreſenting the Diſc of the Moon, the ſame with the former, made by the Section of the Moon with a Plane erected upon *AB*, and conſequently deſcribed upon the ſame Center *L*, and with the ſame Diameter. Becauſe the Moon is at a conſiderable diſtance, each point of its Diſc will be ſeen by Right lines parallel to the Right line *LT*. And therefore *BE*, and *AE*, will be the greateſt Latitudes of the inlightened and obſcure parts in the Diſc of the Moon. Draw the Diameter *MN* perpendicular to *AB*; and deſcribe the Semi-Ellipſe *MEN*, whoſe greater Axe is *MN*, and leſſer equal to twice *LE*, *MANEM* will be the inlighten'd Part, and *MBNEM* the obſcure Part of the Moon's Diſc *AMBN*.

For the boundary of illumination on the Spherical Surface of the Moon, by the Sun, which is likewiſe Spherical, is a Circle; and this Circle ſeen at a diſtance obliquely, appears like an Ellipſe, whoſe Semiaxes are *LM*, *LE*, as is evident from Optics: 'Tis ſeen obliquely by *TL*, becauſe the Right line *SL* perpendicular to its Planes, is inclined to *LT*; excepting when the Moon is Full, where they coincide.

In the Conſtruction of this Problem, we ſuppos'd half the Globe of the Moon to be inlighten'd, and half likewiſe to be ſeen by a Spectator, tho' neither of the Suppoſitions are rigorouſly true: For the Sun being bigger than the ore than half the Moon is inlighten'd; Spectator not being at an infinite Di- om the Moon, its intire Hemiſphere e viſible. But the difference in both s is ſo ſmall, that it may very well be : And the Phaſis or Appearance of delineated as above, ſufficiently a- Obſervations. That the Scheme may the Heavens, the Line *B A*, to which necting the Horns is perpendicular, lac'd in ſuch a Situation, as that it may tly to the Sun: For the Right line *L A*, *T*, coincides with the line *L S* tend- Sun.

aſes of the inferior Planets and Mars, in *Prop.*vi. and ix. are to be delineated ame manner; But then if *L* repreſent Planet, the Right lines *T S*, *L S* are clin'd to one another, and form a

PROPOSITION XVIII.

n the Phænomena of the Moon ariſing from acity of the Earth caſting a Shadow, that ſe of the Moon.

rth is an Opake Body, conſequently inlighten'd by the Sun, it caſts a Sha- ds thoſe parts which are turn'd from and this Shadow muſt always be in of the Ecliptic; ſince both Sun and always there. [*Fig.* 17.] If at any Moon when Full, ſhould happen to Plane of the Ecliptic, or pretty near it will be, when the Full Moon hap- near a Node,) 'tis evident the Moon mers'd in the Shadow; and conſe- prived of the Sun's Light, by which it ſhines; that is, it will ſuffer an *Eclipſe* which will be *Total* or *Partial*, according as the whole or only a part of the Moon's Body enters the Shadow.

The Earth's Shadow terminates at laſt in a Point, and does not reach ſo far as Mars: For Mars, tho' in the Plane of the Ecliptic, and oppoſite to the Sun, is not Eclipſed; which it neceſſarily would be, if it were immers'd in the Earth's Shadow: For Mars is an opake Body, as is ſhewn in *Prop.* ix. 'Tis evident at firſt ſight, from this Figure of the Earth, that the Sun is bigger than the Earth: For if the lucid Body be bigger than the opake one, the Shadow will be equally thick and cylindrical; But if the lucid Body be leſs than the opake, the Shadow indeed will be conical, but growing bigger and bigger, and the farther off it is from the opake Body, the thicker it is; and in both caſes will be extended infinitely. As the Sun is bigger than the Earth, ſo the Earth is bigger than the Moon; becauſe the Moon ſometimes is totally Eclipſed by entering into the Earth's Shadow; but the Earth's Shadow is much ſmaller where the Moon enters it, than nearer to the Earth, as is evident from what has been already ſhewn.

Let the Circle *V M* [*Fig.* 18.] repreſent the tranſverſe Section of the Earth's Shadow, where it croſſes the Moon's Orbit. Let *L* ☋ *F* be the Orbit of the Moon, *C E* the Ecliptic. The duration of ſome Eclipſes is found to be ſo long (for inſtance, four hours,) as to let the Moon go the length of three of its Diameters in the Shadow totally Eclipſed. This happens when the Center of the Moon paſſes thro' the Center of the Earth's Shadow or Circle *V M*; And ſuch an Eclipſe is call'd a *Total* and *Central* Eclipſe.

Some-

times the Moon is not arrived at, or has
paſſed the Node, when it is entering the
Shadow: Notwithſtanding, if at that
be near the Node, it will be totally im-
and conſequently there will happen a
tho' not a Central Eclipſe, nor of ſo
ontinuance [*Fig.* 19.] For the way of
n thro' the Earth's Shadow, is leſs than
meter of the Shadow, becauſe it does
thro' its Center; and conſequently, *ca-*
bus, will be paſs'd thro' ſooner.

the Node be ſo far off from the Sha-
ig. 20.] as that only part of the Moon is
, as it paſſes in its Orbit, near the
the Eclipſe will be a partial one, and is
e of ſo many *Digits* as there are twelfth
the Moon's Diameter darken'd, when
ſe is greateſt. For the Diameter of the
ke any other Integer) is imagin'd to be
nto twelve Parts or Digits: Which is to
ſtood likewiſe of any other Phaſis of an

In all theſe Caſes, in the beginning
ipſe, the Moon enters the Weſtern part
hadow, with the Eaſtern part of her
d in the end of it, ſhe leaves the Ea-
of the Shadow with the Weſtern part
mb; and all the intermediate time is
into the Eclipſe; but only ſo much
otal immerſion, as paſs'd while the
as totally in darkneſs. The Eaſtern
e Moon in its acceſs to the Shadow,
nter preſently into the thickeſt Dark-
grows darker and darker as it ap-
nigher to the Shadow; and this ariſes
Penumbra, which always accompanies
which is made by a lucid Body, that
der a ſenſible Angle. And this Pe-

numbra is diffuſed all about the Shadow. Let ☉be the Sun, [*Fig.* 21.] *E* the Earth, and let Right lines be drawn juſt as in the Scheme: All about the Shadow *TVMR*, where no part of the Sun's light can come, a Penumbra *VTPMRN* is ſpread, ſome part of the Sun's light not been ſtop'd therein: And this Penumbra is darker towards *TV*, *RM* which are the extremities of the perfect Shade; becauſe fewer Rays can arrive thither, the portion of the Sun, from which they are emitted, being ſmaller; but leſs obſcure towards *TP*, *RN*, where more Rays can reach: And beyond which limit all the Rays of the Sun can reach without any hindrance at all, and inlighten according to the degree of their vigour.

The Red colour, that the Moon in the middle of the compleat Shadow is affected withal, and makes her look like a Brick (for in ſome Eclipſes the Moon intirely diſappears,) ſeems to ariſe from the Sun's Rays, either refracted in their paſſage thro' the Earth's Atmoſphere, or reflected by ſome particles of Matter flying about the Earth's Shadow, to the Moon; or from the light of the Stars, or all theſe taken together.

Some kind of Eclipſe or other of the Moon, happens generally twice a Year at leaſt: For there being two Nodes in which the Moon's Orbit croſſes the Ecliptic, and they moving *in antecedentia*, (by *Prop.* xvi.) and the Sun appearing to go thro' the Ecliptic *in conſequentia*, (by *Prop.* ii.) the Sun muſt meet one or other of theſe Nodes twice every Year; and conſequently the Earth's Shadow muſt perforate, as it were, the Moon's Orb in the other Node. If therefore a Full Moon happens juſt at that time, the Moon muſt neceſſarily be totally and centrally Eclips'd,

emonſtrated above. And tho' a Full
es not happen juſt as the Sun and Node
:t the inclination of the Orbit of the
› the Ecliptic, and the depth of the
hadow is ſo great,that tho'theFull Moon
above ten Days from the aforeſaid time,
after it, and it can be diſtant but fifteen
t the Moon will ſcarce get clear of the
But if the abovemention'd meeting of
and Lunar Node, happen on the very
e New Moon, or a Day or two before
which can happen but ſeldom, the
ll be far enough off from the Earth's
n the next Full Moon, whether pre-
following, and ſo will eſcape an E-
›nſequently there will be no Eclipſe
Year.

PROPOSITION XIX.

lain the Phanomena of the Sun ſeen from
Earth, and ariſing from the Opacity of the
Eclipſes of the Sun accounted for.

Moon upon the interpoſition of the
eprived of the Sun's light, and ſaid to
:d; ſo, in like manner, if the Earth
robbed of the Sun's light, by the in-
of the Moon, this Phænomena ought
d an Eclipſe of the Earth. But the
of it being on the Earth, and allow-
loſs of Light or Eclipſe, nor Motion,
ther thing that ſeems to argue an im-
in his place of abode, calls this Phæ-
an Eclipſe of the Sun; for the ſame
an Inhabitant of the Moon, would ſay
as eclipſed, when the Moon is really
nto the Earth's Shadow.

'Tis evident an Eclipſe of the Sun will happen in every ſuch New Moon, as has the Moon at or near any of the Nodes. For then the ſhadow of the Moon lying directly betwixt the Sun and Earth, reaches to the Earth, and cauſes a total Eclipſe to the Inhabitants of the tract *C D* [*Fig.* 22.] that are immers'd in the thickeſt ſhade. But the Moon's Shadow not being large enough to cover the whole Earth, the circular tract *BC*, *ED*, which ſurrounds the former *CD*, is cover'd with the Penumbra, and its Inhabitants ſee only a partial Eclipſe of the Sun; which is greater towards *C* and *D*, becauſe a greater portion of the Sun will be cover'd by the Moon; but leſs towards *B* and *E*, the extremities of the Penumbra, and the defect of light is ſcarce ſenſible. At the ſame time, in other places, as *E F*, the bigneſs of the Earth is the reaſon why there is no Eclipſe at all; the Sun inlightning it without any hindrance or impediment.

All the preceding account happens in Nature, juſt as it has been related. But if we look upon the Sun and Moon from our Earth, the Moon will ſeem to cover the Sun more or leſs according as the Spectator is nigher or farther off from the total Shadow [*Fig.* 23.] And becauſe the Moon which covers the Sun can't be ſeen, the Sun will appear to be darken'd in that part which the Moon covers, the reſt only remaining lucid.

Sometimes a central Eclipſe happens not to be total; but a bright Ring ſeems to ſurround the Moon: The reaſon is, at that time the Moon's ſhadow is ſo ſhort that it cannot reach the Earth; either becauſe the Moon is then ſo far off from the Earth; or becauſe the Sun's Rays that graze along the extreme parts of the Moon

re more than ordinarily inflected, and
n the Moon's ſhadow. The bigneſs of
Eclipſe is to be eſtimated juſt as in the
by the digits of the Sun's diameter, that
en'd by the Moon at the time given.

vident likewiſe that the Moon moving
the Eaſt, or from ♎, thro' ♏ to ♐,
] the weſtern part *A* of the Earth will be
adow firſt, which will paſs along the
iſc, like a Spot, thro' *B*, *C*, *D*, *E*, to *F*,
leaves the Earth. But if the Moon be
on from the Earth, the eaſtern limb of
n will firſt cover the weſtern of the Sun,
weſtern of the Moon will laſt uncover
rn limb of the Sun: And the greateſt
that happens in a total Eclipſe is ſoon
d; ſome part of the Sun's disk being
uncover'd, almoſt ſo ſoon as the whole
er'd: and that part, tho' never ſo little,
htily inlighten the Air.

the Moon muſt be in a Node the very
of the New Moon, to cauſe the biggeſt
f the Sun that is poſſible, and that the
of the Moon may go along the middle
arth: Yet if ſhe be not far off from
the ſhadow of the Moon, or at leaſt
he Penumbra, will fall upon ſome tract
arth, being ſo big, and there cauſe a
at leaſt a partial Eclipſe: And in this
re are more Eclipſes of the Moon than
un. But Eclipſes of the Sun, in any
ace, are much fewer than the Eclipſes of
on; becauſe the Moon's ſhadow is leſs
Earth's, and conſequently it does not
any given place upon the Earth, ſo
the Earth's ſhadow does ſome part of
on.

SCHOLIUM.

The ſame Phænomena will appear in Jupiter and Saturn: For their Satellites or Moons, will be eclipſed, by being immers'd in the ſhadow of their primary Planet; and thoſe Eclipſes of them are obſerved by us, juſt as the Eclipſes of our Moon may be obſerved from them. In like manner, every Satelles coming between the Sun and its Primary, caſts a ſhadow upon the Primary, which ſeems to move along the Disk of the Primary from Eaſt to Weſt like a Spot. But the Duration, Phaſes, Periods, *&c.* of theſe Phænomena are various, and differing from the like ſeen by us, and ariſing from our Moon, according to the diverſity of the Shadows, Motions and Magnitudes, both of the Primary and Secondary Planet.

PROPOSITION XX.

Each of the Secondary Planets mention'd in Prop. XV. *is urged by a Force compounded of a Centripetal Force, tending to the center of the Primary, about which it revolves, and of all the Accelerating Force with which the Primary is urged. And therefore the Forces whereby the Satellites are retain'd in their Orbits about the Primary ones, tend towards the centers of their Primary ones reſpectively.*

Becauſe the Satellites of Jupiter and Saturn revolve equably in circular Orbits, concentric with Jupiter and Saturn, they deſcribe Areas about theſe Centers reſpectively, proportional to the Times. In like manner, if the Moon's Orbit differ from a Circle concentric with the Earth, the leſs the Moon appears, (that is, the farther it is from the Earth) the ſlower it moves round the Earth, ſo that ſtill the Area, that the Radius drawn to the center of the Earth de-

equably augmented. So that univer-
h of the abovemention'd Satellites de-
eas, by a Radius drawn to the center
mary Planet, about that Center, pro-
to the Times. If therefore the Syſtem
rimary Planet, and its revolving Sa-
e ſuppos'd to be urged along parallel
a Force equal and contrary to that,
the Primary Planet in that Point of
where it then is, is urged towards the
Primary Planet will no longer deſcend
e Sun; and the Secondary will con-
eſcribe the ſame Areas about the Pri-
efore; that is, proportional to the
For if the Bodies move any how in
one another, and are urged by equal
g Forces along parallel lines; or,
ll one, if the Space in which they per-
r Motions be moved uniformly in a
, they will move all after the ſame
s they would do if thoſe Forces were
the Space were at reſt, in which they
ed). So that each Satelles, only ur-
difference of the Forces, will go on
e, about the center of its Primary,
portional to the Times. Therefore,
i. the difference of thoſe Forces tends
nary Planet as a Center. But before
hole Syſtem was urged along parallel
a Force equal and contrary to that,
he Primary Planet is urged towards
hat is, in its natural State, each Satelles
y a Force compounded of the Centri-
e tending to the center of its Primary,
the accelerative Force that the Pri-
rged by. Conſequently the 'foreſaid
of the Forces, is that whereby the

Satelles is retained in its Orbit about the Primary, the remaining accelerative Force, whereby the Primary Planet is urged in any Point of its Orbit, conferring nothing to this: But this difference has been ſhewn to tend to the Center; therefore the Forces whereby the Secondary Planets are retained in their Orbits about the Primary, tend towards the Centers of the ſaid Primary reſpectively. Which was to be demonſtrated.

SECTION IV.

Of the Periods of the Primary Planets about the Sun, and Secondary about their reſpective Primary, compar'd together, and of their Diſtances likewiſe compared together, with their mutual relation to one another, the reaſon and cauſes of it.

PROPOSITION XXI.

THE Motion of the Secondary Planets revolving about a Primary, is ſuch, as that the Squares of the Periodic Times, are in the ſame ratio as the Cubes of their Diſtances from the Center of the Primary.

This Theorem is abundantly evident from the Obſervations of Aſtronomers: And by the Squares of the Periodic Times, you muſt underſtand, the Squares of Right lines, or of Numbers, having the ſame ratio as the Periodic Times; in this Senſe alſo are the Squares of the Velocity, and the like.

Thus in the Satellites of Jupiter, where the Periodic Times of the inmoſt, ſecond, third, and

ſt, are reſpectively as $1\frac{3}{40}$, $3\frac{1}{2}$, $7\frac{1}{6}$, and
ſt; and their Diſtances as $5\frac{2}{3}$, 9, $14\frac{1}{4}$
the Square of the Periodic Time of
ſt, namely 3, is to 13 the Square of
dic Time of the ſecond Satelles, as 170,
of the diſtance of the inmoſt from
er of Jupiter, to 736, the Cube of the
of the ſecond from thence. In like
is to 51, the Square of the Periodic
the third Satelles, as 170 to 2890, the
the diſtance of the third from the Cen-
piter. And 3 is to 280, the Square of
dic Time of the outmoſt Satelles, as
800, the Cube of the diſtance of the
rom Jupiter's Center. And therefore,
the ſame ratio holds between any o-
of them compar'd together; as the ſe-
h the third or the laſt, or the third with
This Ratio comes out more exact, if
ances and Periodic Times are taken
urately.

ame will be found to hold in the Satel-
aturn, if you take the numbers laid
Prop. xv, that their diſtances from Sa-
d their Periods about him may be eſti-
But the Moon being a ſolitary Satel-
Propoſition can't be applied to her.

PROPOSITION XXII.

Motion of the Primary Planets about the Sun,
ch, as that the Squares of their Periodic
re in the ſame ratio as the Cubes of their
from the Sun.

, for inſtance in round numbers, the Pe-
Saturn is 30 Years, Jupiter 12; their
are 900 and 144. The diſtance of Sa-
om the Sun, is, by Obſervation, to the
diſtance

diſtance of Jupiter from the ſame, as 9 to 5 almoſt; their Cubes are 729 and 125. But 'tis evident thoſe Squares are nearly as theſe Cubes. Alſo the Period of the Earth is a little more than four times greater than the Period of Mercury; conſequently their Squares are as 17 to 1 almoſt: Suppoſing the Diſtance of the Earth from the Sun to be ten parts, the Diſtance of Mercury is, by Obſervation, about 4, or 3, 9; and their Cubes as 1000 to 59: But 17 is to 1 almoſt as 1000 to 59; And ſo on in the other Planets. The more correct the Diſtances and Periods are taken, the nearer you'll come to this Proportion. What has been now ſaid, may ſuffice at preſent: The Propoſition being more exactly to be made out in the third Book.

PROPOSITION XXIII.

THE Spaces that a Body deſcribes, acted upon by any kind of finite Force, (whether it be determinate and unchangeable, or the ſame in continual increaſe, or continual decreaſe) are in the very beginning of the Motion, in the duplicate ratio of the Times.

Let the Times be expreſs'd by the Right lines *AD*, *AE*; [*Fig.* 24.] and the Velocities acquired at the end of thoſe Times, by the Ordinates *DB*, *EC*: And the Spaces deſcribed with theſe Velocities will be as the Areas *ABD*, *ACE*, made up, as it were, of theſe Ordinates; namely, the Space deſcrib'd in every ſmalleſt particle of Time, being as the Velocity and that Particle of Time conjunctly. But, in the beginning of the Motion, the Ordinates *DB*, *EC*, are very near the Point *A*; and in that caſe, the Trilineal Figures *ADB*, *ACE*, are ſimilar rectilineal Triangles, that part of the Curve

ich belongs to the infinitely ſmall
gures, not extending it ſelf beyond
line *AG*, which is a Tangent to the
A. Now theſe ſimilar rectilineal Tri-
in the duplicate ratio of their homo-
s *AD*, *AE*. Conſequently the Spaces
the beginning of the Motion, are
icate ratio of the Time. Which was
nſtrated.

COROLLARY.

nce we may gather, that the Errors
deſcribing ſimilar parts of ſimilar Fi-
oportional Times, generated by any
rbing Forces ſimilarly applied to the
d meaſur'd by the diſtances of the
n thoſe places of the ſimilar Figures,
heſe Bodies would arrive, in the ſame
al Times, without thoſe Forces, are
as the Squares of the Times where-
generated. For theſe Errors are the
ich the Bodies acted upon by the di-
orce deſcribe. But if theſe diſturbing
not equal, but in a given ratio; the
as thoſe Forces and the Squares of the
ɔjunctly; ſuppoſing the Forces ſimi-
ied.

PROPOSITION XXIV.

ent or evaneſcent ſubtenſe of the Angle of
t, in any Circle, is in the duplicate ratio
rminous Arc.

C [*Fig.* 25.] be a Circle, the right line
ngent in *A*, conſequently the Angle
t *BAD*. I ſay, any Subtenſe that is
t to the Point *A*, for inſtance, *BD* is
are of the Arc *AD*; that is, the ſub-
is to another ſubtenſe *bd*, in the ſame

Circumſtances, as the ſquare of the Arc AD to the ſquare of the Arc Ad.

Draw the Diameter AC, which will be perpendicular to AB, join the Right lines DA, dA, DC, dC, and draw DE, de parallel to AB.

Firſt, let the Subtenſe BD be parallel to AC. Now (by 8 and 17. *Elem.* vi.) $AD^q = CA \times AE$, and $Ad^q = CA \times Ae$. Wherefore $AD^q : Ad^q ::$ $(AE : Ae ::)$ $BD : bd$. But ſince BD, bd are neareſt to the Point A, or in their naſcent condition, the Arcs AD, Ad, and their Subtenſes do not differ from one another; that is, they are equal. Conſequently, in this caſe, BD is to bd, as the Square of the Arc AD, to the Square of the Arc Ad.

Secondly, if the Subtenſe BD of the Angle of Contact be not parallel to AC, but to AG, [*Fig.* 26.] the ſame ſtill holds: For, drawing DF, df parallel to AC; then, becauſe DFB dfb are ſimilar Triangles, $BD : bd :: DF : df$; but it has been already ſhewn that DF is to df, as AD^q to Ad^q. Wherefore BD is to bd, as AD^q to Ad^q.

Thirdly, if BD be ſuppos'd to be drawn according to any other certain Law, (for inſtance converging toward the Center;) ſince BD, bd are as near as may be to the Point A, the Angles B, b will be equal; and conſequently, in that caſe, $Bd : bd :: (DF : df ::) AD^q : Ad^q$. Which was to be demonſtrated.

SCHOLIUM.

This is true likewiſe in any other Curve, to which a Circle equi-curve may be drawn; ſuch as all the Conic Sections are. For the Points D, d (being the neareſt that can be to A) muſt be in that other Curve, as well as in the Circle, which is equi-curve to it, by Hypotheſis: Conſequently

he Properties of *B D*, *b d*, which a-
Circle, agree likewiſe to this Curve,
he Circle is equi-curve. And ſince
ing Propoſition is true of all Cir-
uſt likewiſe be true of all Curves,
have equi-curve Circles drawn to

PROPOSITION XXV.

eſcribing different Circles with an equable
m, are acted upon by Centripetal Forces
he Centers of thoſe Circles. And the For-
e another as the Squares of the Arcs, de-
e ſame time, applied to the Radii of the
his likewiſe is true of the Centrifugal For-
odies thus moved.

the Bodies, by their *Vis inſita* alone,
cribe Tangents, 'tis evident that they
off from their Rectilineal Motion, and
n Circular Orbits by Forces tending
within the Circles. But ſince, by
n, they are carried in the Circumfe-
an equable Motion; the Areas de-
a Radius drawn to the Centers are
ugmented; and therefore are propor-
the time which flows equably; and
ntly theſe Forces, by *Prop.* xii. tend to
enters.

Bodies *M* and *T* [*Fig.* 27.] revolving
rcumferences of the Circles *M A*, *T R*,
the infinitely ſmall Arcs *M F*, *T D*;
eir extremities *F*, *D*, draw the Right
D C as far as the Tangents, either pa-
S M, *S T*, or diverging from *S*: (For it
the ſame, ſince *M F*, *T D* are only na-
s.) Theſe Right lines are the effects
ccelerating Centripetal Forces; and
con-

conſequently proportional to them, as their adequate Cauſes; that is the Centripetal Force of the Body M is to the Centripetal Force of the Body T, as EF to CD. Make the Fig. MGH ſimilar to the Fig. TCD; wherefore GH will be a naſcent Right line: And ſo CD is to GH, as the Arc TD, to the Arc MH. And, by the præceed. GH is to EF as MH^{q} to MF^{q}: And therefore, *ex æquo*, CD is to ET, as ($TD \times MH^{q}$ to $MH \times MF^{q}$; that is, as $TD \times MH$ to MF^{q}, or as) $TD \times MH \times ST$ to $MF^{q} \times ST$: And ſince TCD, MGH are ſimilar Figures, the Arcs TD, MH are ſimilar; that is, $ST : SM :: TD : MH$. Wherefore $MH \times ST = TD \times SM$. Then by Subſtitution $CD : EF :: (TD^{q} \times SM : MF^{q} \times ST ::) \frac{TD^{q}}{ST} : \frac{MF^{q}}{SM}$.

But the Centripetal Forces of the Bodies T and M were ſhewn to be proportional to CD, EF. Therefore the Centripetal Force of the Body revolving in TR, is to the Centripetal Force of the Body revolving in MA, as $\frac{TD^{q}}{ST}$ to $\frac{MF^{q}}{SM}$. And, (becauſe the Motion in both is equable) TD and MF, being deſcrib'd at the ſame time, have the ſame ratio with any others deſcrib'd at the ſame time. Wherefore the Centripetal Force of a Body revolving in TR, is to the Centripetal Force of a Body revolving in MA, as the Square of an Arc deſcribed in any time in TR applied to the Radius ST, is to the Square of an Arc deſcribed in the ſame time in MA applied to the Radius SM.

If the abovementioned Bodies be imagined to be tied to S, by two Strings, and kept revolving

e Circles by the help of them; the
, whereby the String is ſtretched,
as it is in the String pulling the Body,
ie Centripetal Force, but conſidered
he Body ſtretching the String, may
he Centrifugal Force: And conſe-
l that has been ſaid concerning the
Force, is true alſo of the Centri-
. Which was to be demonſtrated.

COROLLARY 1.

nce it follows, that if a Body ſhould
owards the Center, being acted upon
tripetal Force, that kept it in the
s that this Force be neither augment-
iſhed in its deſcent, but continue the
o the firſt impulſe continually ſuper-
one equal to it, (which is the *Galilean*
of Gravity:) In the time that the
deſcribes an Arc in the Circle, it
its deſcent towards the Center, de-
ht-line, equal to that which is pro-
e Square of that Arc applied to the
of the Circle. For by this *Prop.* 'tis
done in the very beginning of the
n the preſent Hypotheſis of deſcent,
un are in the duplicate Ratio of the
d ſince the Circle is equably deſcri-
iares of the Arcs deſcribed are alſo in
te Ratio of the Times of deſcrip-
the application of theſe Squares, to
er of the Circle, being a conſtant
does not alter the Ratio. Conſe-
holds perpetually. So that in this
of an uniform Centripetal Force, a
l in half the Time that it deſcribes
Circle in, deſcend the length of a
to that produced by the application

of the Square of half that Arc to the Diameter of the Circle; namely, the ſubquadruple of the Right-line, it would deſcend in double the time, *viz.* in which it deſcribes the intire circular Arc.

COROLLARY 2.

From hence likewiſe it follows, that the centripetal Forces of Bodies, deſcribing different Circles by an equable Motion, are in a ratio compounded of the duplicate ratio of the Velocities directly, and of the ſimple ratio of the Radii inverſely. For the Motion in any Circle being equable, the ratio of the Velocities will be the ſame with that of the Arcs deſcribed in the ſame time; that is, of the Spaces run in the ſame time: But (by this Propoſition) the centripetal Forces are in a ratio compounded of the duplicate ratio of the Arcs deſcribed at the ſame time directly, and of the ſimple ratio of the Radii inverſely: Wherefore the ratio of theſe Forces is compounded of the duplicate ratio of the Velocities directly, and of the ſimple ratio of the Radii inverſely.

PROPOSITION XXVI.

THe ſame things being ſuppoſed, I ſay that the centripetal Forces are reciprocally as the Squares of the Periodic Times applied to the radii of the Circles; or that they are in a ratio compounded of the ſimple ratio of the Radii directly, and of the duplicate ratio of the Periodic Times inverſely. The ſame alſo is true of the centrifugal Forces of theſe Bodies.

Let the centripetal Force of one of the Bodies, moving in one of the Circles, be call'd *V*, in the other *v*; the Celerity in one Circle *C*, in the other *c*; the Arcs at the ſame time deſcribed in the firſt Circle *A*, in the ſecond *a*; the Radius,

or

e of the Body from the center, in the
D, in the ſecond d; the Periodic
ne firſt Circle T, in the ſecond t.
ll known, that in comparing the mo-
odies together, the Spaces run are in
und ratio of their Times and Celeri-
y. But the Radii of the Circles are
ial to the intire Circumferences, which
aces run in the Periodic Times. And
$D : d :: C \times T : c \times t$, ſo that $C \times T \times d$
): therefore $C^2 \times T^2 \times d^2 = c^2 \times t^2$.
refore, dividing by $D^2 \times d^2$, 'tis $\frac{C^2 \times T^2}{D^2}$
. Wherefore $\frac{t^2}{d} : \frac{T^2}{D} :: \frac{C^2}{D} : \frac{c^2}{d}$. But,
. of the prec.) $V : v :: \frac{C^2}{D} : \frac{c^2}{d}$. And
$V : v :: \frac{t^2}{d} : \frac{T^2}{D}$. But the ratio of $\frac{t^2}{d}$ to
e ſame with that of $t^2 D$ to $T^2 d$; or
^{2}, and D to d. Therefore the ratio of
equal to the ratio of t^2 to T^2, and of

as been ſaid of the Centripetal Forces
dies revolving about a Center, is true
entrifugal Forces; ſince they differ
the manner of conception; as was
the foregoing Propoſition. *Q. E. D.*

COROLLARY 1.

ence (and from Corol. 2. Prop. preced.)
, that if the Periodic Times are equal,
vell the centripetal Forces, as the Velo-
ill be as the Radii; and *vice verſa*.

For, by this Propoſition, $\frac{V}{v}=\frac{D\times t^2}{d\times T^2}$. Wherefore if $T=t$, or $T^2=t^2$, then will $V:v::D:d$. And *vice verſa*, if $V:v::D:d$, ſince $V:v::Dt^2:dT^2$, then $D:d::Dt^2:dT^2$; and conſequently $T^2=t^2$, or $T=t$. The ſecond part of the Corollary appears from hence. In the compariſon of Motions, the Velocities are as the Spaces run in the ſame Time. But, becauſe the Periodic Times are equal, the Spaces run in the ſame time are the Circumferences of the Circles; but they are as their Radii. Conſequently the Velocities are as the Radii. The converſe of this Propoſition is drawn after the ſame manner.

COROLLARY 2.

Hence alſo it follows, that if the Squares of the Periodic Times are as the Radii, the centripetal Forces are equal, and the Velocities are in the ſubduplicate ratio of the Radii; and *vice verſa*. For from this Prop. $\frac{V}{v}=\frac{D\times t^2}{d\times T^2}$: But by ſuppoſition, $T^2:t^2::D:d$, or $D\times t^2=d\times T^2$. Therefore V and v, proportional to them, or the centripetal Forces themſelves, are equal. And *vice verſa*, if $V=v$, the quantities proportional to them will likewiſe be equal, namely, $D\times t^2=d\times T^2$: And therefore $D:d::T^2:t^2$. In like manner, becauſe by Cor. 2. of the preceding Prop. $\frac{V}{v}=\frac{C^2\times d}{c^2\times D}$, and in the preſent caſe, $V=v$, then $C^2\times D$ will be equal to $c^2\times D$, or $C^2:c^2::D:d$; that is, $C:c::\sqrt{D}:\sqrt{d}$. The converſe of this is drawn after the ſame manner as that of the former.

COROL-

COROLLARY 3.

ircles deſcribed are equal, the centri-
es are reciprocally as the Squares of
ic Times: For the ratio of the Radii,
e, being that of equality, makes no
in the compoſition.

PROPOSITION XXVII.

ſame things being ſuppoſed, if the Bodies
ve in ſuch a manner, as to have the
the Periodic Times as the Cubes of the Ra-
e Centripetal Forces are reciprocally as the
he Radii: And the Celerities of the Bodies
lly as the Square Roots of the Radii; and

g the Characters uſed in the former
, 'tis evident from it, that $V : v ::$
From whence it follows, that $\frac{VT^2}{D}$
r $VT^2 d = vt^2 D$. And therefore Dv:
t^2. And by ſuppoſition, $D^3 : d^3 ::$
erefore $Dv : dV :: D^3 : d^3$. Conſe-
$Dvd^3 = dVD^3$; or $vd^2 = VD^2$.
fore $V : v :: d^2 : D^2$.
in the demonſtration of the forego-
ition, it was ſhewn that $C^2 \times T^2 \times$
$^2 \times D^2$. Wherefore $c^2 \times D^2 : C^2 \times$
. But by ſuppoſition, $D^3 : d^3 :: T^2$:
efore $c^2 \times D^2 : C^2 \times d^2 :: D^3 : d^3$.
$: C^2 :: D : d$ (or by extracting the
t, and inverting the Proportion) $C : c ::$
hat is, $C : c :: \sqrt{d} : \sqrt{D}$. Which were
nſtrated.
nverſes of theſe are evident. For as
tting $T^2 : t^2 :: D^3 : d^3$, it follows that
$: D^2$, and that $C : c :: d^{\frac{1}{2}} : D^{\frac{1}{2}}$; ſo

converſely, from the ſuppoſition of any one of theſe, you may return to the former Proportion; *viz.* $T^2 : t^2 :: D^3 : d^3$; and from hence again to the other Concluſion. And conſequently any one of theſe three being ſuppoſed, the other two will follow.

SCHOLIUM.

The three præceding Propoſitions (and their Corollaries) are true, not only of Bodies revolving in concentric Circles, but of ſuch alſo as deſcribe Circles whoſe Centers are intirely different, (for the identity of Centers was not ſuppoſed in the Demonſtration;) and of thoſe that deſcribe ſimilar parts of any Figures, having their Centers ſimilarly poſited. For what was aſſumed in the Demonſtration of theſe Theorems in Circles, are equally true of all other Figures like thoſe we but juſt now deſcribed; namely, that the naſcent lineola *GH* (in *Fig.* 27.) is to the naſcent lineola *EF*, as MH^q to MF^q (for this is demonſtrated of any Figure that a Circle is equicurve to, in *Prop.* 24.) and that *MH* is to *TD* as *SM* to *ST*, is true of any ſimilar Figures; provided the Center *S* be ſimilarly poſited in both Figures, and *MH*, *TD*, ſimilar, and ſimilarly poſited parts of thoſe Curves in reſpect of the Center. And inſtead of an equable Motion in Circles, if the Figure be different from a Circle, the Motion of the Body in the Perimeter of it, muſt be ſuch, as that the Areas deſcribed, by rays drawn to their Centers ſimilarly poſited, be proportional to the Times; otherwiſe (by *Prop.* 12.) the Forces, whereby the Bodies are drawn aſide from their Rectilineal Motion, and kept in their Curvilineal Orbits, do not ſo much as tend to thoſe Centers.

PROPOSITION XXVIII.

Forces, whereby the Primary Planets are
petually retracted from their Rectilineal Mo-
retained in their Orbits, are reciprocally as
s of their Diſtances from the Center of the
h they reſpect.

heſe Forces reſpect the Center of the
been ſhewn in *Prop.* 13. But that the
eſe Forces is ſuch, as that they are re-
as the Squares of their Diſtances from
r, or that they increaſe as the Squares
ance decreaſes, and decreaſe as the
f the diſtance increaſes: As for in-
at the Force whereby the Earth is made
wards the Sun, and is retained perpe-
its Orbit, ſo as not to fly off in a Tan-
, is to the like Force in Mars, as 5 to
in Jupiter as 27 to 1; becauſe the di-
the Earth from the Sun, is to the di-
Mars from the ſame, as 2 to 3, and
ance of Jupiter, as 5 to 26; and that
ipetal Force in the Earth is but half
enus, becauſe the Earth's diſtance
un is to the diſtance of Venus as 10 to
on in the others, is thus demonſtra-
Motion of the Planets revolving a-
un, and tending towards it as a Cen-
ch, as that the Squares of the Pe-
nes are as the Cubes of the diſtances
Sun, by *Prop.* 22. Now by the pre-
p. the Centripetal Forces of Bodies
according to this Law, are recipro-
e Squares of their diſtances from the
Conſequently 'tis evident that the For-
ting the Sun, by which the Planets are
n, and retain'd in their Orbits, are re-

ciprocally as the Squares of their diſtances from the Center of the Sun.

PROPOSITION XXIX.

THE Forces whereby the Secondary Planets are retained in their Orbits, are reciprocally as the Squares of their Diſtances from the Center of their reſpective Primary, to which theſe Forces tend.

For ſince (by *Prop.* 21.) the Motion of the Circum-Jovials revolving about Jupiter, and Circum-Saturnials revolving about Saturn, is ſuch, as to have the Squares of the Periodic Times in the ſame ratio with the Cubes of their Diſtances from the Center of Jupiter and Saturn reſpectively; and ſince (by *Prop.* 23.) the Centripetal Forces of Bodies revolving by this Law are reciprocally as the Squares of their Diſtances from their Center: 'Tis evident that the Centripetal Forces, whereby the Secondary Planets are reſpectively urged towards Jupiter and Saturn, are reciprocally as the Squares of their Diſtances from the Center of their reſpective Primary Planet.

But this Force in any Secondary Planet is only one part of the whole Force, whereby the Satelles is urged; and the other Force is the whole accelerating Force whereby the Primary is urged; as has been ſhewn in *Prop.* 20.

As for the Moon, the Earth's attendant, becauſe it is ſolitary, it has no other Secondary to be compared withal, as to Diſtance and Periodic Times. So that the Law of its Centripetal Force, whereby it is urged towards the Earth, can't be deduced after this manner. Yet hereafter you will find it drawn from the Figure of the Orbit that the Moon deſcribes about the Earth; which hitherto we have conſidered as

ng ſenſibly from a Circle. But if it
e a Circle concentric to the Earth,
f the Forces whereby it is retained in
could not be from hence diſcovered:
an Orbit may be deſcribed by a Body,
w of the Centripetal Force be what it

COROLLARY.

ws from theſe two Propoſitions, that
e Primary Planets, that neareſt the
among the Secondary, that next the
Primary, moves faſteſt. For it has
nſtrated, that in both of them, their
l Forces are reciprocally as the Squares
Diſtances from their Center: And in
was demonſtrated that the Celerities
e are reciprocally as the Square Roots
iſtances from the Center of Motion;
mean proportional between the Di-
that which was nearer, and that which
remote, is to the diſtance of the near-
Velocity of the nearer, to the Velo-
e more remote. And ſince the firſt of
ntities is greater than the ſecond, the
be greater than the fourth; that is,
ty of the nearer is greater than the
f the more remote.

SECTI-

SECTION V.

Of the Motion of the Primary Planets about their proper Axes, and of the Phænomena thence ariſing.

PROPOSITION XXX.

TO explain the Motion of the Celeſtial Bodies about their proper Axes, given in Poſition, and the Revolutions of them.

When we ſay a Body is not moved, we mean by it, that every line, and every point in it, is at reſt. But if we ſay a Body is moved along a line, without any other Motion, we intend to ſignify, that the Center of Gravity of the Body deſcribes that line, while the parts of the Body, in reſpect of the whole, remain at reſt; that is, every line in that Body retains a ſituation parallel to that it had before it was moved; and conſequently every line in the Body thus moved, remains always parallel to it ſelf. For after this manner, and this manner only, will it happen, that the Sum of all the Motions will be the leaſt, (that is, the motion of the whole the leaſt, as is ſuppoſed) and that any point of the Body thus moved, is as much moved as any other, and deſcribes a line ſimilar and equal, and only differing in ſituation: For the line deſcrib'd by the Center of Gravity, is ſuppos'd to be deſcribed by the Body it ſelf, from the known nature of a Center of Gravity.

If a Body be ſaid to be moved about a given Axe, being in other reſpects not moved, that Axe is ſuppos'd to be unmov'd, and every point out

to deſcribe a Circle, to whoſe Plane
perpendicular. And for that reaſon,
be carried along a line, and at the
be revolved about a given Axe; the
ll the time of the Body's motion, will
parallel to it ſelf. Nor is any thing elſe
o preſerve this Parallelіſm, than that
Motion beſides theſe two be impreſſed
Body; for if there be no other third
it, its Axe will continue always pa-
he Right-line, to which it was once

things being premiſed, 'tis evident
ervation, that the greater Celeſtial Bo-
ot all, yet at leaſt of every kind) beſides
ons ſpoken of in general above, do
bout their Axes. And, firſt, from ob-
, of Spots in the Sun, 'tis evident
un, the only Body of its kind, in our
revolves from Weſt to Eaſt about an
lin'd to the Plane of the Ecliptic, in
of about 87½ Degrees, in the ſpace of
Days. And ſince the Sun has no pro-
Motion, its Axis continues unmoved.
he Primary Planets, the Earth revolves
equable Motion, from Weſt to Eaſt,
Axe, inclin'd to the Plane of the
in an Angle of 66½ Degrees, in the
a natural Day. And ſince the Earth
arried about the Sun in the ſpace of a
s Axe muſt however continue parallel to
ho' it can't be intirely at reſt. And this
e to paſs by means of theſe two Moti-
e, without the aſſiſtance of a third,
me have groundleſly invented and call'd
name of a *Parallelіſm*. But the Axis of
h, tho' it continue parallel to it ſelf, as

to ſenſe, during one Revolution, yet after ſeveral, it changes this Site. Of the Cauſes and Phænomena whereof, we ſhall ſpeak in a proper place.

After the ſame manner Jupiter is revolved, from Weſt to Eaſt, about an Axe in the ſpace of about 10 Hours; Mars in $24\frac{2}{3}$ Hours, and Venus in 23 Hours, about their Axes; every one of which therefore continues always parallel to itſelf, if theſe Bodies are not moved by any other kind of Motion beſide their Revolution about the Sun, and Circumrotation about their own Axes.

Among the Secondary Planets, the Moon, the Earth's attendant, beſides its monthly Motion about the Earth, and annual about the Sun, (by which alone each line in it would be always parallel to its ſelf,) it likewiſe revolves about itſelf in the ſame ſpace of a Month, ſo as to turn always the ſame Face to the Earth.

We are not ſo certain yet of the other Planets, both Primary and Secondary, from Obſervations. Notwithſtanding, 'tis probable that they, like the former, revolve about a certain Axe; for this reaſon, that every part of them may be brought under, or removed from, the Rays of the Sun, more than once in one Revolution about the Sun, and may undergo Changes and Viciſſitudes ſuitable to their Nature.

PROPOSITION XXXI.

TO *deſcribe the Figure of the Sun and Planets, revolving about their own Axes.*

If the Sun and Planets were deprived of all Motion about their Axes, they would put on a Spherical Figure; ſuppoſing their parts like the parts of the Earth, to gravitate towards their Center,

s really they are, as we ſhall afterwards
e,) and either are now or were for-
. This is demonſtrated by *Archimedes*
atics. From a Motion about their Axes,
paſs, that their Parts receding from
s much as they can, endeavour to
ut the Circle that lies betwixt the
by a Centrifugal Force ariſing from
lar Motion. And therefore if the
he Sun or Planet be fluid, it will in-
Diameters of that middle Circle be-
two Poles, by its aſcent; and leſſen
wards the Poles by its deſcent, and
ly conſtitute an oblate Spheroid, ge-
the rotation of an Ellipſe about its
being depreſſed towards the Poles,
ed towards the Circle that lies ex-
en the Poles, more or leſs according
tity of the Centrifugal Force. This
al Figure is obſerved in Jupiter by
a Teleſcope; and is proved to be in
y Experiments, (ſince we can take a
at a diſtance.)

SCHOLIUM.

pheroidical Figure follows upon the
the Body about its own Axis, the
s Parts; ſo, on the contrary, from
d Figure, we may juſtly infer its Mo-
its Axis, and the (former) fluidity of

PROPOSITION XXXII.

ain the Phænomena ariſing from the Di-
Motion of the Earth about its Axis, ſeen
rth.

oregoing Propoſitions I have ſhewn,
d be the Phænomena of the Motion
of

of the Sun and Planets ſeen from the Earth, if the Earth were a Point; as that the Sun would ſeem to move among the Fixt Stars from Weſt to Eaſt; that the Planets would ſometimes ſeem to go forwards, at another time backward, according to the ſituation of the Sun and each Planet. But ſince the Earth is an opake Body, and large enough, in reſpect of the Obſerver, that ſmall part of its Surface, tho' Spherical, which comes at the ſame time under his confin'd view, will ſeem to be extended like a Plane. And the Eye taking a view of the Heavens all around itſelf, defines a concave ſpherical Superficies, concentric to the Earth, or rather to the Eye; which the abovemention'd Plane of the Earth's Superficies, becauſe drawn thro' the Center, will divide into two equal Parts, one of which will be viſible, but the other, by reaſon of the Earth's Opacity, will lie conceal'd. Becauſe the Earth moves upon its own Axis, the Spectator ſtanding upon it, together with the ſaid Plane he ſtands upon, call'd his *Horizon*, dividing the viſible from the inviſible Hemiſphere of the Heavens, is carried round the ſame way, *viz.* to the Eaſt. From whence it is, that Objects ſituated towards the Eaſt, and inviſible, become viſible, the Horizon ſinking as it were below them; and Objects ſituated towards the Weſt, are hid and become inviſible, the Horizon being elevated above them. And the former Objects therefore will appear to the Spectator, who uſually reckons the Place he ſtands in, to be immoveable, to aſcend above the Horizon or riſe, the latter to deſcend below the Horizon or ſet.

Since the Earth and the Horizon of the Spectator fix'd to it, continues to move always towards

ame Quarter and about the ſame
y; all Bodies, and all Appearances
artake of that Motion (that is, all
as are entirely ſeparate from the
ſeem to move in the like time uni-
towards the oppoſite parts, *viz.*
Weſt, excepting ſuch things as the
produced will meet withal, they
at reſt. And every one of theſe
rding to Senſe, will deſcribe the
ce of a Circle, to whoſe Plane the
Earth is perpendicular at the Cen-
ecauſe all theſe Circles, together
ble Objects deſcribing them, ap-
the concave Spherical Superficies
ens, concentric to the Earth; eve-
ject will ſeem to deſcribe a greater
cle, according to its greater or leſ-
from the Points above deſcribed,
reſt. In ſhort, the Celeſtial Sphere
be moved from Eaſt to Weſt, in
f a Day; about the ſame Axe in
out which the Earth really moves
rom Weſt to Eaſt: And thoſe Points
Poles of this Motion, in which the
Earth produced meets with its Super-
at will be the middle Circle betwixt
d conſequently the greateſt, (call'd
tial and *Equator*, for reaſons to be
the next Propoſition,) where the
at Circle of the Earth, which lies
ween the Poles, being produced,
concave Sphere. And univerſally,
on the Surface of the Earth, mark'd
t of it, in the diurnal Motion of the
correſpondent Circle in the Cele-
, made by the interſection of the
Surface

Surface of the Celeſtial Sphere with the Superficies of a Cone, whoſe Baſe is a Circle marked on the Earth, and Vertex the Center of the Earth. The Axis of the World, and the above-mention'd Celeſtial Circles, have always the ſame ſituation among the Fix'd Stars; becauſe their primitive Terreſtrial ones, from whence they have their original, produced ſo far, fall upon the ſame Stars, as to Senſe, in what Point ſoever of its Orbit the Earth be placed.

PROPOSITION XXXIII.

TO explain the Phænomena of the Sun ſeen from the Earth, ariſing from the Diurnal Motion of the Earth about its Axis, in conjunction with its Annual Motion about the Sun; that is, to explain the changes of Day and Night, and their variety in different Places, and different Seaſons of the Year.

Tho' each place of the Earth be inlighten'd by every Star that is above its Horizon, yet the illumination of the Sun being ſo very great above all others, its being above the Horizon alone is called *Day*; and its abode below the Horizon is called *Night*: And becauſe the Sun is intirely ſeparate and remote from the Earth, and ſo does not partake of its Diurnal Motion, it will Riſe and cauſe Day, in reſpect of a given place; and being carried thro' the midſt of the Heavens, deſcribing a Circle to appearance, having the Axis of the World perpendicular to its Plane, will at laſt Set again; making the Night to continue, till by the apparent univerſal Revolution of all the Celeſtial Bodies, it come up [illegible] gain above the Horizon of the ſaid Place.

To explain the variety of the Days and N [illegible] and the Seaſons of the Year thence ariſin [illegible] gine the Circle ♈ ♋ ♎ ♑ [*Fig.* 28.] to r [illegible]

deſcrib'd by the Earth, in a Year,
an plac'd in the Center at *S*. The
his Scheme is deſignedly reclined,
o appear in the form of an Ellipſe,
re commodious delineation of the
tions of the Earth.

rbit, let the Earth *A* ♋ *B E* be ſup-
carried after the manner deſcribed
from ♈, ♉, to ♊, *&c.* revolving at
e about its own Axis *B A*, towards
arter, or from ♋ upwards towards *E*.
ities of the Axis of the Earth will be
A the Southern Pole, and *B* the
E ♋ the Equator, whoſe Plane is
he Plane of the Ecliptic, by an An-
Degrees; *viz.* the Complement of
es to a right Angle, by which the
Earth (perpendicular to the Plane
tor) is inclin'd to it.

E repreſents the Earth, ſo it may
to repreſent the Celeſtial Sphere,
the preced. Prop.) concentric to the
ſurrounding it as it were, and car-
ith it; for this Sphere may be de-
y diſtance. A [illegible] Place of the
tar or Celeſti [illegible] thi [illegible]
, where a [illegible] jo [illegible]
e Earth [illegible]

and conſequently declines neither to the North Pole, nor the South, but ſeems to deſcribe, in its diurnal Motion, the Equinoctial it ſelf.

The Sun always ſeems to be in the Plane of the Ecliptic, (as was ſhewn in *Prop.* 2.) it will therefore appear to be in the common interſection of the Circles of the Ecliptic and Æquator deſcribed in the Celeſtial Sphere; namely, in ♈. Now in this Situation, the illumination of the Earth, made by the Sun, reaches both the Poles *A* and *B*; (becauſe the Boundary or Circle of illumination is a great Circle on the Earth, to whoſe Plane, the Right line joining the Centers of the Sun and Earth, is perpendicular) conſequently half the Terreſtrial Equator *E* ♋, or any other Circle parallel to it, is illuminated by the Sun, while the other half continues in darkneſs. And therefore every Plane of the Earth being carried round by the equable diurnal Motion, will be as long in darkneſs as it was in the light, that is, the Day, all over the Earth, will then be equal to the Night. From whence the Circle that the Sun then deſcribes by its Diurnal Motion, lying between the two Poles in a Celeſtial Sphere, is called the *Æquinoctial Circle.*

The Earth moving a little forwarder towards ♏ and ♐ in its Annual Motion, has the Plane of its Equator *E* ♋ no longer directed towards the Sun; but ſubſiding below it towards the South: From whence the Sun ſeems by little and little to decline from the Celeſtial Equator, towards the North Pole. For the Earth being at reſt to all appearance, its Equator muſt likewiſe be apparently at reſt. Conſequently the Celeſtial Equator correſponding to it, is moved only by the apparent Diurnal one: And therefore

n; which changes its Situation to it,
o move. And the light of the Sun,
re reached to both Poles, *A* and *B*,
grees be ſpread beyond *B*, and fall
. But when the Earth arrives at ♑
n hereupon appears in ♋, where it
o have its greateſt declination North-
al to the inclination of the Ecliptic
tor,) and will afterwards return to-
Ecliptic. The Circle in the Celeſtial
llel to the Equator, and expreſs'd by
ich the Sun in its North declination
ve arriv'd, and deſcribe by its Diur-
at that time, is call'd the *Tropic of*
rowing its name from the Sign of the
e Sun is then in; with which name
e correſpondent Circle on the Earth
The Earth being in this Situation,
that the rays of the Sun, inlightening
will reach beyond the North Pole *B*,
ſtop ſhort on this ſide the South Pole
o as that the Arc *B L* or *A F*, is equal
ET; the meaſure of the Sun's greateſt
n, or of the Inclination of the Ecliptic
uator. If Circles be imagined to be
o' *L* and *F*, parallel to the Equator
nely *K L*, *F G*,) they will be the *Polar*
e one the *Arctic*, and the other the
These things being ſuppos'd, 'tis
all that Tract of the Earth, which is
n the Arctic Polar Circle *V L*, notwith-
he Diurnal revolution, is all the while
d, and injoys therefore a continual
l on the contrary, that all within the
f the Antarctic Circle *F G*, have con-
ght or Darkneſs covering it. 'Tis fur-
ent (from *Prop.* 19. *Book* 2. of the *Sphærics*

of Theodoſius,) that the greater part of any Circle, parallel to the Equator *E Q*, lying between it and the Arctic Circle *K L*, is inlightened; the greater part of any one lying between the Equator and the Antarctic Circle *FG*, is in darkneſs; and that ſo much the more (by *Prop.* 20. of that Book) as the Circle is farther diſtant from the Equator; and (by *Prop.* 11 *Book* 1.) one half of the Equator is always in darkneſs, and the other half always inlightened. And therefore in that Situation of the Earth, wherein the Sun appears in ♋, to the Inhabitants of the Northern parts of the Earth, the Days are longeſt, and Nights ſhorteſt; conſequently it is Summer: But to the Inhabitants of the Southern part, the Days are ſhorteſt, and Nights longeſt, conſequently 'tis Winter. And theſe longeſt Days are ſo much the longer, and ſhorteſt Nights ſo much the ſhorter, according as the Place is more Diſtance from the Equator: But to ſuch as inhabit the Equator, the Days and Nights are ſtill equal; and conſequently are ſo all the Year long.

The Earth going forward thro' ♒, ♓, to ♈; during which time, the Sun ſeems to move thro' the Signs ♋, ♌, and ♍, the Sun returns towards the Equator; till at laſt at ♎, the Sun appears in the Equator of the Heavens, (becauſe at that time, the Plane of the Earth's Equator produced does again paſs thro' the Sun,) making the Days equal to the Nights all over the Earth. And now again the extremities of illumination reach to both Poles; ſo that the Day under the Pole *B*, which has been all the while inlightened, is equal to the ſpace of half a Year, (the time the Earth has ſpent ſince its departure from ♎;) and the Night under the Southern Pole *A*, as long.

The

h moving forwards thro' the Signs
the Sun will ſeem in the mean while,
♎, ♏ and ♐, and to decline gradual-
e Equator towards the South; till
Earth being really in ♋, and the
ng to be at ♑, all the ſame Appear-
happen to the Inhabitants of the
emiſphere, as have happen'd to the
of the Northern Hemiſphere, when
as at ♑; and what happen'd to the
of the Southern Hemiſphere before,
appen to them of the Northern.
he Earth is carried thro' ♋; ♌, and
will ſeem to move thro' ♑, ♒, and
uator, and ♈, compleating the Year:
n will cauſe the ſame Phænomena
Night, the former decreaſing, the lat-
g, to the Inhabitants of the Southern
e, as it did to the Inhabitants of the
Hemiſphere, while it went thro' the
ite Signs.

SECTION VI.

bits of the Planets and Comets, r Figure; together with the Law tripetal Forces, requiſite to make ove in ſuch Orbits.

ROPOSITION XXXIV.

imary Planet deſcribes the Perimeter of an
, having the Sun in one of its Focus's.

hitherto been treating of what was
was ſufficient to conſider the way of
each

each Planet, as if it were the Circumference of a Circle, having the Sun for a Center, ſince it does not differ very much from a Circle. He that conſiders the Phænomena of the Planets mov'd about the Sun more cloſely, will find, that they are not every where equally diſtant from the Sun: For inſtance, the Earth is more diſtant from the Sun in Summer than in Winter; as is evident from the apparent Diameter of the Sun, which is greateſt about the middle of *June*, and leaſt about the middle of *December*. 'Tis further evident from the ſwifter Motion of the Earth about the Sun, in the midſt of Winter, than in the midſt of Summer, near a fifteenth Part. And for this reaſon it is, that there are about eight Days more from the Vernal Equinox to the Autumnal, the Summer Seaſon, than from the Autumnal Equinox to the Vernal, the Winter Seaſon; tho' in both caſes the Earth or Sun ſeems to have moved but a Semicircle. Since therefore the Earth moves ſwifter about the Sun in Winter than in Summer, it muſt then be nearer to the Sun, by *Prop.* 12. to make the Area, deſcribed by a Right-line drawn from the Sun, augment equably as the Time, the greateſt length of the Baſe compenſating for the leſſer Altitude of the Trilineal Figure. The ſame holds likewiſe in the Motion of the other Primary Planets about the Sun: Nay the Orbit of any of them (except Venus) is more different from a Circle, having the Sun for its Center, than the Orbit of the Earth, as ſhall be ſhewn hereafter.

This deviation of the Planets in their Motions about the Sun, from the Circumference of a Circle concentric with the Sun, is ſo great and ſenſible, that all Aſtronomers have been oblig'd to

ſuppoſe

ntric Orbs: And becauſe they thought
iſpute, that all Celeſtial Motion muſt
Circular, they imagined each Planet
in a Circle, as *AIPR* [*Fig.* 29.] ha-
ter *C*, at ſuch a diſtance from the Sun
gree with its Motions. The remoteſt
this Excentric Orb from the Sun,
he *Aphelium*; *P*, the *Perihelium*; both
the Right-line *AP*, connecting the
d in which they placed the Sun, the
pſides; the part of it *CS*, the *Ex-*
And by the help of ſuch a *Theory* of
as this ſuppoſe, the Places of the
nd by Calculators, agreed exactly e-
h their true ones in the Heavens;
f them whoſe Orbits were very little
om Circles; for inſtance, the Earth.
hey came to Mars, its Place, by com-
om this Theory, was ſo different from
y obſervation, that they concluded
was not Circular, but ſomething de-
the Points *I* and *R*, which are re-
m the Apſides, and therefore, as it
Oval. This Oval the ſagacious *Kepler*,
mentaries upon Mars,) after indefatiga-
n Calculations, found to be a perfect
ving the Points *A* and *P* for its Verti-
were the Apſides in the Circular Or-
ſtill retain the ſame Names, and the
the Sun for one of its Foci: Becauſe
e alone exhibits, the Diſtances from
(or Right-lines *SE*) agreeable to ob-
, as it does the true Equation for de-
g the Angle *ASE*, from the mean
of the Sun given: That is, only the
entioned Ellipſe is of that nature, as
the Periodic Time, or intire Revolu-

tion, in the ſame ratio to the Time ſince its departure from the Aphelium in *A*, as the intire Area *A E P L A*, is to the Area *A S E Q A*, intercepted between the line of the Apſides *S A*, and the line *S E* (known by obſervation) drawn from the Sun to the Planet: But that this is a circumſtance in the Motion of every Planet about the Sun, is evident from *Prop.* 13. Beſides, the Place of the Earth in an Elliptic Orbit is ſuch, as that the Places of the other Planets about the Earth, found in their reſpective Orbits by Calculation, and ſeen from the Earth, agree very exactly with their Places found by Obſervation in the Heavens; which happens ſo exactly in no other Orbit whatever. Since therefore the Places of the Planets, found by a Calculus accomodated to Elliptic Orbs, agree ſo exactly with their Places by Obſervation; but do not at all agree with a Calculus founded upon Orbs of any other Figure, but upon the account of their being ſomething near, or approaching to an Elliptic form; 'tis evident that the Orbits deſcribed by the Planets are Elliptic.

The Elliptic Orbits of the Primary Planets deſcribed about the Sun in the common Focus, are almoſt immovable; that is, when the Planets are carried to the Apſides of their Orbits, or any other given Points, they return to the ſame Points of the Mundane Space nearly: For ſuch kind of Orbits only, ſolve the obſerved Phænomena of the Places of the Planets among the Fixt Stars, ſeen from the Earth.

PROPOSITION XXXV.

A Comet moves in a Conic Section, that has the Sun in one of its Foci.

For ſuch Orbits, and ſuch only, agree with the

of the Comets obſerved in the Hea-
true that *Kepler*, and ſeveral Philo-
er him, ſuppoſed the Trajectories of
be Right lines; and have Calculated
es upon that Hypotheſis, agreeing
ctly with their Places by Obſervation.
ight have been done, tho' the Comet
d in a Conic Section, if the Obſer-
e made only while it is in that part of
which is not much different from a
Let *A P V B C* [*Fig.* 30.] be a very
ection of a Cone; that is, the Di-
whoſe Foci is almoſt equal to its
Axe: Let *S*, the Center of the Sun,
the Foci; it may be the Comet is ob-
e it is deſcribing the part *A P* of its
during the reſt of the time, while
rom *P*, thro' *V*, *B*, and *C*, (where it
and goes off into very diſtant Re-
ies hid by the Rays of the Sun, in re-
e Obſerver upon the Earth, moved
Sun *S*: Or it may be the Comet
ring its Motion along *A P V B*, under
of the Sun, (the Situation and Mo-
e Earth allowing of it, and concur-
then at laſt is obſerved, when it is
and is about to deſcribe the line *B C*;
h caſes, the Path of a Comet differs
ly from a Right-line. In the former
Comets will ſeem to be ſwallow'd by
becauſe they were ſeen before their
he Sun, and afterwards were ſeen no
d conſequently are look'd upon as de-
But in the latter caſe, they will ſeem
t of the Sun, becauſe they were firſt
eir aſcent from the Sun, towards the
gions. Beſides, when a Comet in its
deſcent

deſcent towards the Sun (as in *AP*, comes under our Obſervation, approaching afterwards to the Sun, and lying hid for ſome time in his Rays (namely, while it deſcribes *PVB*,) and unexpectedly emerges on the other ſide of the Sun, with a Tail conſiderably increaſed, (for reaſons afterwards to be aſſigned in their proper place;) it is commonly taken for a new Comet, and different from the former; the Trajectory of the one being the Right line *AP*, but of the other *BC*. Now the difference and ſeeming contrariety of their Trajectories, have deceived ſome Aſtronomers, who have aſſerted that it was not the ſame Comet that deſcribed them both. Nor is this Miſtake eaſily corrected in a Comet that never returns to the ſame place, as we know of, by Obſervation. From all which, the reaſon is plain, why Rectilineal Trajectories, for the moſt part, agree with the Motion of Comets; namely, becauſe only one portion of it, is taken for the whole Trajectory: But if the whole be conſidered together, as well in its deſcent towards the Sun, as in its aſcent from it, no other line but a Conic Section would be found to agree with it.

Of the three Sections of a Cone, an Ellipſe, having the Center of the Sun in one of its Foci, is moſt agreeable to the Motion of a Comet, as ſhall be demonſtrated in its proper place: And if the Comets be durable Bodies, and return, none but an Ellipſe will ſuit their Motion. But inſtead of a portion near the Vertex of a very excentric Ellipſe, which the Motion of a Comet requires, a portion of a Parabola, having the ſame Vertex and Focus, may be ſubſtituted; becauſe not ſenſibly differing from it. As the Parabolic Curve is its true Trajectory, if it never

or elſe perhaps an Hyperbolic one,
any ſuch Comet, whoſe Motion an
Trajectory will better agree to,
abolic.

PROPOSITION XXXVI.

condary Planet revolves about its Pri-
in the Perimeter of an Ellipſe, having
the Primary in one of its Foci.

he Moon, the Earth's Secondary, it
ce, in its Revolution about the Earth,
at the ſame time ſlower; and in the
int greater, and ſwifter: 'Tis plain
that its Orbit is excentric to the
at it is an Ellipſe having a Focus in
Center, may be proved by reaſons
by which (in the præced. *Prop.*) we
ed the Orbits of the Primary Pla-
the Sun, to be Elliptic: namely, be-
ning an Elliptic Orbit for the Moon,
s from the Earth, come out agreeable
ion, and the Æquations of its Motion,
not to be found from the Suppoſition
er kind of Orbit. All this agrees
ith the Orbits of the Satellites of Ju-
aturn; tho' they are nearly Circular
tric with Jupiter and Saturn.

Elliptic Orbits of the Primary Pla-
reſt, as was ſaid in the præceding
, yet 'tis quite otherwiſe in the Se-
For their Motions are ſo diſturbed by
e explain'd hereafter, that the ſame
e not deſcribed again, abſtracting
Motion of the primary Planet. But
heir Orbits to Ellipſes, we muſt ima-
llipſe of each Secondary Planet, to
ſo, as that the Plane of the Ellipſe,

and Focus fixed in the Center of the Primary, continuing the ſame, the Axis of the Ellipſe is carried, *in conſequentia*, by an angular Motion, round the immoveable Focus. But this Motion is ſo ſlow, and inſenſible, (being in the Moon an hundred times ſlower than the Moon it ſelf,) that it may very well be neglected in this place; and the Ellipſes deſcribed by the Secondary about their Primary, may be look'd upon as unmoved, abſtracting from their Motion round the Sun in common with their Primary.

PROPOSITION XXXVII.

If a Body be projected according to the direction of any Right line, and at the ſame time urged by a Centripetal Force tending to the Center S, [*Fig.* 31.] *ſo as by a Motion compounded of both, it deſcribes the Curve* APp; *and the Right line* PR *be a Tangent to it, in a point* P; *and from the point* B, *neareſt to* P, *a line* BD *be drawn perpendicular to the Right line* SP, *and the Right line* BR *parallel to the ſame line* SP; *and if the like conſtruction be made at any other Point* p, *of the Curve: I ſay the Centripetal Force in* P, *is to the Centripetal Force in* p, *as* $\frac{Sp^{q} \times bd^{q}}{br}$ *is to* $\frac{SP^{q} \times BD^{q}}{BR}$: *Or that the Centripetal Force in* P, *is reciprocally as the Solid* $\frac{SP^{q} \times BD^{q}}{BR}$, *when the Figure* PRBD *is indefinitely ſmall.*

Let the Force in *P*, that tends to the Center *S*, be called *V*. Let the Time wherein the indefinitely ſmall Arc *PB*, is run, or in which a Body, by its *Vis inſita* alone, would run the indefinitely ſmall Tangent *PR*, be called *T*. Let the Centripetal Force acting in *p* be called *v*: and

ne that $p\,b$, or the Tangent $p\,r$, is
r. Take an Arc $P\,\beta$, run in the ſame
he Arc $p\,b$; and draw $\beta\,\rho$ parallel to
e things being ſuppoſed, 'tis evident
io of the lineola $B\,R$ to the lineola $b\,r$,
ded of the ratio of $B\,R$ to $\beta\,\rho$, and
of $\beta\,\rho$ to $b\,r$: But (by *Prop.* 24.) $B\,R$
he duplicate ratio of the Arc $P\,B$ to
; and theſe Arcs, ſince they are in-
mall, are as the Triangles $B\,S\,P$,
e ſame Altitude; that is, (by *Prop.*
Times in which they are deſcribed by
wn to S; or, by Conſtruction, as the
ein the Arcs $P\,B$, $p\,b$, are deſcribed.
lineola $\beta\,\rho$, is to the lineola $b\,r$, as
roducing them; that is, as the Cen-
ce in P, to the Centripetal Force in
refore the ratio of the lineola $B\,R$
la $b\,r$, is compounded of the dupli-
f the Times wherein $P\,B$ and $p\,b$
d, and of the ratio of the Centripe-
n P to the Centripetal Force in p:
Symbols, $\frac{B\,R}{b\,r} = \frac{T^2}{t^2} + \frac{V}{v}$; or
$\frac{\times V}{v}$. And conſequently $\frac{V}{v} = \frac{BR \times t^2}{b\,r \times T^2}$
. 11.) T is to t, as the Area $S\,B\,P$ to
$b\,p$; or as its double $S\,P \times B\,D$ to
double of the latter: Subſtituting
ſtead of the ratio $\frac{t^2}{T^2}$, its equal
and then $\frac{V}{v} = \frac{B\,R \times S\,p\,q \times b\,d\,q}{b\,r \times S\,P\,q \times B\,D\,q}$,
$B\,R \times S\,p\,q \times b\,d\,q$ to $b\,r \times S\,P\,q \times$
is, (dividing them by $B\,R \times b\,r$)
as

as $\frac{Sp^{q} \times bd^{q}}{br}$ to $\frac{SP^{q} \times BD^{q}}{BR}$. The Force therefore in *P* tending towards *S*, is reciprocally as the naſcent or evanaſcent Solid $\frac{SP^{q} \times BD^{q}}{BR}$. Which was to be demonſtrated.

COROLLARY.

From whence, if any Figure (*A P p*) be given, and in it a Point *S*, to which the Centripetal Force is directed; the Law of the Centripetal Force, whereby a Body drawn from its rectilineal courſe, ſhall be made to keep in the Perimeter of that Figure, and deſcribe it in its revolution, may be found; provided this Force be compounded of a proper one impreſs'd along the Tangent of the Figure. For the Law ſought (by this *Prop.* 37.) is, that the Force be reciprocally as the naſcent Solid $\frac{SP^{q} \times BD^{q}}{BR}$, which is to be computed from the property of the Figure being given. A remarkable inſtance of which, we ſhall give, taken from nature.

PROPOSITION XXXVIII. LEMMA.

IF *a Right line* ZPR [*Fig.* 32.] *touch the Ellipſe* API, *whoſe Foci are* S *and* F, *in any Point* P, *and a Diameter as* I K *be drawn parallel to it, thro' the Center* C; *the portion* EP, *of the Right line* SP, *joining either of the Foci (as* S) *and the point of Contact, is equal to half the greater Axe* C A.

Thro' the other Focus *F*, draw *FH* parallel to *P R*, meeting *S P* in *H*. Becauſe (by *Prop.* 48. *lib.* 3. *El. Conic. Apollonius*) the Angles *FPZ*, *HPR*, are equal, and conſequently their alternate ones *PFH*, *PHF* are equal; therefore *PH* is equal to *PF*. Again, (by 2. *Elem.* 6.) *S E* is equal to *E H*, becauſe

s equal to CF; and ſince SH is the
of the Right lines PS, PH, it will
the difference of the lines PS, PF;
half this difference. And there-
made up of the leſſer and half the
is equal to half the Sum of PS, and
y *Prop.* 52. *lib.* 3. *El. Conic.*) the Right
F, taken together are equal to the
: Wherefore half their Sum (that is,
l to half the greater Axe CA. Which
emonſtrated.

ROPOSITION XXXIX.

ly revolve in an Ellipſe; the Law of the
etal Force tending to the Focus of the
uired.

A [*Fig.* 33.] be the Ellipſe, whoſe
s deſcribed by the revolving Body;
r Umbilical Point S, to which the
rected, whereby the Body drawn
ctilineal Motion is retained in the
bit: required the Law of this Force.
e conjugate Axes of the Ellipſes TA,
g one another in the Center C. And
Point P, in its Perimeter, draw a
Z, and a Diameter PM, and a con-
ICK parallel to PZ, to which, from
he perpendicular PN. Compleat
logram $PCIH$, and the Right line
ch the Ellipſe: (by 17. *Book* 1. *Conic.*)
tting IK in E. Thro' B, the neareſt
le to P, draw BR parallel to PS,
PZ, meeting the Right line SP in
P in O.

tripetal Force tending to S, is (by
eciprocally as the ſolid $\frac{SP^q \times BD^q}{BR}$,

Which therefore muſt be computed from the nature of the Ellipſe. In order to which, let the *Principal Parameter* of the Ellipſe, or that which belongs to the greater Axe TA, be call'd L. The Ratio of $L \times BR$ to BD^q is compounded of the Ratios of $L \times BR$ to $L \times PO$, $L \times PO$ to $MO \times PO$, $MO \times PO$ to BO^q, BO^q to BX^q, and BX^q to BD^q: But the Ratio of $L \times BR$ to $L \times PO$, or (by *Prop.* 1. *El.* 6.) of BR or PX to PO, is equal to the Ratio of PE to PC, (by *Prop.* 2. *El.* 6.) becauſe XO is parallel to EC; and (by præced.) PE is equal to AC: Therefore the Ratio of $L \times BR$ to $L \times PO$ is equal to the Ratio of AC to PC. The Ratio likewiſe of $L \times PO$ to $MO \times PO$ is the ſame with the Ratio of L to MO. And the Ratio of $MO \times PO$, or the rectangle MOP to BO^q, is the ſame (by *Prop.* 21. *B.* 1. *Conic.*) with the Ratio of the rectangle MCP (that is, CP^q) to IC^q. And in the preſent caſe, the Ratio of BO^q to BX^q is the Ratio of equality; for the Point B coinciding with the Point P, BO becomes equal to BX. Laſtly, the Ratio of BX^q to BD^q is equal to the Ratio of PE^q to PN^q: (For the Triangles BDX, PNE are equiangular, the Angles at D and N being right; and BXD, PEN, being alternate, and therefore equal, becauſe of the two Parallels BX, EN.) And PE^q is equal to CA^q, becauſe the lines themſelves are equal (by the præceding Propoſition.) Therefore the Ratio of BX^q to BD^q is the ſame with the Ratio of CA^q to PN^q. Beſides, (by *Prop.* 72. *Book* 4. of *Gregory St. Vincent*,) the rectangle under GC and CA is equal to the Parallelogram $PCIH$, that is, to the rectangle under IC, PN: Wherefore (by 16. *El.* 6.) CA is to PN as IC to GC; therefore CA^q

is

s IC^q to GC^q: The ratio therefore
BD^q, is the ſame with the ratio of
^{q}. Since therefore the ratio of
D^q is compounded of the ratios of
$L \times PO$, $L \times PO$ to $MO \times PO$,
o BO^q, BO^q to BX^q, and BX^q
compounded alſo of ratios reſpe-
l to theſe; namely, of the ratios
, L to MO, CP^q to IC^q, and IC^q
d therefote will be equal to the ra-
$\times L \times PC^q$ to $PC \times MO \times GC^q$:
13. *Book* 1. *Conic.*) $AC \times L = 2\,GC^q$.
therefore the one in the room of
e ratio of $L \times BR$ to BD^q, is equal
f $2\,GC^q \times PC^q$ to $PC \times MO \times GC^q$;
e ratio $2\,PC$ to MO: But (in the
, ſince the point B is the neareſt
,) the point O muſt be the leaſt
ible from the point P; that is, MO
PC; and therefore, in this caſe,
qual to BD^q: Becauſe therefore,
$BD^q = L \times BR$, 'tis $\frac{SP^q \times BD^q}{BR} =$
$\frac{BR}{}) = SP^q \times L$: The Centri-
therefore is reciprocally proportio-
lid; and L in it is a conſtant inva-
line.

therefore of the Centripetal Force
e Focus of an Ellipſe, (whereby a
off perpetually from its Rectilineal
pt in the perimeter of that Figure,
it in its Motion,) is, that it is reci-
portional to the Square of the di-
he Place P from the Center S.
be found.

COROLLARY 1.

If the Focus *S*, of the Ellipſe, to which the Force tends, together with the next Vertex *T*, continue as before; [*Fig.*34.] but the Focus *F*, approach it, and at laſt coincide with it; the Body *P*, will now be kept revolving in the Perimeter of a Circle, by the Centripetal Force tending to *S*, after the ſame manner, and by the ſame Law, as it was in the Perimeter of the Ellipſe. And as a Circle may be deſcribed by a Body, acted upon by any Centripetal Force tending to a point ſituated without the Right line, according to the direction of which it would have moved by its *Vis inſita* alone, (as was ſaid in *Prop.* 29.) ſo a Right line will be deſcribed by a Body that is urged by no Force tending to a point ſituated without the ſaid Right line: For if the two Vertices continue the ſame, and the Foci approach to them, the Ellipſe will be turned into a Right line. But to deſcribe any other Lines, beſides a Right one and a Circular, a particular Law of the Centripetal Force is neceſſary.

COROLLARY 2.

If the Focus *S*, [*Fig.* 35.] of the Ellipſe, to which, as a Center, the Force be directed, together with the next Vertex *T*, remain as before, and the other Focus *F*, go farther off *in infinitum*, the Body *P* will be kept in the Perimeter of the Figure *T P*, by the Centripeaal Force tending to *S*, after the ſame manner as it was before, and by the ſame Law. But in this caſe, the Ellipſe will be turn'd into a Parabola: The Law therefore of a Centripetal Force tending to the Focus of a Parabola, retaining a Body in the Perimeter of ſuch a Figure, is this, that it is reciprocally proportional to the Square of the Diſtance from the Focus.

CO-

COROLLARY 3.

reſt continuing as before, the Focus
] go off to a more than infinite
that is, if it appear on the other ſide
make the Vertices *A* and *T* lie be-
Foci *S* and *F*; whereas before, the
F were between the Vertices *A* and
h caſe, the Ellipſe is turn'd into an
the Body acted upon by the ſame
Centripetal Force as before, (that
increaſe, as the Square of the Di-
the Center decreaſes,) will then
Perimeter of this Hyperbola.

COROLLARY 4.

e Focus *F*, [*Fig.* 37.] of the for-
, and the Vertex *A* continue fixt,
mean while the other Focus *S*, to
orce is directed, goes off further
, to an infinite Diſtance, (that is,
be turned into a Parabola;) the Cen-
ce directed to the Point *S* at an in-
ice, or according to Right lines pa-
Axe *AF*, by which the Body moves
ter, will become equable: For Di-
an infinitely diſtant Center, whoſe
finite, are equal. And therefore
hich is reciprocally as the Square of
e, is equable. And converſely, a
pon by an equable Force, tending
infinitely diſtant, and which con-
directed according to Right lines
given one, will deſcribe in its Mo-
abola, whoſe Axis is parallel to the
line. And this is the famous The-
lileo, concerning the path or tract
ile.

COROLLARY 5.

All things continuing as before, if the Focus *S*, (or Center to which the Force is directed) go off to more than an infinite Diſtance, as in *Corol.* 3. it happen'd to the Focus *F*, then the Ellipſe will be turned into an Hyperbola, [*Fig.*38.] and the Body made to move in the Perimeter of the Figure thus changed, by a Force acting according to the direction of the Right line *S P*, that is, reciprocally proportional to the Square of the Diſtance of the Place from the Center *S*: and that conſequently is a Centrifugal Force, in regard of the Point *S*, becauſe the Curve *A P*, is convex towards *A*.

As an Ellipſe is deſcribed by a Body urged by a Force tending to either of the Foci (becauſe within it,) and acting according to the Law of the preceding Propoſition; and an Hyperbola, while the Centripetal Force of the ſame Law tends to a Focus within it, or while the ſaid Centripetal Force is changed into a Centrifugal one, tending to a Focus, without the Curve: So a Parabola, which, as it were a mean betwixt theſe two Sections, partakes of the nature of both; and is deſcribed by the action of a Centripetal Force of the ſame Law, tending to a Focus within the Curve, (as in both the preceding,) and by the action of a Centripetal Force tending to one Focus, or a Centrifugal Force tending from the other, (after the manner of an Ellipſe and Hyperbola reſpectively,) according as that other Focus is ſuppos'd to be placed towards the convex or concave parts of the Curve: For both ways it may be conſidered as infinitely diſtant.

COROLLARY 6.

From this Propoſition and its Corollaries, it follows, that if any Body be projected from any Point

cording to the direction of any Right
vith any Velocity, and be acted upon
time by a Centripetal Force, which
lly proportional to the Square of the
the Place from the Center *S*, ſitu-
ut the abovemention'd Right line;
noved in one of the Conic Sections,
Focus, as the Center to which the
rected. For the Ellipſe of the for-
ition can be changed into no other
Right line and a Conic Section; (by
of either Focus in the Axe pro-
eed be.) And the principal Para-
is Section or *L*, by the demonſtrati-
opoſition, (where it was ſhewn that
BD^q,) is a third proportional to *BR*
nſidered as naſcent.

PROPOSITION XL.

Bodies revolve about a common Center,
Centripetal Force be reciprocally as the
he Diſtance from the Center; I ſay the
he Periodic Times in Ellipſes are as the
Tranſverſe Axes.

be any two Orbits *APT*, *apt*, about
ocus *S*, (*fig.* 39.) in which the former
n is ſuppos'd to continue. Their
L and *l*, (by Corol. 6. of the pre-
poſition) are reſpectively equal to

; the Points *B* and *P*, likewiſe

nciding. But *RB* is to *rb* (being
n the ſame time) as the Centripetal
to the Centripetal Force in *p*; that
ſition, as Sp^q to SP^q: And there-
$\left(\frac{BD^q}{Sp^q} : \frac{bd^q}{SP^q} ::\right) BD^q \times SP^q :$

$bd^q \times Sp^q$. And $BD \times SP$ is to $bd \times Sp$, as their halves; that is, as the Areas deſcribed at the ſame time by the Bodies P and p: And therefore L and l are as the Squares of the Areas, that are deſcribed in the ſame time, by Radii drawn to the common Center S; that is, theſe Areas, deſcribed at the ſame time, are in the ſubduplicate ratio of the Parameters. And ſince the intire Areas of the Ellipſes are as any leſſer parts of them, deſcribed at the ſame time, drawn into the reſpective Periodic Times; the intire Areas will be to one another, in a ratio compounded of the ſubduplicate of the Parameters, and ſimple ratio of the Periodic Times.

Since (by *Prop.* 191. *Book* 4. of *Gregory St. Vincent*) the Ellipſe $AGTY$ is to $agty$, as $AT \times GY$ is to $at \times gy$: And therefore the rectangles under the Axes of the Ellipſes, are in a ratio compounded of the ſubduplicate ratio of the Parameters, and of the ſimple ratio of the Periodic Times; that is, as $L^{\frac{1}{2}} \times$ Periodic Time in APT to $l^{\frac{1}{2}} \times$ Periodic Time in apt. And ſince from the nature of an Ellipſe, GY is a Geometric mean between L and AT, and gy betwixt l and at; or $GY = L^{\frac{1}{2}} \times AT^{\frac{1}{2}}$, and $gy = l^{\frac{1}{2}} \times at^{\frac{1}{2}}$; Therefore $L^{\frac{1}{2}} \times AT^{\frac{3}{2}} : l^{\frac{1}{2}} \times at^{\frac{3}{2}} :: (GY \times AT : gy \times at ::)$ $L^{\frac{1}{2}} \times$ Periodic Time in $APT : l^{\frac{1}{2}} \times$ Periodic Time in apt. Dividing therefore the Antecedents by $L^{\frac{1}{2}}$ and the Conſequents by $l^{\frac{1}{2}}$, and ſquaring the Quotients you will have the Cube of AT, to the Cube of at, as the Square of the Periodic Time in the Ellipſe APT, to the Square of the Periodic Time in the Ellipſe apt. Which was to be demonſtrated.

COROLLARY I.

Becauſe a Circle is a Species of an Ellipſe; where both the Foci coincide with the Center, and the greater Axe is equal to any Diameter,

that the Squares of the Periodic
odies revolving in an Ellipſe and
t the ſame Center, are as the Cubes
ſverſe Axes: And therefore, if the
eir Cubes are equal, the Squares of
Times, and conſequently the Pe-
s themſelves, will be equal: that is,
enter of the Forces is the ſame, the
imes in Ellipſes, will be the ſame
in Circles, whoſe Diameters are e-
greater Axes of the Ellipſes.

COROLLARY. 2.

nt from the Demonſtration of the
that the Parameters of any Orbits,
Bodies revolving about a common
acted upon by a Centripetal Force,
iprocally as the Squares of the Di-
the Center, are as the Squares of
eſcribed in the ſame time, by Radii
e Center: For this was demonſtra-
lly of any Orbits, whether Elliptic,
or Hyperbolic, before the reaſoning
to Ellipſes, in which only, the Pe-
has any place.

PROPOSITION XLI.

aw of the Centripetal Force being as
; I ſay the Velocities of any Bodies
ut the ſame Center, in any points what-
a ratio compounded of the ſubduplicate
principal Parameters belonging to the
ſcribe, and of the reciprocal ratio of the
s let fall from the common Center upon
bat are Tangents at the points where the

odies P and *p* [*Fig.* 40.] revolve a-
ommon Center *S*, in any kind of
b. Becauſe the Bodies, by ſuppoſi-

tion, are acted upon by a Centripetal Force, reciprocally proportional to the Square of the Diſtance of the Place from the Center; theſe Lines will (by *Corol.* 6. *Prop.* 39.) be Conic Sections, having S for a Focus. Let their principal Parameters be call'd L and l. From S, let fall SN, sn, perpendicular upon the Right lines PR, pr, Tangents in P and p. I ſay the Velocity of the Body P, is to the Velocity of the Body p, in a ratio compounded of the ratio of $L^{\frac{1}{2}}$ to $l^{\frac{1}{2}}$, and of the ratio of Sn to SN; or as $L^{\frac{1}{2}} \times Sn$ to $l^{\frac{1}{2}} \times SN$.

Take the Arcs PB, pb, deſcribed in the ſame infinitely ſmall time; and thro' B draw BX parallel to RP, and BD perpendicular to SP; let the ſame be done at b. The Velocities in P and p are as the naſcent Arcs PB, pb; becauſe they are the Spaces run in the ſame time; that is, as the Lines PR and pr equal to them. The right angled Triangles BDX, SNP, having the Angles BXD, SPN equal, becauſe BX, NP are parallel; ſo that $SN : SP :: BD : BX$; and therefore BX, or what is equal to it $PR = \frac{SP \times BD}{SN}$. And $pr = \frac{Sp \times bd}{Sn}$, becauſe of ſimilar ratios. But $SP \times BD$, $Sp \times bd$ are as their halves, that is, SPB, Spb; or the Areas deſcribed at the ſame time by the Bodies P and p, which again (by *Cor.* 2. *Prop.* preced.) are as the Square Roots of the Parameters L and l; or as $L^{\frac{1}{2}}$ to $l^{\frac{1}{2}}$. And therefore PR is to pr, or the Velocity of the Body in P, is to the Velocity of the Body in p, as $\frac{L^{\frac{1}{2}}}{SN}$ to $\frac{l^{\frac{1}{2}}}{Sn}$, or as $L^{\frac{1}{2}} \times Sn$, to $l^{\frac{1}{2}} \times SN$: That is, in a ratio compounded of the ſubduplicate ratio of the Parameters to the Figures PB, pb, and of the reciprocal ratio of the

the

diculars, let fall from the Center of
S, upon the Tangents. Which was
nſtrated.

COROLLARY 1.

t follows, that the Velocities of the
heir greateſt and leaſt Diſtances from
on Focus, about which they revolve,
tio compounded of the ſubduplicate
e Parameters, and of the reciprocal
Diſtances from the common Focus.
ir greateſt and leaſt Diſtances from
the Perpendicular to the Tangent is
ce it ſelf. If theſe Diſtances from
are equal, the Velocities, in that
time, are in the ſubduplicate ratio
ameters; that other ratio of the di-
coming a ratio of equality. And if
Figures be a Circle, the Velocity in
Section, in the greateſt or leaſt di-
n the Focus, will be to the Velocity
cle, as the Square Root of the Para-
he Conic Section is to the Square of
te.

COROLLARY 2.

ame or different Figures, having e-
eters, the Velocity of the Body is
y as the Perpendicular let fall from
Center of the Forces, on the Tan-
the ſubduplicate ratio of the Para-
this caſe, is a ratio of equality.
ore the Velocities of a Body revol-
Ellipſe, in the greateſt and leaſt di-
n the Focus, are reciprocally as the
For, in this caſe, the diſtances are
diculars to the Tangents of the Or-

COROLLARY 3.

The apparent Motions of a Body in the principal Vertices *A* and *P*, [*Fig.* 41.] viewed from the Center of the Forces *S*, are reciprocally as the Squares of the Diſtances *S A*, *S P*. Take at the Vertices *A* and *P*, the indefinitely ſmall Arcs *A a*, *P p*, deſcribed in equal indefinitely ſmall Times, by a Body revolving about *S*; and join *S a*, *S p*. The apparent Motions of the Body from *S*, are the Angles *A S a*, *P S p*. The ratio of theſe Angles *A S a*, *P S p*, is compounded of the Ratio of *A a* to *P p* (ſince theſe Arcs do not differ from Circular Arcs deſcribed upon the Center *S*,) and of the ratio of *S P* to *S A*; or the reciprocal of the Radii. But becauſe *A a*, *P p* are deſcribed in equal Times, they will be as the Velocities in the points *A* and *P*, which (by *Corol.* preced.) are as *S P* to *S A*. And therefore *A S a* is to *P S p*, as SP^{q} to SA^{q}.

COROLLARY 4.

Hence it follows, that, if the Orbit be very nearly a Circle, the apparent Motions of the Body viewed from the Center *S*, are very nearly reciprocally as the Squares of the Diſtances from it. For, in this caſe, all the Right lines that fall upon the Orbit from *S*, are nearly perpendicular to it.

COROLLARY 5.

From *Corol.* 2. it follows, that if a Body, moved in any Conic Section *A D S* [*Fig.* 42.] as before, ſhould leave the Curve, and go on uniformly in a Right line *P Q*, touching the Curve in *P*, with the Velocity it had in the point *P*; the Area *S P Q*, which it would deſcribe by a radius drawn to *S*, would be equal to the Area *S A D*, which (in the ſame or equal time,) it would deſcribe, being kept ſtill moving in

ic Section. Take the lineola *A a*,
bed in the ſame indefinitely ſmall
vo Bodies; and from the Focus *S* of
, to the Tangents *A H* and *P Q*, let
rpendiculars *S H*, *S N*. By *Corol.* 2.
y in *A* is to the Velocity in *P*, as *S N*
t as the Velocities in *A* and *P* are,
paces run in the ſame time, by the
mely, *A a* and *P R*. And therefore
R, as *S N* to *S H*. Therefore the
A a is equal to the Triangle *S P R*.
his holds in the indefinitely ſmall
hat conſtitute the Trilineal Figures
S P Q, and the Spaces *S A D*, *S P Q*
o of an equal number of Triangles,
of running *A D* and *P Q* being ta-
the Triangle *S P Q* is equal to the
.

PROPOSITION XLII.

rimary Planets and Comets moves about
n according to this Law, that the Mo-
is compounded of an equable one along the
the Orbit, and another tending towards
the Sun, in which the accelerating Force
y proportional to the Square of the di-
hat Center.

the Primary Planets (by *Prop.* 13.) is
by a Force tending toward the Cen-
un; and (by *Prop.* 34.) deſcribes the
of an Ellipſe, having the Sun in its
d all this is true alſo of any Comet,
itions 14 and 35;) or at leaſt it de-
e other Conic Section having the
, ſeeing it is acted upon by a Force
wards the Sun. And therefore (by
d its *Corollaries*) every Comet is acted
n equable Force impreſs'd along a
Tangent

Tangent to the Orbit, and another whereby it is drawn off from the Tangent and kept in a Curvilineal Orbit, which tends towards the Sun's Center, placed in the Focus of the Section, and is reciprocally proportional to the Square of the diſtance from it. Which was to be demonſtrated.

The Planets and Comets therefore are to be conceiv'd as ſo many Projectiles, which are acted upon by two Forces. For in this manner, and this only, do they deſcribe Orbits, ſuch as Obſervation finds them to be. And the Law of the Force that draws off theſe Projectiles from their Rectilineal Motion, is, that it increaſes as the Square of the diſtance of the Comet or Planet projected, from that Center of the Sun, increaſes: Which is the Law whereby the Planets tend to the Sun, as we have demonſtrated, otherwiſe than from the Figure of its Path, in *Prop.* 28.

PROPOSITION XLIII.

THE Nodes and Apſides of the Orbits and Planets are at reſt.

For, by *Prop.* 40. every Planet revolves in an immoveable Plane, and the common Interſection of any two Planes continues immoveable: But the Interſection of the Plane of any Planet, with the Plane of the Earth's Orbit, is the Line of the Nodes of that Planet. The Nodes therefore of that, and in like manner of all the Planets, are at reſt. Beſides, every Planet deſcribes, in a Plane that is at reſt, always the ſame Ellipſe, which therefore is at reſt, (by *Prop.* 34.) and conſequently its Apſides are at reſt. Which was to be demonſtrated.

COROL-

COROLLARY.

e office of every Fixt Star is like Sun, (as far as Men are capable of namely, to have ſeveral leſſer Bodies ay Planets revolving round its vaſt other affections likewiſe will be ſi- that its Motion, like the Sun's, will inſenſible; which is farther evident nſtant and always the ſame mutual the Fixt Stars. Conſequently the the Nodes and of the Apſides of the Orbits, in reſpect to the Fixed Stars, he ſame.

SCHOLIUM.

poſition is true, if the Centripetal e Planets towards the Sun be only as has hitherto been done: But if Actions of the Planets and Comets nother, be taken into the conſidera- will be a little otherwiſe; as ſhall its proper place. But the Effects Actions, by reaſon of the ſmall- the Forces that produce them, are and as it were none at all, and there- neglected in this place. Nay, the the Nodes and Apſides of the Pri- s are ſo ſmall, that no notice at all aken of them by very conſiderable s, in their Calculations, and the Apſides look'd upon to be at reſt.

PROPOSITION XLIV.

Planets and Comets move in the Hea- with the greateſt Freedom; and con- r Motion may be preſerved for a very long

Prop. 39. (where we determined the Centripetal Force, whereby a Body is urged, that revolves in the Perimeter of an Ellipſe, or any other Conic Section, having the Center to which the Force tends for a Focus,) we ſuppoſed both the Motions, of which the Motion in an Elliptic, Parabolic, or Hyperbolic Curve is compounded, to be perfectly free, and having nothing to hinder or reſiſt it. Since therefore the Planets deſcribe in their Motion, perfect Ellipſes, whoſe common Focus is the Sun, and continue their revolutions in the ſame Ellipſes; 'tis manifeſt from the converſe, that the Motions of the Planets are intirely free from all reſiſtance; that is, that the Planets move in the Heavens very freely. And if the Planets move freely in the Heavens, without any reſiſtance, the Comets will do ſo too; becauſe they are moved along the regions of the Planets, during their being viſible to the Inhabitants of the Earth, in Orbits that are either true Ellipſes or ſpecies of Ellipſes; namely, Parabola's or Hyperbola's, having the Sun in the Focus; to the deſcription of which the ſame freedom from all reſiſtance is neceſſary. And that the reſiſtance of the medium in the regions of the Solar Syſteme, beyond the Planets, is not greater than in the Planetary regions, is very probable. For if there were the leaſt ſenſible reſiſtance in the Heavens, the ſame Orbits could not be deſcribed by the Planets carried about the Sun, like ſo many Projectiles, as have been demonſtrated to agree to them, moving freely: But, juſt as in our Air, the way of a Projectile is found to be very different from a Parabola, which it would deſcribe, if there were no reſiſtance; ſo the ways of the Planets at leaſt after ſeveral revolutions, would be found to be very different from Ellipſes; and the Ellipſes the ſame, for Magnitude and Poſition, with ſuch as are to be de-

y Planets, moved in free Spaces,
o exactly agree with the Places ob-
rmer times, as well as in the preſent
find by a due Calculation they do.
tly there is no reſiſtance at all: But
and Comets move in the Heavens
and therefore their Motion may be
ſo as to endure for a very long time.
to be demonſtrated.

PROPOSITION XLV.

econdary Planets move about the Primary
ling to this Law, that beſides the accele-
of the Primary, the Motion of each is
of an equable one, along the Tangent to
nd of another tending to the Center of
in which the accelerative Force is reci-
portional to the Square of the diſtance
ter.

ſide all the accelerating Force where-
ary is urged, (as was done in *Prop.* 20.)
dary Planet (by *Prop.* 37.) revolves a-
mary in the Perimeter of an Ellipſe,
ocus in the Center of the Primary,
by *Prop.* 20.) the Centripetal Force
which it is drawn off from its rectili-
n: (For the deformity of the Ellipſe
n the Motion of the Apſides, or from
elſe, is ſo ſmall that it may well be
in this place:) And therefore (by
he Law of the Centripetal Force
the Center of the Primary, is, that
ocally proportional to the Square of
e from the Center: For by this Force
Body drawn off from a Rectilineal
a Tangent, ariſing from its *Vis inſita*,
n an Elliptic Orbit. But theſe two
he one equable in the Tangent, the
other

other tending towards the Center of the Primary, and increaſing as the Square of the diſtance of the Satellite from it leſſens,) being compounded, make the Secondary Planet to deſcribe an Elliptic Orbit like a Projectile.

SECTION VII.

The Planets and Comets are kept in their Orbits by Gravity, which is propagated thro' the whole Solar Syſteme according to the ſame Law.

PROPOSITION XLVI.

THE Force whereby the Moon tends to the Center of the Earth, is the ſame with the Force of Gravity, whereby all Bodies about the Earth tend to the ſame Center.

Let *R A E* [*Fig.* 43.] repreſent the Earth, *T* its Center, *V L* the Orbit of the Moon, *LC*, a part of it run by the Moon in the ſpace of a Minute. Becauſe (by *Prop.* 15.) the Moon finiſhes its Courſe or whole Circle in 27 days, 7 hours, 43 minutes; that is, 39343 minutes of Time; therefore *L C* is $\frac{1}{39343}$ of the whole periphery, or 33 Seconds of a degree. Now the Circumference of the Earth (according to the late menſuration of Mr. *Picart*) is 123249600 *Paris* Feet; conſequently its Semidiameter *TA* is 19615800 Feet: wherefore *TL* the Semidiameter of the Moon's Orbit is 1176948000 Feet; or ſixty times *TA*, (by *Prop.* 15.) And the verſed Sine *L D* of the Arc *L C*, of 33″, and *B C*, which is equal to it, is 15 $\frac{1}{15}$ Feet, nearly. The Force therefore, whereby the Moon tends toward the Center of the Earth (as by which it is drawn

rom its Rectilineal Motion along the
B, and retained in its Orbit,) is ſuch,
els the Moon towards the Center of
about 15$\frac{1}{12}$ *Paris* Feet in a Minute.
ceding Prop. this Force in the Moon
, as the Square of the diſtance
enter of the Earth is leſſen'd; if
deſcend to the Surface of the Earth,
quare of the diſtance from the Cen-
ſhed 60 × 60 times, it will be increa-
times, and therefore 'twould impel
laced on the Surface of the Earth,
5$\frac{1}{2}$ Feet, towards the Center in a
. But Gravity is that Force, which
Body placed on the Surface of the
ength of 15$\frac{1}{2}$ *Paris* Feet towards the
the ſpace of a ſecond of time, as
has determined by very accurate
, in *Prop.* 25. *Part* 4. of his *Ho-*
Therefore the ſame Force of Gra-
ſpace of a minute, impells a Body
$\frac{1}{12}$ Feet towards the Center; be-
ces run by a heavy Body, in its
the Squares of the Times. Since
Force, whereby the Moon is kept
bit, and the Force of Gravity, per-
and equal things, and would pro-
ſame Effects, in the ſame Circum-
end toward the ſame point, (name-
er of the Earth,) they are the ſame
is, the Force whereby the Moon is
m its Rectilineal Motion, and kept
is the ſame Force with that we call
hich was to be demonſtrated.

of the Moon, a little greater than
he Semidiameter of the Earth,
preceding Calculation better, as
number likewiſe is more agreeable

to Aſtronomical accounts: Though we have thought it ſufficient to give the meaſure in round Numbers.

SCHOLIUM.

After the ſame manner as the Moon revolves about the Earth, any other heavy Body projected with a Force ſtrong enough, from a given Point without the Surface of the Earth, according to the direction of an Horizontal Right line, would deſcribe an Orbit, and compleat a revolution, like a Planet, without touching the Earth; and that continually. Let the Circle *ABCD* [*Fig.* 44.] repreſent the Earth, *T* its Center. Take any Point without the Surface of the Earth, as *G*, from which, let any heavy Body be projected according to the direction of the Line *GH*, perpendicular to the Line *TG*. 'Tis evident, that if the Body be projected with little or no Force, (that is, if it be let fall freely) it will fall down to the Point *A*, directly under *G*; but if with ſome Force, it will reach the Earth at the Point *B*, diſtant from *A*, towards *H*; if with a greater, the Body projected will meet the Earth at *C*, beyond *B*. If the projectile Force be increaſed, ſo as that the Body does not reach the Earth, till it has got beyond the Point *E*, oppoſite to the Point *A*, then aſcending again towards *G*, it will compleat the Ellipſe *GKMF*; and, if not hindered, will perpetually deſcribe it, and ſo become a Planet. If the Projectile in the Point *M* of its Way, oppoſite to *G*, be leſs diſtant from the Earth, than at *G*; then the Center of the Earth, *T*, is the Focus of the Ellipſe, more remote from *G*. If the projectile Force increaſed ſtill, till the Points *M* and *G* are equally diſtant from *T*, the Path or Orbit will be a Circle; and, ſtill augmenting the projectile Force, (all other things continu-me,) it will become an Ellipſe, in the Focus that is neareſt *G*, and the s will be removed from *T*, proporti- augment of the projectile Force, till it turns into a Parabola, and then in- rbola. The ſame things will happen not perpendicular to *TG*, excepting caſe, *G* will not be the principal he Orbit. The higher the Point *G* Earth, the leſs Force is needful to jectile into a Planet; and the lower eater: So that if the Moon ſhould with the ſame Celerity as it is car- its Orbit, from a height above the a mile, like a common Projectile, et and ſtrike the Earth before it had tance of 15 miles. For the Arc oon runs in $20''$, is leſs than 15 miles, Body in $20''$ near the Earth in fal- $\times 20 \times 15\frac{1}{12}$ or 6033 Feet; that is, mile; therefore the Moon would rth, before it could be projected to of 15 miles. But if, the action continuing the ſame, and the Earth nnihilated, all Projectiles (like the ld deſcribe Ellipſes, or ſome other ns, whoſe common Focus would which now is the Center of the

ROPOSITION XLVII.

ondary Planets of Jupiter gravitate to- Jupiter, the Satellites of Saturn towards oth the Primary and Secondary Planets ards the Sun; and all of them are om their Rectilineal Motions, and kept by the Force of their Gravity.

of that Simplicity of Nature which

is obſervable in all her Works, more Cauſes ought not to be admitted, than are ſufficient to explain the Phænomena of Natural things; and the natural Effects of the ſame kind are to be looked upon as having the ſame Cauſes: Since therefore the revolution of the Medicean Stars about Jupiter, and Saturn's Attendants about Saturn, and of the Primary Planets about the Sun, are Phænomena of the ſame kind with the revolution of the Moon about the Earth; their Cauſes will be the ſame. For the Forces whereby the Secondaries of Jupiter and Saturn, and the Primary Planets, are drawn off from their Rectilineal Motions, and kept in their Orbits, reſpect the Centers of Jupiter, Saturn, and the Sun, reſpectively, after the ſame manner, as the Force, whereby the Moon is kept in its Orbit, reſpects the Center of the Earth; and the Forces whereby they are urged, increaſe and decreaſe by the ſame Law, according to their different Diſtances from the Centers of Jupiter, Saturn, and the Sun, reſpectively, as the Force acting upon the Moon, placed in various Diſtances from the Center of the Earth; as has been demonſtrated before at large. Conſequently, ſince this Force in the Moon has been ſhewn (in *Prop.* preced.) to be the ſame with the Force, which we call Gravity; that Force, whereby the abovemention'd Planets, both Primary and Secondary, tend reſpectively towards the Sun, and their Primary Planets, muſt be the ſame with the Force of Gravity. Beſides, all the Secondaries gravitate alſo towards the Sun, and their Gravity towards the Sun, is the accelerative Force that is common to each Secondary and its Primary; concerning which, ſee *Prop.* 20. and 45. Which was to be demonſtrated.

COROLLARY.

is evident, that there is ſuch a thing
towards the Sun and all the Primary
or ſince the Moon and all Terreſtrial
itate towards the Earth, and the Sa-
upiter and Saturn, towards Jupiter
reſpectively; and ſince the three
ary Planets, Mercury, Venus and
odies of the ſame kind with Saturn,
the Earth, (which alſo is evident
Gravity towards the Sun, before de-
) 'tis certain, from the ſimilitude of
he like natural Effects before laid
ſtabliſhed, that there is a Gravity al-
them, whoſe accelerative Force is
proportional to the Square of the
n the Center. And becauſe to any
on, there is a Re-action, equal and
e Sun likewiſe muſt gravitate to-
Planets, both Primary and Secon-
es, becauſe it is certain, from Ex-
ccurately made, that all kinds of
he ſame Diſtance, and in equal
qual Spaces in their Falls towards
nd that the quantity of Motion of
ies, equally accelerated, is as the
Matter. All Bodies, of what Na-
in equal Diſtances, gravitate to-
rth proportionally to their quan-
ter. And ſince no doubt but the
avity towards the Sun and Planets,
towards the Earth, (which is ſuf-
ifeſt from what has been already
rom hence, that the Primary Pla-
Satellites, that is, Bodies very un-
d toward the Sun with an equal
Force, juſt as Bodies unequal in
nd Denſity fall with equal Celeri-

ties towards the Earth ;) 'tis evident that all Bodies gravitate towards the Sun, and each of the Planets; and their Gravities towards any one, at equal Diſtances from its Center, are proportional to the quantities of Matter in each. And becauſe all Action is mutual, the Sun and Planets gravitate according to the ſame Law towards all Bodies, and conſequently all Bodies towards one another.

PROPOSITION XLVIII.

THE Force or Efficacy of any Virtue propagated from a Center, or to a Center, in Right lines every way round about, in different places, is reciprocally proportional to the Square of the Diſtance of the place from the Center.

Let *S*, [*Fig.* 45.] be the Center, from which, or to which, the Virtue is propagated, round it deſcribe two Spherical Surfaces *TE*, *MA*, at any diſtances, *ST*, *SM*. I ſay the Force or Efficacy of the Virtue in *T*, is to the Force of the ſame in *M*, as *SM*q, to *ST*q.

The ſame Virtue equally diffuſed and ſpread thro' a double Space, will be twice as little in any given part: And if it be diffuſed thro' a triple Space, it will be three times as little: And, univerſally, the Efficacy of the Virtue is reciprocally as the Space into which it is diffuſed; becauſe 'tis directly as the conſtipation of the Virtue. But any Virtue, which in the diſtance *ST* from the Center, is equally diffuſed thro' the Spherical Surface *TE*, in the diſtance *SM*, is diffuſed thro' the Spherical Surface *MA*, after the like manner. The Efficacy therefore of that Virtue, in the diſtance *ST*, is to the Efficacy of the ſame in the diſtance *SM*, as the Spherical Superficies *MA*, is to the Sphe-

Superficies *TE*; that is, as SM^q
Which was to be demonſtrated.

kind are the Virtues or Effects of
eat, *&c.* and alſo of Gravity, as is
om what has been already demonſtra-
ſome other Virtues, as thoſe of the
and Amber, are of another kind.

PROPOSITION XLIX.

*accelerative Gravities towards different
es, in equal diſtances, are as the Bodies
toward which they tend.*

e be any two Bodies *A* and *B* [*Fig.*46.]
es *A* and *B*; they (by *Corol. Prop.* 47.)
ate towards one another: And ſince
on is equal and contrary to the Action,
y or Motive Force of the Body *A*
, is equal to the Gravity or Motive
B towards *A*. But the Motive Force
om the Quantity of Matter or Maſs
to the Accelerative Force, juſt as the
of Motion from the Quantity of Mat-
into the Celerity: The Quantity of
erefore, or Maſs of the Body *A*, drawn
ccelerative Force towards *B*, is equal to
of the Body *B* drawn into its Accele-
ce towards *A*. Conſequently the Ac-
Force of the Body *A* towards *B*, is
ccelerative Force of the Body *B* to-
as the Maſs of the Body *B*, to the
he Body *A*: But the Forces (treated
are the Gravities, and the Maſſes of
s are the Quantities of Matter or the
emſelves: And therefore the Accelera-
vity of the Body *A* towards *B*, is to
lerative Gravity of the Body *B* to-
as the Body *B*, to the Body *A*. But
t has been demonſtrated above,) all

Bodies placed in an equal diſtance from the Body *B*, with that of the Body *A*, have an accelerative Gravity towards *B*, equal to that of *A*; and all Bodies placed in a diſtance from the Body *A*, equal to that of *B*, have an Accelerative Gravity towards *A*, equal to that of *B*; and the diſtance of the Body *A* from the Body *B*, is equal to the diſtance of the Body *B* from the Body *A*, and the Diſtances and Bodies are taken at pleaſure. Therefore the Accelerative Gravities towards different Bodies, in equal diſtances, are as the Bodies themſelves. Which was to be demonſtrated.

COROLLARY.

From hence it follows, that if Gravity be conſider'd as the Attractive Force of the Body towards which it is directed, propagated as in the preceding Propoſition; the Abſolute Forces of the attracting Bodies are as the Bodies to which they belong. For the Abſolute attractive Force of the Body *A*, is to the Abſolute attractive Force of the Body *B*, as the Accelerative attraction of all Bodies towards *A*, is to the Accelerative attraction of all Bodies towards *B*, in equal diſtances. But, in equal diſtances, the Accelerative attraction of all Bodies is the ſame with the Accelerative attraction of any Body, and theſe Accelerative attractions, by this Propoſition, are as the Bodies toward which they tend; that is, as the Attracting Bodies themſelves: And therefore, *ex æquo*, the Abſolute Forces of Attracting Bodies are as the Bodies themſelves, to which they belong. And hence again, and from what goes before, (by *Prop.* 23. *El.* 6.) we may conclude that the Accelerative Force of the Body *A*, gravitating towards *B*, is to the Accelerative Force of the Body *C*, gravitating towards *D*,

ompounded of the ratio of the Body
dy *D*, and of the duplicate ratio of
e between *C* and *D* to the diſtance
and *B*: And that the ratio of the
the Body *A* to the weight of the
compounded of the abovemention'd
ratio, together with the ratio of the
the Body C.

SECTION VIII.

lotion of Bodies attracting one , and the Affections; which are ed to the Syſtem of the Sun and Planets.

PROPOSITION I.

ies attracting one the other, and revolving
one another, deſcribe ſimilar Figures, both
elves, and about the common Center of

9. and the following ones, we have
e Motions of Bodies attracted to an
Center; for it is moſt proper to
the moſt ſimple Caſes. But becauſe
ions are made to Bodies, and Bodies
another; as has been before demon-
hen a Body revolves about another,
ng Body it ſelf can't be at reſt; but
acted upon by a mutual Attraction,
about the common Center of Gra-
if there were more Bodies that mu-
acted one another, they would ſo
ng themſelves, that the common
Gravity might be either at reſt, or
rm direct Motion. For it is well
Geometricians, that the common
Gravity does not change its ſtate of
Motion

Motion or Reſt, by reaſon the action of Bodies upon one another; as has been demonſtrated by Mr. *Wren* and Mr. *Hugens.*

Method therefore requires us to conſider the Motions of Bodies, as was ſaid, attracting one another; namely, the Orbits they deſcribe, and the other generals, wherein a Syſtem of Bodies acting upon one another mutually differs from the Syſtem ſuppoſed above; and that in the firſt place univerſally, whatever the Law of mutual Attraction be.

Let *S* and *P* [*Fig.* 47.] be the two attracting Bodies, which, revolving mutually about one another, deſcribe the Curves *Sſ*, *Pp*, while their Center of Gravity *C*, is either at reſt, or (which does not diſturb the Demonſtration,) moves uniformly in a Right line: I ſay that the four Figures, namely, *SſC*, *PpC*, that which *S* deſcribes about *P*, conſider'd as immovable, and that which *P* deſcribes about *S*, conſider'd after the ſame manner, are ſimilar.

Becauſe *S* and *P* continuing the ſame, the ratio of *CP* to *CS*, during the Motion, will likewiſe continue the ſame; becauſe, from the nature of the Center of Gravity, it is the ſame with the reſpect to the Bodies *S* and *P*: And ſince the Angles *PCp*, *SCſ*, are every where equal, (becauſe *C* is in the Right line connecting the Bodies,) the Figures *SſC*, *PpC* will be ſimilar. For the Figures *PpC*, *SſC*, are ſimilar, which the Right line *pCſ*, deſcribes with an Angular Motion about the Point *C*, conſider'd as at reſt, provided every moment, the ratio of *Cp* to *Cſ*, be the ſame with that of *CP* to *CS*.

Again, becauſe the ratio between *SC* and *Cp* is always given, by compounding, the ratio of *ſp* to *Cp* is likewiſe given, and the Line *pCſ* continues a Right one: And therefore (by what

has

id) the Right line C*p* about the Point
Right line ſ*p* about the Point ſ, con-
t reſt, deſcribe ſimilar Figures. Be-
igures whereof one is deſcribed by *p*
nſidered as at reſt, the other deſcribed
p, conſidered likewiſe as at reſt, are
equal, (as is demonſtrated in Optics,)
is the ſame in Magnitude and Poſi-
p ſ. Wherefore the four Figures pro-
imilar. *Q. E. D.*

PROPOSITION LI.

odies S *and* P *attract one another by any*
and in the mean while revolve about the
ter of Gravity C; *I ſay, that a Figure*
ibed, by the ſame Forces about either of
onſider'd as at reſt, ſimilar and equal to
which the Bodies ſo moved deſcribe about

the Orbits *PR*, *SQ* [*Fig.* 48.] to be
firſt about the Point *C* at reſt, and
ſ and *p* any places at pleaſure, to which
S and *P* arrive at the ſame time. Let
line ſ C *p* be ſuppos'd to move parallel
till the point ſ coincide with *S*, and
done in all the correſponding points
rves *SQ*, *PR*: 'Tis evident, that
mes to *S*, the other extreme of the
ſ*p*, thus moved parallel to it ſelf, will
the point ϖ, in the Curve *P*ϖ, which
P would deſcribe about the point *S*, if
t reſt) *P* ſhould revolve about it, be-
ted by the ſame Force as the Bodies did
tually attract each other: For the pa-
tion of the Right line ſ*p*, makes no
in the Forces, with which the Bodies
ch other. But the ſame *P*ϖ is the Line
eſcribes about *S*, conſidered as at reſt:

For thus it really continues at reſt; and the length and inclination of the Right line $S\varpi$ to SP, is the ſame of that with $ſp$. Since therefore theſe Figures are congruous, they are equal and ſimilar. And what has been demonſtrated concerning the Figure, which P deſcribes about S, is true of the Figure, that S deſcribes about P, looked upon as at reſt, namely, that it is equal and ſimilar. But if the Center of Gravity C go forward uniformly, by ſuperinducing upon the Syſteme of the Bodies S and P, an equal and contrary Motion, all things will be reduced to the former Caſe: But there is no change made by this uniform Motion along parallel Right lines, in the Forces of the Bodies S and P. Therefore the Propoſition is certain in both Caſes. Which was to be demonſtrated.

COROLLARY 1.

Two Bodies mutually attracting one another, by any Forces, and revolving about the Center of Gravity, deſcribe, by Radii drawn to that Center and to one another, Areas that are proportional to the Times.

COROLLARY 2.

Becauſe a Body revolving about another immovable one, and attracted to it by a Force reciprocally proportional to the Square of the Diſtance, would deſcribe (by *Prop.* 39.) an Ellipſe, or ſome other Conic Section, one of whoſe Foci coincides with the Center of the Body that is at reſt; it follows, from this Propoſition, that two Bodies attracting one another, by Forces reciprocally proportional to the Square of the Diſtance, deſcribe, both about the common Center of Gravity, and about one another, Conic Sections, having their Foci in the Center, about which they are deſcribed. And conſequently, if ſuch Figures are deſcribed, the Centripetal

rces are reciprocally proportional to
s of the Diſtance.

PROPOSITION LII.

odies attracting one another by any Forces
er, mutually revolve about one another,
will be the ſame, as if they did not at-
other, but both were attracted with the
by a third Body placed in the common
ravity. And the Law of the attractive
be the ſame, in reſpect to the diſtance of
rom that common Center, and in reſpect to
ſtance between the Bodies.

Forces, whereby the Body P [*Fig.*47.]
are directed to any point whatever,
t line PS produced towards S. And
nner, the Forces of the Body S, are
any point whatever, of the ſame
uc'd towards P. But C is the only
he Right line SP, produced both
continues at reſt, in every ſituation
ies; and therefore C is that point, to
Bodies P and S tend, with the ſame
they attract one another.
becauſe the Bodies that revolve are
the ratio of the diſtance of either
ies from the common Center, to the
the ſame Body from the other. And
ly the ratio of any Power of the
the one, to the like Power of the
the other, is given; as alſo the ra-
Quantity, which is any way com-
f one diſtance and any given Quan-
nother Quantity, which is in the like
mpounded of the other diſtance and
iven Quantities, having that given ra-
Diſtances to the former. Wherefore,
ce, whereby one Body is attracted by
the

the other be directly or inverſly as the diſtance of the Bodies from one another, or as any Power of this diſtance; or laſtly, as any Quantity compounded any how of this diſtance and the Quantities given; this ſame Force, whereby the ſame Body is attracted to the common Center of Gravity, will be in like manner directly or reciprocally as the diſtance of the attracted Body from that common Center, or as the ſame Power of that diſtance, or laſtly, as a Quantity compounded after the ſame manner of this diſtance and of the analogous given Quantities: And therefore there will be the ſame Law of the attractive Force in regard of the diſtance from the Center of Gravity, as there is in regard of the diſtances of the Bodies. Which was to be demonſtrated.

PROPOSITION LIII.

THE *Periodic Time of two Bodies,* S *and* P, [Fig. 49.] *revolving about the common Center of Gravity* C, *is to the Periodic Time of any other Body* P, *revolving about the other immovable point* S, *(attracting with the ſame Forces,) in the ſubduplicate ratio of the Body* S, *to the aggregate of the Bodies* S *and* P.

Let the Body *p*, equal and ſimilar to *P*, be imagined to deſcribe the Orbit *p d r*, about *ſ*, at reſt, and ſimilar and equal to *S*. The Figures *C P D R*, *ſ p d r*, (by Prop. 50.) are ſimilar; and the Forces whereby the Bodies being in any Points (*D* and *d*) ſimilarly poſited, tend toward the Centers *C* and *ſ*, are equal; becauſe, (by *Prop.* 51.) the Force whereby the Body *D* tends to *E*, is equal to the Force whereby the Body *d* tends to *ſ*. And, by the preceding Propoſition, the Body *D* tends to the *C*, with the ſame Force as to *E*. And therefore, (by *Corol.*

. 26, and *Schol. Prop.* 27.) the Square
dic Time of the Body *P*, moving
PDR, is to the Square of the
me of the Body *p*, in the Orbit
Right line *CP* to the homologous
p; that is, as *CP* to *SP*: that is,
ature of a Center of Gravity) as the
+ *P*. Wherefore the Periodic Time
y *P* in the Orbit *PDR*, (or of the
the Orbit *SEL*) is to the Periodic
Body *p* in the Orbit *pdr*, in the
ratio of the Body *S*, to the aggre-
Bodies *S* and *P*. Which was to be
d.

PROPOSITION LIV.

dies S *and* P, *attracting one another by*
eciprocally proportional to the Square of
s, revolve about the common Center of
ſay, that the greater Axe of the Ellipſe,
Body P *deſcribes by this motion about the*
he manner deſcribed in Prop. 50.) *is to*
Axe of the Ellipſe, which the ſame Body
cribe about the other S *at reſt*, (*as in*
in the ſame Periodic Time, in the ſubtri-
f the ſum of the Bodies S and P, *to*
Body S.

lipſes deſcribed about the movable
nt Body *S*, by the Body *P*, were e-
eriodic Times in them (by *Prop.* 51,
be in the ſubduplicate ratio of the
the aggregate of the Bodies *S* and *P*.
e laſt Ellipſe to be changed, till the
ime of the Body *P* (deſcribing this
ut the quieſcent Body *S*,) become e-
Periodic Time of the Body *P* revol-
the Body *S*; (as we ſuppoſe in this
;) that is, till the Periodic Time in
this

this being leſſen'd may be in the ſaid ſubduplicate ratio of the Body *S*, to the ſum of the Bodies *S* and *P*. But becauſe (by *Prop.* 40.) in the preſent caſe, the Squares of the Periodic Times are as the Cubes of the Tranſverſe Axes, the Periodic Times themſelves will be in the ſeſquiplicate ratio of the ſaid greater Axes. The greater Axes therefore of the latter Ellipſe, being leſſen'd, is in a ratio, whoſe ſeſquiplicate is the ſubduplicate ratio of *P* to $S+P$; that is, in the ſubtriplicate ratio of *S* to $S+P$: (For $\frac{3}{2}$ of $\frac{1}{3}=\frac{1}{2}$:) and conſequently is to the unvaried Axe (equal to this before it was leſſen'd) of the other Ellipſe, deſcribed about the Body *S*, in the ſubtriplicate ratio of *S*, to $S+P$. And therefore inverſly, the greater Axe of the Ellipſe deſcribed about the movable Body *S*, is to the greater Axe of the Ellipſe deſcribed about the quieſcent Body *S*, in the ſame or an equal Time, in the ſubtriplicate ratio of the ſum of the Bodies $S+P$, to the quieſcent Body *S*. Which was to be demonſtrated.

PROPOSITION LV.

THE Law of Attraction being the ſame as before, I ſay the Orbit PR, *(Fig. 50.) of the Body* P, *comes nearer to an Ellipſe, whoſe Focus is* C, *the common Center of Gravity of the Bodies* S *and* M, *than to an Ellipſe, whoſe Focus is the Center of it* S; *and that the Areas deſcribed by Radii drawn to* C, *are more proportional to the Times, than the Areas deſcribed by Radii drawn to the Centre of it* S.

For the attractions of the Body *P* toward *S* and *M*, compoſe its abſolute Gravity, which is directed more towards *C*, the common Center of their Gravity, than towards the greater Body it ſelf *S*, and which is more nearly reciprocally proportional to the ſquare of the diſtance *PC*,

to the ſquare of the diſtance PS:
ſt would then hold preciſely, when
nothing; and the ſame thing would
M, if it ſhould increaſe till it im-
ped *S*. 'Tis certain therefore, that
in the preſent Caſe, are directed to
lying betwixt *S* and *M*, and nearer
of them, as is the common Cen-
r Gravity *C*; which alſo is that
the Natural Forces of any kind
odies placed round about it, are
concenter'd: for the aggregate
eavy Bodies is moſt juſtly looked
here, where the common Center of
y is; as it is well known by Philo-
hat therefore was propoſed is evi-
tain. Which was to be demonſtra-

PROPOSITION LVI.

*he ſame Laws of the Attraction, I ſay the
Body* P *deſcribes Areas more nearly pro-
Times, about the common Center of Gra-
e internal Bodies* S *and* M, *by Radii
Center, and an Orbit coming up nearer
an Ellipſe, having a Focus in that Cen-
internal and biggeſt Body* S *be agita-
ttractions, juſt as the others are, than
uld be at reſt, not being attracted at
more or much leſs attracted, or much
leſs agitated.*

p. preced. it is evident, that the
ich the external Body *P* is urged,
Center of Gravity of the internal
M. If that Center coincided with
Center, and the common Center
the three Bodies *S*, *M*, and *P*,
P on one ſide, and the common

Center of *S* and *M* on the other, would deſcribe accurate Ellipſes about the common Center of all three, being at reſt, and as it were the common Focus, by *Corol.* 2. *Prop.* 51. And therefore the perturbation of the ſaid Motions (in the Ellipſes) will be very ſmall, when the common Center of Gravity of three Bodies, *S*, *M*, and *P*, is at reſt: And it is at reſt, if the Body *S* be agitated after the ſame manner as the others are; as was ſaid before. And therefore the perturbation will be the leaſt, when *S* is attracted by the ſame Law as the others are. Which was to be demonſtrated.

COROLLARY.

After the ſame manner, if more leſſer Bodies revolve about the greateſt, it may be inferred, that the Orbits deſcribed come nearer to Ellipſes, and the deſcription of the Areas more equable, if all the Bodies mutually attract one another with accelerative Forces, which are as their Maſſes directly and the Squares of their diſtances inverſely, (for this is to be agitated, juſt as the others are, ſuppoſing the Law of the attractions to be the ſame with that in the Propoſition;) and the Focus of each Orbit be placed in the common Center of Gravity of all the internal Bodies, than if the greateſt Body were at reſt, and made the common Focus of all the Orbits.

PROPOSITION LVII.

SEveral Bodies, whoſe Accelerative Forces are to one another reciprocally as the Squares of the diſtances from their Centers, may be moved in Ellipſes about a great one, and deſcribe Area's nearly proportional to the Times, by Radii drawn to that Body.

Let

ſeveral leſſer Bodies P, p, π, [*Fig.*51.]
ed to revolve about a great Body, as
ecauſe the common Center of Gravi-
n all does not change its ſtate of Mo-
eſt, by the mutual actions of the Bo-
ne another; (as was explain'd before,)
ppoſe them free from all other exter-
on or impediment; that Center may
If now we ſuppoſe the leſſer Bodies
be ſo little in compariſon to the
S, as that it is never ſenſibly diſtant
id common Center of Gravity, then
ne *S* will be likewiſe very nearly
t the leſſer Bodies will revolve about
in Ellipſes, having the great Body
non Focus; and therefore they will
eas, by Radii drawn to it, proporti-
Times; bating the Errors ariſing
the digreſſion of the greateſt Body
mmon Center of Gravity, or from
attractions of the leſſer Bodies upon
. But theſe leſſer Bodies may be
that digreſſion or thoſe mutual at-
leſs than any given ones. And
veral Bodies may move about a
n Ellipſes, and by Radii drawn to
eſcribe Areas proportional to the
s that the Errors committed (or
them from being exact) may be
y aſſignable; that is, indefinitely
h was to be demonſtrated.

PROPOSITION LVIII.

tual action of the Sun and any primary
the Planet deſcribe an Ellipſe, whoſe
mmon Center of Gravity of the Sun and
t. But by the action of the primary
ne another, the Orbit of each is nearly

an Ellipſe, whoſe Focus is the common Center of Gravity of the Sun, and all the inferior Planets.

If the primary Planets gravitated towards the Sun, and not the Sun towards the Planets, each primary Planet would deſcribe an Ellipſe about the Sun's Center at reſt, as a Focus; as was demonſtrated in *Prop.* 42. And if the Maſſes of the Planets were inſenſible in compariſon to the Maſs of the Sun, the ſame would hold very nearly, by the preceding: And that this is almoſt the caſe of the Solar Syſtem, is evident from *Prop.* 34. But becauſe (by *Corol. Prop.* 47.) not only the Sun gravitates towards a Planet, but a Planet alſo towards the Sun, and beſides a Planet is not a Maſs intirely inſenſible in regard of the Sun; each Planet (by *Corol.* 2. *Prop.* 51.) deſcribes an Ellipſe, whoſe Focus is the common Center of Gravity of the Sun and the ſaid Planet: And (by *Prop.* 54.) the greater Axe of this Ellipſe, exceeds the greater Axe of the Ellipſe which the Planet would deſcribe about the Sun being at reſt, in the ſame or an equal Time, in the ſubtriplicate ratio of the ſums of the Maſſes of the Sun and Planet, to the Maſs of the Sun.

Beſides, becauſe (by the abovementioned *Corol. Prop.* 47.) the Planets gravitate towards one another, and the Sun is agitated by the ſame attractions as the Planets; Mercury, (by *Prop.* 50.) will deſcribe very nearly an Ellipſe having the Focus in the Center of the Sun; Venus, an Ellipſe whoſe Focus is in the common Center of Gravity of the Sun and Mercury, and ſo on; each primary Planet will deſcribe an Ellipſe, whoſe Focus is in the common Center of Gravity of the Sun and all the inferior Planets. And after this manner, the actions of the primary Planets upon one another, are conſidered, and reduced to a ſtandard.

SECTION IX.

lotion of a Syſtem of Bodies, revolves about another Body; all is applied to the Syſtem of the Planets Primary and Secondary.

PROPOSITION LIX.

mmon Center of Gravity of a Syſtem of
more heavy Bodies, revolving about one
llipſes, may move in an Ellipſe, or any
ction, about another greater placed in the
Ellipſe, and may deſcribe, by Radii
Focus, Areas proportional to the Times,
ear.

ppoſe a Syſtem of two Bodies re-
ually about one another (as in *Cor.*2.
d another Syſtem of leſſer Bodies
olving (after the manner deſcribed
about a great one P, [*Fig.* 52.] to
rmly in a Right line, which is evi-
le: (that is, for its common Center
o move along Right lines, for in-
and in the mean while to be urged
the Force of another great Body
at a great diſtance, and forced to
Curve *PH*, being drawn off from
ne *PR*. And becauſe equal Acce-
es impreſſed along parallel Right
lter the ſituation of the Bodies to
but cauſe the whole Syſtem, pre-
Motions in regard of one another,
r'd at the ſame time together: 'Tis
here will ariſe no alteration in the
e Bodies mutually attracted, from

the attractions, towards the greateſt Body *S*, but from the inequality of the Accelerative attractions, or from the Inclination of the Lines to one another, according to which the Attractions are made. Now the diſtance of the greateſt Body may be increaſed, till the differences of the Right lines drawn from it to the Bodies of the Syſtem in Motion (namely, the great one *P*, and the leſſer *L*, *l*, λ) in reſpect to the Lines themſelves thus drawn, and the Inclinations to one another, become leſs than any given ones. And ſince all the Accelerative attractions towards *S*, by the Hypotheſis of Gravity, are reciprocally as the Squares of the Diſtances; theſe (in this caſe) differ from equal Forces, by a difference which is leſs than any given one: And therefore the Motions of the parts of the Syſtem *P*, with its Bodies *L*, *l*, λ, will perſevere after the ſame manner as before, when the greateſt Body *S* did not attract that Syſtem; and the whole Syſtem attracted after the manner of one Body, will deſcribe by its Center of Gravity, (in which the whole Syſtem of the heavy Bodies *P* and *L*, *l*, λ, is ſuppoſed to be contracted and united, as in other Philoſophical caſes,) a Conic Section whoſe Focus is *S*; and by a Radius drawn to that greateſt Body, will deſcribe Areas proportional to the Times: And the errors which are over and above, may be leſſen'd at pleaſure, by increaſing the diſtance of the Body *S*. Therefore the Center of Gravity of a Syſtem of Bodies *P*, and *L*, *l*, λ, may move in an Ellipſe very nearly. *Q. E. D.*

COROLLARY I.

When a Syſtem of two Bodies revolving about one another, or of a great Body with leſſer Bodies revolving about it, is moved about a greater; the Motion

f the parts of the Syſtem among
will be ſo much the more diſturbed,
ater Body approaches nearer the ſaid
ecauſe the inclination of the Lines
n the greater Body to thoſe parts, is
d the inequality of the ratio likewiſe
nd, in one word, becauſe there is a
ceſs made from that caſe, wherein
perturbation at all; namely, when
is at an infinite diſtance.

COROLLARY 2.

ence, on the contrary, if the parts
m of the Body *P* and the leſſer re-
lies *L*, *l*, *λ*, move in Ellipſes or Cir-
t any conſiderable perturbation; 'tis
hat the ſame are only urged very
any Accelerative Forces tending to-
r Bodies, or that they are urged e-
along parallel Lines, indefinitely near.

PROPOSITION LX.

Body L. [*Fig.* 53.] *revolve about a great*
as in Corol. 2. *Prop.* 51.) *and the Syſtem*
revolve about a greater S, (*as in the*
nd in theſe three Bodies, if the Accele-
tions of any two, towards the third, be
s the Squares of the Diſtances from it;
oſed to ſhow in general the Errors com-
Body L, *together with the cauſes and*
em; namely, why the Body L *does not,*
drawn to T, *deſcribe Areas exactly pro-*
e Times, and run in the Perimeter of an
f whoſe Foci is T.

he Body *L* is ſometimes more, ſome-
iſtant from the Body *S*, let *SK* be
ſtance; which may likewiſe expreſs
ative attraction of the Body *L* to-
that mean diſtance. Let the place *L*

of the leaſt Body in its Orbit *B C G D*, be taken at pleaſure: And in *S L*, produced if need be, let the Right line *S A* be taken, ſo as that it be to *S K* in the duplicate ratio of *S K* to *S L*, which conſequently will expreſs the Accelerative attraction of the Body *L* towards *S*, when it is in the point *L*, thus aſſumed; becauſe, (by ſuppoſition,) the Accelerative Force in *L* is to the Accelerative Force in *K*, in the duplicate ratio of *S K* to *S L*; that is, (by conſtruction,) as *S A* to *S K*. Connect *T L*, and through *A* draw *A M* parallel to it, meeting *S T* in *M*.

The Attraction *S A* of the Body in *L* towards *S*, may be reſolved (as is well known,) into the Attractions *A M*, *M S*. Wherefore the Body *L* will be urged by a threefold Accelerative Force; one tending to *T*, and ariſing from the mutual attraction of the Bodies *L* and *T*; by this Force alone, the Body *L*, by a radius *L T* drawn to the Center of the Body *T*, would deſcribe Areas proportional to the Times, and an Ellipſe, whoſe other Focus is in the ſaid Center. The other Force, whereby *L* is urged, is expreſſive by *A M*, and is parallel to the direction of it; which becauſe it tends from *L* to *T*, adding the former Force, will with it compoſe another, whereby the Areas deſcribed, will ſtill be proportional to the Times, (by *Corol.* 1. *Prop.* 51.) But becauſe this Force, as *A M*, is not reciprocally proportional to the Squares of the diſtance *L T*, the Force compounded of this and the former, will not be ſuch; but will recede from that proportion, and that ſo much the more, as (other things being alike) this new Force added to the former, and repreſented by *A M*, is greater. Conſequently the Body *L*, which is urged by this compound Force, will not deſcribe the Perimeter of an Ellipſe, whoſe

Focus

; (for to this a Force tending to *T*,
ocally proportional to the Square of
e from it, is required by *Prop.* 39.)
auſe its Orbit to have ſome digreſſion
Ellipſe, whoſe Focus is *T*; and that
will be ſo much the greater, as this
l Force (whereby *L* is urged,) recedes
rce reciprocally proportional to the
the diſtance from *T*; that is, (as has
n,) the greater the ſuperadded Force
d by *AM*, is, in reſpect to the Force,
L is attracted to *T*, other circumſtan-
alike. Again, the third Force, where-
ly *L* is urged, which the Right line
ſents, acting according to the directi-
namely according to the direction of
el to *M S*) ſuperadded to the former
ill compound another, whoſe directi-
rom *L* towards *T*; but along a Right
ing from thence towards *F*, more or
ding to the ratio of this third Force
n of the former Forces. And there-
hole Force, compounded of all three,
is urged, ſince it does not tend to *T*,
o.) will make the Body *L* deſcribe
proportional to the Times, by the Ra-
And the digreſſion from that ſimilitude
os between the Areas deſcribed and
in which they were deſcribed, will
h the greater, as the ratio of this third
ng according to the direction of *L F*,
mer Forces, is greater. Beſides, the
ion of this third Force will induce
ty in the Elliptic Figure of the Orbit,
le account, both becauſe it is not di-
n *L* to *T*, and becauſe it is not reci-
roportional to the Square of the di-
nd the deformity is ſo much the great-

er, as, other things being alike, this Force *MS* is greater. In the Right line *ST*, (from *S* towards *T*) take *S N*, to *S K* in the duplicate ratio of *SK* to *S T*; the Accelerative attraction of the Body *T* towards *S*, will be expreſs'd by *SN.* If the Accelerative attractions *SM*, *SN*, be equal, (as in the 2d of theſe *Fig.*) they will not alter the ſituation of the Bodies *T* and *L* to one another, by acting upon them equally, and according to parallel Right lines: But the Bodies *S* and *T* approaching nearer to one another, the motions of *T* and *L*, in regard of one another, will be juſt the ſame as if theſe attractions were intirely abſent. But if the Accelerative attraction *S N*, be leſs than the Accelerative attraction *S M*, (as in the firſt of theſe *Fig.*) the former, and a part of the latter equal to it, will, (as was ſhewn in the preceding caſe,) reduce themſelves mutually to that State, as if both of them were abſent, and the difference of them, on the part of the Accelerative Force *S M*, expreſſed by *M N*, remain; which therefore diſturbs the proportion of the Areas and Times, and the Elliptic Figure of the Orbit, *G C B D*. But if *S N* exceed *S M*, (as in the 3d of theſe *Figures*) their interval *M N*, will denote that difference of the Accelerative Forces, whereby the proportion of the ſaid Areas deſcribed, and Times of deſcription, together with the Elliptic form of the Orbit *G C B D*, are diſturbed. And therefore by the attraction *N S*, is the third attraction *S M* always reduced to *M N*, which exerts its Force from *M* to *N*, or from *L* towards *F*, the former Forces being intirely unchanged. And theſe are the Forces that generate the Errors of the Body *L*. *Q. E. I.*

PRO-

PROPOSITION LXI.

ſame things being ſuppoſed, let it be pro-
l to give an account of ſuch Errors of a
L. [Fig. 54.] *revolving about a great*
in the ſame plane wherein T *revolves*
ater Body S, *ariſing from the inclination*
Planes; as alſo of the Cauſes and Reaſons

he demonſtration of the preceding
ıewn, that all the errors of the Body
rom the Forces repreſented by the
s *AM*, *MN*. But becauſe one of them
ıccording to the direction *LT*, which
plane of the Orbit of the Body *L*, the
ng according to its direction, will not
Motion of the Body *L*, as to its La-

ituation of the Orbit of the Body *L*,
that the Body *S* is found in its com-
on with the plane of the Orbit *ETE*,
leſcribes about *S*: (that is, to ſpeak
ınguage of an Aſtronomer, if the
the Orbit of the Body *L*, be in the
f the Body *S*;) *MN* alſo lies in the
ıe Orbit of the Body *L*; becauſe in
on Section: And therefore the Force
ording to the direction of it, does not
ſe Motions.

her things remaining the ſame, let
GCBD of the Orbit of the Body *L*,
ed to be inclined to the Plane *SET*,
rt of it *CGD*, may be elevated above
rt *CBD*, depreſſed below it; Upon
Center *T* and equal Semidiameter, let
CbDg, be ſuppoſed to be deſcribed
ıne *SET*, and the Nodes will be *C*
uppoſe them when ſeen from *T*, to be
tile aſpect with the Body *S*; that is,
ſup-

ſuppoſe the Angles *STD*, *STC* right. In the caſe of the motion of the Body *L*, along the Semicircle *C G D*, (*ſee the firſt of theſe Fig.*) the Force as *MN* of the Body *L*, acting with the direction *MN*, (that is, from *L* to *F*) attracts the Body from its Orbit *CGD* towards *C g D*; that is, leſſens the Latitude of the Body *L*, (reckoned from the Plane *C g D*): And the Body *L*, in its continual receding from the Plane *C G D* towards *S*, till it has arrived to the next Node, does not paſs the Plane *ETS* or *C g D* in the point wherein it paſſed before, but a little on this ſide of it; and makes the Node of the Orbit, now deſcribed by it, a little more *in antecedentia*, in regard of the Node *D* of the Orbit *CGD*, which it would have deſcribed without this Force *MN*. In like manner in the paſſage from the Node *D* to the Node *C*, through the Semicircle *D B C*, (to which caſe the ſecond of theſe *Figures* is accommodated,) the Force as *M N*, acting according to the direction of *MN*, or from *L* towards *F*, makes the Body *L* to go backwards perpetually from the Orbit *D B C*, and deſcribe another inclin'd to *D b C*, in a leſſer Angle, and to paſs the Plane *D b C* ſooner than in *C*; that is, than it would have done if the Force *MN* had not been preſent; and by this means there will be made a new place for the Node tending *in antecedentia*; and again, during its paſſage, the part *CGD* of its Orbit towards the conjunction with the Body *S*, the neareſt Node will recede *in antecedentia*, as before.

Beſides, (the Eye being placed in the Line of the Nodes produced,) let *S g T b* [*Fig.* 55) repreſent the Plane of the Orbit of the Body *T* revolving about *S*, *GDB* the Plane of the Orbit of the Body *L*, while the latter is inclined to the

r, by the ſmalleſt Angle, namely, *GDg*
which happens (as has been ſhewn)
Body *L* is ſeen from *T* in the Syzygies
dy *S*. And ſince it has already been
at the Nodes of the Orbit of the leaſt
ends *in antecedentia*; the Orbit which
L, departing from *G*, where it was
onjunction with *S*, further deſcribes,
g Db on this ſide *D* in reſpect of *S*, and
Line *G d*. And the Orbit which the
ſcribes, departing from *B*, where it
oſition to *S*, namely, the Line *B δ*,
the Plane *g D b* in *δ*, *viz.* beyond *D*
of *S*. And it is evident that the Line
contains a greater Angle with *g D b* at
n the Angle made by the Line *G D*
th the ſame *g D b*; being about equal
hich the Orbit of the Body *L* made
, when *L* was very near the Node.
therefore being in the Quadratures, go
; but in the Syzygies are at reſt; and
lways whether they are Retrograde or
,upon the whole go backwards; but in
ediate places, partaking of both con-
backwards ſlower; and ſo much the
ey are nearer the Syzygies. But when
are in the Quadratures, the inclina-
Plane of the Orbit of the Body *L*, is
the paſſage of the Body *L* from the
es to the Syzygies, and augment-
paſſage of the ſame from the Sy-
the Quadratures: (For the parts of
Gd, B δ*, towards *d* and *δ*, are in-
a greater Angle to *g D b* than their
rds *G* and *B*; and the Orbit of the
ery moment of time is to be looked
lineola, deſcribed by it in that mo-
that is the Plane of the Orbit, which
paſſes

paſſes thro' the ſaid lineola, and the Center of the Body *T*.) And therefore the Body *L* being in the Syzygies, the inclination becomes the leaſt poſſible, and returns to its former magnitude, or thereabouts, when the Body comes to the neareſt Node. And theſe are the principal Errors ariſing from the inclination of the Planes. *Q.E.I.*

SCHOLIUM.

The ſame holds, tho' the Body *L* [*Fig.* 56.] revolves not about *T*; but both *T* and *L* about *S*, as a Center. For in this caſe the outmoſt *T*, produces the Effects of the Body *S*; and *S*, together with *L* revolving about it, ſupplies the places of *T* and *L* revolving about it, in the foregoing caſe. For the Demonſtration is the ſame, whether the diſturbing Body moves or not.

COROLLARY.

By the ſame Laws, as the Body *L* [*Fig.* 54.] revolves about *T*, let us imagine ſeveral Bodies to move about the ſame *T*, at equal diſtances from it; and a Ring *CBDG* rigid and concentric with the Body *T*, made up of them, being multiplied, and at length becoming contiguous: And the ſeveral parts of the Ring will perform all their Motions (which are not impeded by the rigidity,) according to the Law of the Body *L*. And therefore the Nodes *C* and *D* of the Ring *CBDG* are at reſt, when they are in the Syzygies of the Body *S*: and will move from them, *in antecedentia*, ſwiftly in the Quadratures, and ſlowly in all other places: And the inclination of the Ring will be changed, (the Axe of it Oſcillating,) as has been ſhewn concerning the Orbit *GCBD*, in the Propoſition.

If now the Globe *T* (having the ſame Axe with the Ring *CBDG*, and compleating its Revolutions about that Axe in the ſame times,) be conceived to be increaſed ſo big, or the Ring

(other

ngs remaining as before) leſſen'd, ſo
Globe touch the Surface of the Ring
, and cling to it, becauſe cloſely en-
by it: The Globe *T* (which we ſup-
ly indifferent to all impreſſions,) will
f the Motion of the Ring, and the
of them both, will oſcillate, and the
the Ring (or the interſections of it
lane of the Orbit of the Body *T* about
backwards.

PROPOSITION LXII.

ody L,[Fig.54.] *by a Radius drawn to* T,
deſcribe Areas more nearly proportional to
and a Figure coming nearer to the Form of
hoſe Focus is T; *if* T *be attracted towards*
ame Law as L *is, than if it were attra-*
deal more or a great deal leſs.

the preced. *Prop.* the pertubation of
rtionality of the Areas and Times,
eformity of the Figure of the Orbit,
ipally from the Force repreſented by
nd therefore the Areas and Times will
reſt to the Proportionality, and the
BD to an Elliptic Figure, when *MN*
, or the ſmalleſt poſſible: That is,
Accelerative attractions *SM*, *SN*,
fference is *MN*) of the Bodies *L* and *T*,
equal; or, which is much the ſame,
Accelerative attractions of the Bodies
owards *S*, approach as near as poſſible
ality: (For the other part of the Acce-
ttraction of the Body *L* to *S*, expreſſed
amely *AM*, tho' it does not diſturb
Proportionality, yet it renders the El-
ure deformed; that is, when *SN*, the
ive attraction of the Body *T* towards *S*,
ne; (for from what has been ſhewn
above,

above, the Attraction *SM* is reduced to a leſſer *MN* by the Attraction *SN*,) nor leſs than the ſmalleſt of all the Attractions *SM*, (otherwiſe *MN* would be too great in every ſituation of the Body *L*,) but as it were a mean between the greateſt and leaſt of all the *SM*; that is, neither much greater nor much leſs than the Attraction *SK*. And therefore the Body *L*, by a Radius drawn to *T*, will deſcribe Areas more proportional to the Times, and a Figure approaching nearer to an Ellipſe, if *T* be attracted towards *S*, by the ſame Law as *L*, than if it were attracted much more or leſs. Which was to be demonſtrated.

The ſame holds, if (other things remaining as before) the Body *L* did not revolve about *T*; but both *T* and *L* about *S*. For, as was ſaid in the preceding, the Demonſtration is the ſame whether the diſturbing Body move or not.

SCHOLIUM.

It may in like manner be infer'd, that if a Syſtem of Bodies *P* and *L*, *l*, λ, [*Fig.* 52.] revolving about *S*; (as in *Prop.* 59.) the Orbits deſcribed will come nearer to Elliptic ones, if all the Bodies acted upon by the ſame Laws of Attractions, than if on any account, the ſame Laws did not hold; and the common Center of Gravity of the Bodies *P*, *L*, *l*, λ, will deſcribe an Orbit whoſe Focus is in the common Center of Gravity of all the inferior Bodies; as in *Cor. Prop.* 56.

PROPOSITION LXIII.

BY reaſon of the mutual action of the Earth and Moon, their common Center of Gravity moves in the Orbis Magnus *about the Sun. After the ſame manner, the common Center of Gravity of Jupiter and its Satellites, and of Saturn and its Attendants, are*

n the Orbits of Jupiter and ;Saturn ; de-
Prop. 58.
(by *Prop.* 46. and 47.) the Moon gra-
ards the Earth, and the Earth towards
; the Satellites of Jupiter towards
d Jupiter in like manner towards its
Saturn's Attendants towards Saturn,
towards its Attendants; what was
evident from *Prop.* 59. And becauſe
rop. 47.) the Sun, every Primary Pla-
s Secondary, are acted upon by the
of attractions, the Orbit of each Se-
very nearly an Ellipſe, having a Fo-
Center of the Primary, by the fore-
the motions of the Satellites are di-
has been ſhewn in *Prop.* 60 and 61.
ected with various inequalities, that
ted of in their proper places here-

PROPOSITION LXIV.

of the Figure of the Earth, the Equi-
Points go backwards, and the Axis of
a every Annual Revolution, twice chan-
tion to the Ecliptic, and twice returns to
lination.

3. we conſidered the Axis of the
emaining always exactly parallel to
uſe then we had only the Phænomena
olution to explain, in which ſpace of
erves its Paralleliſm very nearly. But
he Earth's Figure, which is an Oblate
ariſing from the Cauſes pointed at in
e caſe is now ſomething different.
s ♎ ♑ (*Fig.* 57.) be the Orbit of the
t the Sun ☉, *A E B Q* the Earth it ſelf,
s are *A* and *B*, Equator *E Q*. Becauſe
.) the Earth is an oblate Sphæroid,

K (de-

(depreſſed towards the Poles *A* and *B*, and elevated towards the Equator *E Q*,) it is like a Globe faſten'd to a Ring; for that exceſs of Matter, which is about the Equator, ſupplies the place of a Ring. And therefore, by *Corol. Prop.* 61, the Nodes of this Ring will go backwards: That is, the Earth departing from ♎, (where the common interſection of the Ecliptic and the Equator of the Earth, is directed towards the Sun ☉, and conſequently the Sun ſeen in the Equator in the Heavens, makes the Equinox,) thro' ♑ towards ♈, will arrive at the Node ♈ ſooner than at ♈ the point oppoſite to ♎: And the Earth going forward from ♈ thro' ♋, towards ♎, will arrive at the new Node ♎, ſooner that at ♎, where the Node was was in the former revolution; that is, the plane of the Equator of the Earth produced, will paſs through the Sun ſooner than the Earth's Center arrives at ♎. But then the Equinox is celebrated, when the Sun is found in the plane of the Earth's Equator; (as has been ſhewn at large, *Prop.* 33.)and they are to be looked upon as the Equinoxial Points, in which the Sun appears at the time of the Equinoxes. Wherefore it is evident, that the Equinoctial Points, and together with them, all the points of the Ecliptic, that have received any names as ſuch, (becauſe depending upon the Equinoctial Points and reckoned from them,) will ſeem to go backwards, or move *in antecedentia*; if the Fix'd Stars be taken as immovable. But if the points of the Ecliptic be looked upon as unmoved, then the Fixed Stars will be reckoned to have moved as much forwards or *in conſequentia*.

The Regreſſion of the Equinoctial Points, as has been hitherto explained, proceeds from the action of the Sun upon the Ring compaſſing the Earth

the Equator, or the redundant Matter
quator, as has been ſhewn, *Corol. Prop.*
the Force of the Moon alſo upon this
ng is conſiderably great, as is evident
Schol. to *Prop.* 61, and its *Corollary*: For
is always in the Plane of the Ecliptic,
ar from it, ſhe will conſpire with the
e ſame Effect.

, in a Semi-revolution of the Earth a-
Sun from ♎ thro' ♑ to ♈, the incli-
f the Equator to the Ecliptic thro' the
the Sun, is leſs than when the Earth
; and it is leaſt when the Earth is in
rop. 60.) the Nodes of the Ring being
n's Quadrature. And when the Earth
at ♈, and the Nodes at the Sun's Sy-
e former Inclination is reſtored again;
g from ♈ thro' ♋ to ♎, the like leſ-
the ſaid Inclination happens again;
Axis of the Earth Oſcillates together
Equator. The Axis therefore of the
every Revolution, changes its inclina-
e Ecliptic twice, and twice returns
mer inclination; which will happen
lſo in every Periodic Month, upon
ount of the action of the Moon.
Oſcillation will ſcarcely be ſenſible,
he regreſs of the Points of the Eclip-
wn to every Body, tho' it ariſe from
Forces. For in each Equinox, the in-
of the Axis of the Earth towards the
returns to its former bigneſs, and
s of Years becomes not more ſen-
t the regreſs of the Points of the
is continually onwards, and the Equi-
oints never return to their ancient
ll they have compleated an intire
And ſuch a change as is intirely inſen-

ſible in one Year, or half of a Year, (in which ſpace of time the whole change of the inclination of the Axis of the Earth is finiſhed,) will in a ſeries of Years become ſenſible; being made perpetually the ſame way. On the account therefore of the Figure of the Earth, the Equinoctial Points go backwards, and the Axis of the Earth every Year changes its inclination to the Ecliptic twice, and twice returns to its former ſituation *Q.E.D.*

PROPOSITION LXV.

THE common Center of Gravity of the Sun and all the Planets and Comets is at reſt; and it therefore may be taken for the Center of the Solar Syſtem, and even of the World it ſelf.

It has been demonſtrated by the Philoſophers mention'd in *Prop.*50. that the common Center of Gravity does not change its ſtate of motion or reſt, by the actions of the Bodies upon one another; but that the Law of a Syſtem of Bodies is the ſame with that of a ſingle Body, as to their continuance in the ſaid ſtate; ſince the progreſſive motion of both is to be eſtimated by the motion of the Center of Gravity. The common Center therefore of the Solar Syſtem is either at reſt, or moves uniformly forward; for excluding all external Force, this is the ſtate of a ſolitary Body. Now, the latter can't be aſſerted, ſince of the ſurrounding Fix'd Stars ſome would become nearer than formerly, and others more remote; and conſequently their ſituation and order in length of time would be ſenſibly changed, contrary to Obſervation. And ſince it has already been ſhewn, that the Sun, Planets, and Comets gravitate towards one another, and are therefore in continual motion, (only the common Center of Gravity remaining quiet,)

nt that the movable Center of any one
can't be look'd upon as the Center of
Syſtem, but the above-mention'd im-
common Center of Gravity.
not an inviſible Point, as the above-
'd, but ſome conſiderable Body very
is to be looked upon for the Center of
tem, it will be the Sun; becauſe that,
n of the vaſt Quantity of Matter it con-
compariſon to the others, will ſcarce
ſenſibly diſtant from the common Cen-
avity above-mentioned.
urther, if any Figure agree, and bounds
o the univerſal compages of Bodies (or
Vorld,) no other Syſtem more juſtly
e middle of it, than our Solar; and
no other Center than that above de-
l, is agreeable to the Univerſe. Which
e demonſtrated.

SECTION X.

ng the Cauſes and Reaſon of the ns of the Planets aſſigned by Philo- s.

ng been deſcribing hitherto the Syſtem
the World, and that only in general, ſo
s ſufficient for our purpoſe, and having
d the analogous cauſe of the Motions of
ter Bodies of the World, or rather the
is moſt fitting,) with that which we dai-
ence about us; namely the Gravity of
s towards all Bodies, whoſe Law is the
h that of our Gravity, or of any other
Virtue propagated along Right lines

concurring in one Point; that is, having laid down the principal foundations of a Phyſical Aſtronomy, regularity of Method requires us to conſider the Cauſes and Phyſical Reaſons of theſe ſame Effects as are aſſigned by others; and at the ſame time ſee, whether they ſolve the Phænomena, ſo well as thoſe explained before have done. But by the Cauſes we ſpeak of here, we do not underſtand the ultimate Cauſes, or ſuch as have no Cauſes of themſelves; but ſuch as we experience about us, and can calculate their Forces, and Geometrically inveſtigate their Properties, as we do of any other Quantities; ſuch as Light, to the conſideration of which, various Sciences, as Optics, Catoptrics, Dioptrics, owe their Original; Gravity, the affections of which are conſidered in Statics, and in almoſt all Mechanics: tho' the Cultivators of thoſe Sciences neither underſtood the ultimate Cauſes of Light or Gravity, nor will they do ſo hereafter.

There are two other famous Sects of this Cœleſtial Philoſophy, that attempt to explain the Cauſes of the Motion of the Bodies of the Univerſe, the *Keplerian*, and the *Carteſian*: As for that which brought in Solid Orbs, and faſten'd Intelligences to them to move them, there is no occaſion to take notice of it, ſince there is little of Philoſophy in it, and it has been ſufficiently confuted by the Aſtronomers of the laſt Age. We ſhall give an account of both theſe, and annex the Reaſons why neither of them belongs to the Heavens, by ſhewing that the Laws of Bodies moved as theſe Philoſophers would have them, are not ſuch as are found to be obſerved by the Heavenly Bodies. And firſt we ſhall explain the *Keplerian Phyſics* deliver'd in

Book

pitome of the Copernican *Aſtronomy*, and
he moſt part in his own Words.

PROPOSITION LXVI.

xplain briefly the Subſtance of the Cæleſtial
yſics of Kepler, *or the Cauſe and Reaſon,*
. Kepler *has aſſigned, why the Planets are*
Orbits about the Sun.

great Aſtronomer would have the Sun
aſt Body, made to move about an Axe
Poſition, by the Creating Power of the
eing in the beginning of Things; and
Motion ſhould languiſh by degrees on
unt of the ſluggiſhneſs of Matter, that
tantly maintained by the care either of
ator, or of ſome Mind deſtin'd to it:
e likewiſe he would have to be obſer-
e Earth, and the reſt of the Globes of
verſe. The Sun by this gyration of his
arrying about the immateriated Species
vn Body thro' the vaſt Spaces of the
arries about like a Leaver, at the ſame
Planets which are apprehended by this
This Species is the Virtue of the Sun
in Right lines thro' the whole Mundane
hich, becauſe it is the Species of a Body,
d about together with the Sun, like a
rtex, running thro' the whole Mundane
vith the ſame ſwiftneſs as the Sun turns
out its Axis, and therefore carries along
he Planet, ſwimming as it were in this
e are farther to take notice, that beſides
ory Force in the Sun, there is a natural
eſs in the Planets to all Motion, by
hey are inclined, by reaſon of their
to continue in the place where they
that there is a conflict between the
Power of the Sun, and the ſluggiſh-

neſs of the Planet, ariſing from its Matter. Both have their ſhare in the Victory; the former moves the Planet out of the place it was in; the latter in ſome meaſure diſengages its own, that is, the Planet's Body from the ſhackles it was laid under by the Sun, when apprehended by ſome part or other of the circular Virtue; namely by that, which immediately ſucceeds the part from which the Planet had juſt diſengaged it ſelf. And becauſe this Virtue that carries along the Planet, and flows from the Sun's Body, has the ſame degrees of weakneſs at different diſtances, as Light has; this Vectory Power, like the Sun's Illumination, decreaſes as the diſtance from the Sun increaſes: Wherefore it will carry a Planet placed at a greater diſtance from the Sun more ſlowly, and conſequently will make its Periodic Time, on this ſcore, greater. Beſides, ſince a like Vectory Power is propagated from the Body of each Planet moving about its own Axis; this alſo will carry the ſecondary Planets, which it finds in this River of its immateriated Vortex, after the ſame manner; and they will never leave their reſpective Primary, becauſe they are carried by the Species of the Sun's Body about the Sun, as well as their Primary, and together with their Primary. And upon the ſimple ſuppoſition of theſe things, 'tis evident that the Primary Planets would be carried about the Sun in Circles concentric to the Sun, in the Plane of a great Circle of the Sun lying exactly betwixt its Poles; and that the Secondary Planets would be carried after the ſame manner about their Primary.

PROPOSITION LXVII.

plain how Kepler *deduces from theſe Prin-*
es, that the Periodic Times of the Planets
actly in the Seſquialter Proportion of their
ircles.

was the firſt who obſerved that this Pro-
eld in the Cœleſtial Bodies, to the no
ntage of Aſtronomy and true Phyſics;
that in the Revolution of the Primary
bout the Sun, and of the Secondary a-
reſpective Primary, the Squares of the
Times had the ſame Proportion as the
the Semidiameters of their Orbits or
from the Sun; taking the Mean di-
nd abſtracting from the Excentricity.
he inferred from his own Principles
ollowing manner. Since the Vectory
the Sun, (whereby the Planets are car-
by the immateriated Vortex or River
ar Species,) decreaſes like the Illumi-
ne diſtance from its fountain or the
aſing; (that is, in the duplicate ratio of
ces,) and only exerciſes its Virtue ac-
one dimenſion, namely, in Length.
ory Power in producing this Effect,
carrying the Planet round) will be
nly in the ſimple ratio of the diſtance
Sun increaſed: And ſince (by his Ar-
Proportions) he makes the bulks of the
Bodies to have the ſame ratio as their
from the Sun; the Vectory Power of
vill employ equal Forces in the carry-
each Planet; namely, ſo much great-
account of the greater ſize or magni-
Planet, that the Moving Species lights
the Moving Virtue in that diſtance is
nd in the ſame ratio; namely, of the
diſtance

diſtance from the Sun. Again, induced by the Geometrical concinnity, he ſettles the Quantity of Matter in the Planets compared with one another, in the ſubduplicate ratio of the Bulk or Magnitude; (and ſo the Denſity will be in the ſame ſubduplicate ratio of the Bulk or Magnitude reciprocally,) that is, as was ſaid before, in the ſubduplicate ratio of the diſtance from the Sun.

Since therefore the Periodic Time in a Planet, that is more diſtant from the Sun, is greater than in a Planet that is nearer, becauſe the Way, (that is, the Orbit) is greater, and becauſe the Quantity of Matter to be tranſported is greater, (but in other reſpects equal, becauſe equal moving Forces are employed upon them, as was ſhewn above,) and ſince the greatneſs of the Way is as the diſtance from the Sun, and the Quantity of Matter as the ſubduplicate of that diſtance; on both cauſes conjoined, the Periodic Time of the remoter Planet will be greater, in the ratio compounded of theſe: But the ratio compounded of the ſimple and ſubduplicate, is the ſeſquialter or ſeſquiplicate: Wherefore the periodic Time of the remoter Planet will be greater than that of the nearer, in the ſeſquiplicate ratio of the diſtances from the Sun; that is, the Squares of the Periodic Times are as the Cubes of the diſtances from the Sun. Which things are to be underſtood after the like manner concerning the Secondary Planets revolving about their Primary.

PROPOSITION LXVIII.

TO *explain the Cauſes aſſigned by* Kepler, *why the Orbits of the Planets are Excentric.*

Since each Planet is a Body cognate with the Body of the Sun, having one part (like a Loadſtone)

is, friendly to the Sun, and the o-
te contrary. In the converſion of
of the Sun, this Virtue thereof,
attracts or repels the Planet, accord-
friendly or contrary part is turn'd
is alſo turned about; juſt as in the
or turning a Load-ſtone round, the
r repelling Force is transferred to-
ent parts of the World. And when
hold of a Planet by this Virtue of
hether attracting or repelling, or
ouſly between both, it carries it
with it; for it retains it both by
nd by repelling it, and by retaining
it round. Each of our Load-ſtones
wo Poles or Places of different Vir-
one of which, one Load-ſtone at-
er after a certain direction; but by
repels the ſame. But in the Hea-
are ſomething otherwiſe. For the
s this active or energetic Faculty of
r repelling a Planet, not in this or
r part, but in all the parts of its Bo-
fore it is probable, that the Center
of the Sun anſwers to one extremi-
r pole of the Load-ſtone; and the
ce to the other; ſince it attracts or
net according to a certain direction,
or that place on its ſurface or part
but by the whole.

ings being thus ſettled, 'tis evident,
dy of the Planet will not be carried
un at the ſame diſtance from the
where; but while it turns [*Fig.* 58.]
part to the Sun, it will come nearer
, till the Right lines drawn accord-
irection of this part, (that is, the
which this attractive Virtue is pro-

pagated from the Sun,)being almoſt every where parallel to themſelves, are no more inclined to the Sun, but a Right line drawn from the Sun to the Planet becomes a perpendicular to theſe Fibres; in which caſe, ſince the Sun does attract no longer, the effect of the attraction, (namely, the acceſs of the Planet towards the Sun) will ceaſe, and the Planet will then be in its neareſt approach to the Sun or in its Perihelion. The Planet being ſtill carried about, by the above-deſcribed Vectory Power of the Sun, its unfriendly part, or that which ſhuns the Sun, will by little and little be directed towards the Sun, and hereupon the Planet will be continually repelled from the Sun; and that will be continued till neither of theſe parts or places be inclined towards the Sun, but a Right line connecting the Sun and Planet, would again become perpendicular to the Fibres of the Virtue; and then the Planet will be found in its Aphelion, becauſe at the greateſt diſtance that the repelling force of the Sun can remove the Planet to, while it is carried about. And the Planet being librated by theſe Fibres, ſo as to be attracted by the Sun in one part of its Orbit, and repelled by it in the other, deſcribes an Orbit about the Sun indeed, but not about it as its Center; but Excentric to the Sun. Again, becauſe the Species of the Sun's Body communicating a Motion to a Planet, is thinner and weaker in a larger Circle in the ratio of the diſtance from the Sun, as it produces this effect, (as was ſhewn in the preceding Propoſition) and the Planet in the mean while continues of the ſame Bulk and Denſity, becauſe the ſame Planet: 'Tis evident, that, other circumſtances continuing the ſame, the Celerity of the Planet is leſſen'd, in the ſame ratio with the conſtipation

nſity of the Vectory Power of the
, reciprocally as the diſtance from
rther, the Virtue of the Sun that at-
ropels the Planet iſſuing along vir-
proceeding from the Center of the
oing round together with the Sun,
gth proportional to the Co-ſine of the
e. by the Radius of the Sun and the
dmits the Solar Virtue into the Pla-
herefore this Excentric Orbit will
fect Circle, but an Ellipſe, narrow-
depreſs'd on the ſides, and having
one of the Foci.

m the ſame Principles he ſhews, that
deſcribed about the Sun in the Fo-
a manner as that equal Areas are
equal Times, by a Radius drawn to
ich he calls the *Vector*; and univer-
he Areas, which this ſame Radius
, are proportional to the Times
y are deſcribed.

theſe libratory Fibres, by the help of
lanets are librated upwards, always
were parallel to themſelves while the
ied intirely round, and (by what was
)is ſo inclined in reſpect of the whole
that the line of the Apſides of each
rpendicular to thoſe Fibres, which
ither in the plane of the Orbit, or
et, or in a ſituation parallel to it;
Axis of no Primary Planet, about
urn'd as it were by a diurnal Mo-
s Poſition, and no other Line but
nd Right lines parallel to it, can
ne ſituation in regard of the Uni-
intire Globe of the Planet be ſolid
'tis evident that *Kepler* was obliged
ithin this exterior Cruſt of the Pla-
net,

net, (whoſe Axis keeps a given Poſition, namely, that about which the Planet is turn'd in its Diurnal Motion,) a Globe intirely ſeparate and divided from this Cruſt, and no ways yielding to its Diurnal Motion, whoſe Axis and Right lines parallel to it may ſuſtain the Places of the ſaid *Libratory* Fibres, and conſerve the Poſition neceſſary to this purpoſe deſcribed above; while the exterior Cruſt, according to the nature of each Planet, revolves by a Diurnal Motion peculiar to it ſelf.

PROPOSITION LXIX.

TO *explain the Cauſes aſſigned by* Kepler, *why the Orbits of the Planets are inclined to the Ecliptic.*

In this *Keplerian* Phyſics, which we are now upon, if there were no Fibres (ſuch as we deſcribed in the preceding Propoſition) in the Planets, by which they are librated upwards; or if theſe Fibres turned their ſide to the Sun; that is, neither that part which was friendly to the Sun, nor that part which was the contrary; each Planet remaining at the ſame diſtance from the Sun, would deſcribe a perfect Circle concentric with the Sun. After the like manner, all the Planets (being acted upon only by Forces ariſing from hence) would go round in the ſame Plane in an uniform and direct Stream as it were, in the middle of the Channel, (or in a Circle lying exactly between the Poles of the Revolution of the Sun, called by him the *Via Regia*, or *Royal Way*.) But becauſe they wander variouſly on this or that ſide of the *Via Regia*, the Cauſe of this Effect muſt be ſomewhat like that, whereby a Ship under ſail is thruſt to one ſide of the line, according to which it would have been moved by the Wind alone, namely, by an Oar held

uely to the Stern, or by the Rud-
an oblique poſition. In each Planet
Fibres (which he calls from their Of-
ibres of Latitude) always placed in the
s Orbit, or parallel to it, and which
ways parallel to themſelves in every
or motion of the Planet; ſo that
Planet is in a Node or common in-
of the Plane of the *Via Regia*, with
of the Orbit, the middle of the Fibre
e that paſſes thro' the Center of the
uches the Orbit; and therefore at e-
f theſe Fibres, one part towards one
e *Via Regia*, the other part towards
ide, ſtands without that Plane, incli-
t the ſame Angle, as the Orbit of
is to the *Via Regia*. When therefore
s of the Sun's Body meets with that
e Fibre, which is behind and ſtands
it will impel the Planet that way, as
hich is foremoſt, tends towards; that
ing to the Plane of the Orbit proper
net: But ſince theſe Fibres of Lati-
inue in a poſition parallel to them-
' the whole Revolution, hence it
paſs, that in a Planet placed in its
reateſt diſtance from the *Via Regia*,
tremity of the Fibres go foremoſt, but
directed as it were towards the bot-
s River, (that is, towards the Sun,) and
the impulſe on it's ſide directly oppo-
ive no cauſe for an ejection farther any
a change be made in theſe points of
ſo as that the extremity of the Fibres,
nt foremoſt before the arrival of the
the Limit, may now go behind, the
ng before; and for that reaſon the
ay begin to approach again towards
the

the *Via Regia.* For inſtead of the Power of the Rudder in a Ship, there is in the Planet another Force of the Fibres much more ſuitable; becauſe, as the Libratory Fibres have a natural Power to retain a parallel ſituation in the tranſportation of the Body, ſo alſo there is in the Fibres of Latitude, beſides a like Power of retaining a parallel ſituation, another natural Power of agility, or of preſerving the ſame Line, and of deriving according to the direction of it, a Motion impreſs'd upon themſelves, ſo far as the Motion tends the ſame way with that other extreme of the Fibre. Becauſe theſe Fibres of Latitude are entirely different from the Fibres deſcribed above, by which the Sun attracts or repels a Planet, as well in regard of their nature and office, as in reſpect to their ſituation; for the Fibres of Latitude are every where parallel to a Right line touching the Orbit at the Node in the *Via Regia*: But thoſe others (by preced. *Prop.*) are parallel to a Right Line, which is a Tangent to the Orbit in the Aphelium: Theſe Fibres, according to *Kepler*, ought to be found in different Bodies, ſince (in *Kepler*'s judgment) the Apſides go forward, and the Nodes go backward, tho' it be but ſlowly in the Primary Planets; and 'tis certain that the ſame happens in the Secondary. Wherefore, beſides an exterior Cruſt furniſhed with Fibres, by which the Axis, about which the Planet revolves by its Diurnal Motion, is kept parallel to it ſelf in its whole circuit about the Sun, he found it neceſſary to ſuppoſe an interior Cruſt (which might be like a Nucleus, in regard of the exterior,) intirely ſeparate from the former, and agitated by other Forces; in which there are Fibres, by which the Planet may be librated upward and downwards: And within this Nucleus,

ather Cruſt, he is forced to ſuppoſe
her interior Nucleus ſeparate from
middle one, and having a different
which the Fibres of Latitude are ex-
nce it is impoſſible that theſe diffe-
having different Periods, ſhould be
ne and the ſame Solid.

PROPOSITION LXX.

t out and refute very briefly the miſtakes
re to be found in the Keplerian *Celeſti-*
lain'd above.

undertake to oppoſe *Kepler*'s Cele-
s, yet we don't do it with ſuch a
if we rank'd him a Perſon, whoſe
e Immortal among the common Sy-
s or hunters after Phyſical Cauſes.
ſo far from this, that on the con-
the only Man (excepting perhaps
Ancients, as *Pythagoras*, &c.) who
of the Celeſtial Phyſics in a Mathema-
r. His *Archetypical Ratios*, *Geometri-*
ies, and *Harmonic Proportions*, ſhew ſuch
enius as is not to be found in any
ers of Phyſical Aſtronomy before
t *Jeremiah Horox*, a very competent
eſe matters, tho' a little averſe to
beginning of his Aſtronomical Stu-
aving in vain tried others, entirely
ith *Kepler*'s Doctrine and Phyſical
us addreſſes his Reader, *Kepler is a*
I may juſtly admire above all Mortals
y call him Great, Divine, or even ſome-
ſince Kepler *is to be valu'd above the*
f Philoſophers. Him alone let the Bards
Him alone let the Philoſophers Read;
l of this, that he who has Kepler, *has*

For what greater, or more, like ſomething Divine, could happen to a Mortal in this matter, than by theſe Harmonic Speculations, exactly to determine the proportion of the Orbits of the Planets; than to be the firſt that ſhould find out the true way of the Planet about the Sun (namely the Elliptic;) and laſtly, than to ſhew the true manner of a Planet's Motion in this way; *viz.* that by a Radius drawn to the Center of the Sun, placed in the Focus of the Ellipſe, it deſcribes Areas proportional to the Times? But there are ſome things which I can't approve of in *Kepler*'s Mechanical cauſe of theſe Motions, and they are theſe following.

1. If neither the Friendly nor the Oppoſite part of the Planet be turned to the Sun, or if the Planet (on the account of the weakneſs of the Fibres, whereby it is librated upwards and downwards) be ſo diſpoſed, as neither to be attracted or repelled by the Sun, (as it happens in Venus, according to *Kepler*'s judgment;) then he would have it, that the Planet deſcribes a perfect Circle, by the Species of the Sun which is carried about; whereas 'tis certain, that a Body which is not urged by a Force tending to a Center, will not deſcribe a Circle, but a Right line. Since therefore the Orbits of Planets are Circular even in this caſe, 'tis evident that they are impelled towards a Center by ſome other Force, beſides that which ariſes from the obverting the Friendly part.

2. During the Planet's aſcent from the Perihelium to the Aphelium, that part which is Unfriendly is turned to the Sun, and conſequently is repelled from the Sun, and in this caſe it is urged only by that Force, which is directed along the Line that connects the Sun and Planet. But 'tis well known in Mechanics, that a

ed upon by this Force alone, would
from deſcribing an Ellipſe, having the
Focus, that on the contrary, it would
a Curve convex towards the Sun.
e a Planet, whoſe Orbit is an Ellipſe,
e Sun for a Focus, muſt neceſſarily
even in its aſcent from the Sun, from
lium to the Aphelium, by a Force
the Sun, ſince an Ellipſe is concave
th Foci. *Kepler* therefore, muſt be ve-
ut of the way, when he would have a
ell'd by the Sun, in its aſcent to the
or when he would have a Planet nei-
ted nor repelled, when it deſcribes a
cle about the Sun concentric to it:
in both caſes it be attracted or other-
led towards the Sun, it will not de-
rve concave towards it: For it can't
thus, without ſome kind of attraction.
enquiry into the proportion of the
Orbits, he aſſumes ſome things plain-
to Obſervation, how much ſoever
ey agree with his imagined Archi-
ios: As that the Magnitudes of the
Bodies are as their Diſtances from the
hat their Denſities are reciprocally
e Roots of their Magnitudes. From
ollows, that the Quantity of Matter
ets is directly as the Square Root of
ces from the Sun: Becauſe the quan-
ter is as the Magnitude and Denſity
But every Body knows that Mars is
than Venus, which notwithſtanding
as big again as Venus, according to
oning; becauſe diſtant as far again
un. Obſervation informs us, that
f Jupiter is bigger than Saturn, tho'
the aforeſaid Rule to be as little a-

gain. Notwithſtanding he might deduce from the things that he knew, and firſt diſcovered, (namely, that the Path of a Planet is the Perimeter of an Ellipſe, one of whoſe Focus's is in the Center of the Sun, which it ſo deſcribes, as that its Radius that carries it, ſweeps Areas proportional to the Times,) without the conſideration of the Magnitude and Denſity of the Planet, that the Periodic Times of the Planets revolution about a common Center, are in the ſeſquiplicate ratio of their greater Axes, or mean diſtances from the Sun. But it is probable that *Kepler* deduced the Proportion of the Planetary Orbits, and their Number, from the intervals neceſſary for the interpoſition of five regular Bodies; and that he afterwards aſſigned to them ſuch Magnitudes and Denſities, as he could draw this ſame proportion from by another Method.

4. Not to ſpeak of his Virtue, which has too little of Mechaniſm, and too great an affinity to an Animal Power, which he gives to the Fibres of Libration and Latitude in the Planets; (to the latter, natural power of Agility, but to the former, in one part, an inclination towards the whole Body of the Sun; in the other, a flight from it;) and the diviſion ariſing from hence, of every Planet into a Shell, and a Kernel ſeparate from the Shell, and librated in its cavity, in a different Period, and in ſome another Nucleus within the former.

Not to mention, I ſay, theſe things, which have but little of Mechanics in them, and foreign to the Method which Nature uſes, *Kepler*'s Syſtem about the manner of the Sun's carrying round, attracting, or repelling the Planets, is founded upon what is contrary to one of the chief Laws of Nature and Motion, whereby

ken that there be a Reaction always
ınd equal to any Action. For accord-
pler, in Epit. Book IV. *This Virtue of*
nd Attraction is not in the Planets them-
in the Sun only. And tho' the attractory
y Power of the Sun be extended to, and af-
anets, yet like power of a Planet does not
Sun and affect it. For were it ſo, he
that *the Sun would be moved, out of the*
in the Center of the World, by the Planets,
ocal ratio of their Bodies; or at leaſt it
ibate or reel as it were, being ſometimes
re this way, ſometimes more that way,
more Planets happen to come together on
of the Sun, having ſimilar Powers. But
ırely gets rid of, by denying a mutu-
n. *Kepler* here had forgot the true
f Gravity, that he had laid down a-
ears before, in his *Introduction to his*
es upon Mars, built upon theſe Axioms:
y is a mutual affection of union or con-
Bodies that are a-kin to one another. That
if they are not hindered, will meet in
iate Space, each having moved one to-
her a length reciprocally proportional to the
f the other: So that if the Moon and
not retain'd each in their Circuit, the
aſcend towards the Moon the Fifty fourth
diſtance between them; and the Moon
nd towards the Earth about Fifty three
interval, and they would both meet there;
ſubſtance of both to be of the ſame den-
e Moon draws up the Waters of the Ter-
e, from whence ariſes a flux, where
of the Ocean are broadeſt, and afford
reciprocation. And if the Earth ſhould
ct towards it ſelf its own fluids, the Wa-
would be raiſed and flow towards the

Moon. Nor did he fear himſelf, anſwering the difficulties above-mentioned, about the titubation of the Sun, and its removal out of the Center of the World; where he ſpeaks after the following manner: *For ſo great is the bulk, ſo great the denſity of the Matter of the Body of the Sun, and ſo great the attractive and repulſive Virtue of it; on the contrary, ſuch is the ſmallneſs both of the Planet and its renitence, that the Sun is in no danger of being removed out of its place.*

From what has been deliver'd, 'tis evident, how near this great Philoſopher and Aſtronomer was, to the finding out the true Cœleſtial Phyſics, (who even in *Epit. Aſtron.* Book 5. § 1. had ſaid, *As the Sun attracts the Planet, ſo the Earth attracts Bodies; on the account of which attraction, the Bodies are ſaid to be heavy,)* and how much aſſiſtance he has given in this matter, and how rightly he Propheſied, when he ſaid, *The true Cauſes, Quantities, and Directions of the Motions of the Apſides and Nodes of the Planets, and other matters of this kind, remain in the Pandects of the following Age, not to be underſtood before the Divine Being, the diſpoſer of Times and Seaſons, has open'd this Book to Mortals.*

PROPOSITION LXXI.

TO *explain briefly and ſummarily the* Carteſian *Cæleſtial Phyſics, or the Cauſe and Reaſon, why the Planets are carried round the Sun, which* Renatus des Cartes *invented.*

Such as have endeavored to find out the Phyſical Cauſes of the Cœleſtial Motions, have all of them ſought them in the Sun. For the Rotation of the Sun in his Space about an immovable Axis, the ſame way that all the Planets take and follow in, and that in a ſhorter Period than that of Mercury, which is the neareſt to him, and every other Planet in a greater period,

ther from the Sun ; and the Rotation
imary Planets about their proper Axes ;
e Secondary again about theſe Primary,
oſe Primary revolve about the Sun ;
rds the ſame coaſt of the Heavens, and
n the ſame Plane, have the higheſt
f probability. And theſe Phænome-
lainly beſpeak, either that all this
f the Primary Planets ariſes from the
tion, or at leaſt, that both of them de-
on one common Cauſe. Hence it
ut, that ſeveral Philoſophers thought
ts were carried by the Æther, whirl'd
Sun by a Vortex or Whirl-pool ; and
before *Cartes*, or even *Epicurus*'s Time,
er, who was better acquainted with
ons of the Stars, ſaw well enough that
ld not be carried about by any mate-
ices: For if the Planets ſwimming in
er of the Vortex, and as it were at reſt
e carried about together with the parts
rtex, he ſaw that they would deſcribe
ircles, and have other periods in re-
their diſtances from the Center, than
by Obſervation agree to them. He
rather choſe to ſubſtitute the immate-
ecies of the Sun's Body, inſtead of Mat-
'd about like a Vortex, yet calling it by
of the *Cæleſtial Vortex and River*. But
ds *Cartes*, making light and taking no
f theſe niceties and Aſtronomical
ions, and being reſolved to frame a
rought in Vortices again; together
opular way of repreſenting the moti-
e Stars. And tho' Vortices were intro-
to the Heavens by others before *Cartes*,
been made uſe of by others ſince him,
ed into other ſervice, yet they retain

Cartes's Name. We ſhall therefore briefly deſcribe this Syſtem, ſo far as ſerves our purpoſe, in *Cartes*'s Words, out of the *Third Part of his Principles of Philoſophy.*

Cartes ſuppoſes then, That all the Matter of which the viſible World is made, was, in the beginning of Things, divided by God into Particles very nearly equal, of a moderate ſize, and equally moved, not only each about its proper Center, and ſeparately from one another, ſo as to compoſe a Fluid; but ſeveral alſo about ſome other points, in like manner remote from one another, and ſo they compoſed various Vortices. Afterwards he reſolves theſe equal Particles, worn a little by their inteſtine motion, into Globules of different Sizes, all which he calls Particles of the ſecond Element, and a part of a Fluid form'd out of the irregular minute parts rubbed off from the corners of the Globules, and moved always with a very great rapidity, he calls the Matter of the firſt Element. Now, there being more of this ſort of Matter than was neceſſary to fill up the interſtices of the Globules, what remained was thruſt down towards the Center by theſe Globules receding from their Center, by reaſon of their circular motion, and being there collected into a Spherical Body, conſtitutes the Body of the Sun in the Center of each Vortex. Beſides this equal Motion impreſſed originally upon theſe Particles, about the Center of the Vortex, the Sun, juſt form'd in the center of the Vortex, and revolving about its own Axis the ſame way with the reſt of the Matter of the Vortex, and always emitting ſome of it ſelf along the narrow paſſages that are within the Globules of the ſecond Element towards the Ecliptic, or Circle lying juſt in the middle between the Poles, (receiving as much at its Poles

m the neighbouring Vortices, as it e-
a power of carrying thoſe Globules
h it; thoſe nearer, quicker, but thoſe
ff, ſlower. But ſince theſe lower Glo-
Cœleſtial Matter move quicker than
ior ones, they ought alſo to be leſs.
y were greater or equal, they would
account have more Forces, and there-
ld become uppermoſt by their centri-
ce. And all theſe particulars will be
in any Vortex whatever, within cer-
s, beyond which the upper Globules
d quicker than the lower, being as to
s equal. Theſe limits in the Solar
places about the Orb of Saturn, or
yond.

things being ſuppoſed, if any Star,
ng placed in the center of its Vortex,
office of a Sun, ſhould happen to be
ith Spots, and by that means weak-
to be overcome and carried along by
bouring Vortex of the Sun; if this
t capable of a leſs degree of agitation,
ſs ſolidity than the Globules of the
ement that are near the circumference
rtex, yet ſomething more than ſome
hich are nearer the Sun; we are to
d that this Star, ſo ſoon as 'tis arrived
Vortex of the Sun, ought to deſcend
ly towards its Center, till it arrive to
eſtial Globules as it is equal to in ſoli-
ptitude to perſevere in its motion a-
t lines. And when it has at laſt got
will neither approach nearer to the
yet farther off from it, unleſs it be
propelled from thence, by ſome o-
es afterwards to be related: But being
with thoſe Cœleſtial Globules, will
revolve

revolve about the Sun with them, and become a Planet. Every one of them therefore is at reſt, in that part of the Heaven where it is; and all the variation of ſituation obſerved in them, only ariſe from the motion of the matter of Heavens or Vortex that contains them. According to this Syſtem then, nothing hinders us from imagining the Space, in which at preſent the ſingle Vortex of the Sun is contained, to be at firſt divided into ſeventeen or moreVortices; and that they were ſo diſpoſed, that the Stars, which they had in their Centers, were by little and little at length covered with ſeveral Spots, and then theſe deſtroy'd by other Vortices, the one quicker, the other ſlower, according to their different ſituation. So that ſince theſe four, in whoſe Centers were the Sun, the Earth, Jupiter, and Saturn, were bigger than the reſt, the Stars which were in the Centers of the five leſſer that ſtood about Saturn, deſcended towards Saturn; and thoſe in the Centers of four others about Jupiter, towards Jupiter; in like manner the Moon near the Earth, towards the Earth: Afterwards Mercury, Venus, the Earth, with the Moon, and Mars, (each of which Stars alſo formerly, had a Vortex,) towards the Sun; and at length Jupiter and Saturn, together with the leſſerStars about them,ſunk together towards the Sun, much greater than themſelves, after their own Vortices were deſtroyed: But the Stars of the other Vortices, if they were ever more than ſeventeen in this Space, paſs'd away into Comets, moving almoſt in a Right line out of one Vortex into another, and never returning. So that now, ſince we ſee the Primary Planets carried at different diſtances from the Sun, we are to judge, that this happened, becauſe the ſolidity of ſuch of them as are er the Sun, is leſs than that of thoſe that er off: And that from hence it came to t the ſame part of the Moon is turn- rds the Earth; or at leaſt does not defect om it, becauſe its other part, which is rom the Earth, is ſomething more ſolid; in moving round the Earth ought to a greater compaſs. And ſince we ſee rior of thoſe higher Planets to move uicker, we ſtill judge it to ariſe from hat the Matter of the firſt Element, ompoſes the Sun, by whirling round idly, carries along with it the nearer he Heaven, and the Planets ſwimming which are there relatively at reſt, bet- the more remote. Jupiter, the Earth, *&c.* about their own Axes, becauſe they merly lucid Stars placed in the Centers ortices, and no doubt gyrating there like and even now the Matter of the firſt collated in the Centers of them, ſtill re- ſame Motion, and impels them.

, we are not to think that all the Cen- e Planets are always accurately in the ne, nor the Circles they deſcribe per- compleat: But, as we ſee it happens er natural things, only nearly ſo, and perpetually changing. For ſince all es that are in the Univerſe, are conti- nd act upon one another, the motion depends upon the motion of all the reſt, erefore varied innumerable ways. On ccount we find, that tho' all the Planets Circular Motion, yet none of them ever Circular ones, but always err ſomething m, both in Longitude and Latitude.

PROPOSITION LXXII. LEMMA.

IF two Bodies revolve about the ſame Axe, in the ſame diſtance from the Axe, and two other Bodies likewiſe in any other diſtance; I ſay, the ratio of the difference of the Angular Motion of the former Bodies, to the difference of the Angular Motion of the latter, is compounded of the ratio of the tranſlation of the former Bodies from one another, to the tranſlation of the latter Bodies from one another, and of the ratio of the diſtance of the latter from the Axe, to the diſtance of the former from the ſame.

The *Angular Motion* is that by which the Angle is formed at the Axe about which a motion is: Hereupon they are ſaid to have the ſame or equal Angular motion, that being placed at any unequal diſtance from the Axe, make equal Angles at the Axe, in the ſame or equal Times: And the difference of the Angular Motions is the difference of the Angles formed at the Axe in the ſame Time by the Bodies.

At the ſame diſtance from *S* [*Fig.* 59.] the Axis of Motion, let there be two contiguous Bodies *A* and *B*; likewiſe two others *C* and *D*. Let them afterwards be moved any how, and ſeparated from one another; namely, *A* in α, and in the ſame time *B* in β, *C* in γ, and *D* in δ. 'Tis evident (from *Prop.* 33. *Elem.* 6.) that the difference of the Angular Motions of the two Bodies *A* and *B*, (namely the Angle $\alpha S \beta$) is to the difference of the Angular Motion of the two other *C* and *D* (that is, the Angle $\delta S \gamma$) as the tranſlation of the two former from one another, (or the way, whereby the one is transferred from the other, namely $\alpha \beta$,) to $\delta \gamma$, the tranſlation of the two latter; that is, if the diſtances are equal, the tranſlations are as the differences of the Angular Motions. Again, if at any unequal diſtances from the Axe *S*, the two contiguous odies *A* and *B*, and the two others *E* e ſo moved into the places α, β, and ε, φ, differences of the Angular Motions : Angles $\alpha S \beta$, $\varepsilon S \varphi$) be equal; then will . 33. *El.* 6.) the Tranſlations $\alpha\beta$, $\varepsilon\varphi$, be iſtances *S A*, *S E*. And therefore (by *Elem.* 6.) if neither the diſtances from , nor the differences of Angular Motion , the ratio of the Tranſlations will be ided of the ratio of the differences of ular Motions, and of the ratio of the from the Axis: And therefore (taking om both ſides the ratio of the diſtances Axe,) the ratio of the differences of ular Motion is compounded of the di- of the tranſlations of the Bodies from her, and the reciprocal ratio of the di- rom the Axis of Motion. *Q. E. D.*

POSITION LXXIII. LEMMA.

Curve DCE (Fig.6.) *be ſo related towards ight line* A E, *produced infinitely towards* E, *rpendicular* C B, *let fall from any point of to* A E, *is reciprocally as the Cube of the* A B, *intercepted between* B *and* A *a point the Right line* E A, *the infinite ſpace* B C *tain'd between the Right lines* C B, B E, *urve* C E, *will be reciprocally as the Square ht line* A B.

B, *x*; *B C*, *y*: and becauſe y is as $\frac{1}{x^3}$;

l $y = \frac{1}{x^3}$ drawn into a certain conſtant iable Quantity; let that be a^4, that the Homogenity may be obſerved: that is,

or $y = a^4 x^{-3}$. And therefore, the ſum of all the y's (or the Flowing Quantity whoſe Fluxion is y, or its value a^4x^{-3}) will be $-\frac{1}{2}a^4x^{-2}$: That is, as is known to Geometers, the Area that lies on one ſide of the Line BC, namely the Area $BCEE$, is equal to $\frac{a^4}{2x^2}$. And, becauſe $\frac{1}{2}a^4$ is a given quantity, the Area $BCEE$ will be as $\frac{1}{x^2}$; that is, reciprocally as the Square of the Right line AB: or the Area $BCEE$ is to the Area $bcEE$, as Ab^2 to AB^2. *Q. E. D.*

PROPOSITION LXXIV. LEMMA.

If two contiguous Bodies ſlide one upon the other, and the ſame be done in two other Bodies, preſſed by the ſame Force as the former towards one another; I ſay the impreſſion made by the friction of the former upon one another, is to the like impreſſion of the latter, in a ratio compounded of the ratio of the tranſlation of the former to the tranſlation of the latter, and of the ratio of the Superficies, wherein the former touch one another, to the Superficies, wherein the latter touch one another; namely, wherein the impreſſions are made.

If the contiguous Surfaces were perfectly ſmooth, there would be no impreſſion of the Bodies upon one another, while they ſlide. 'Tis evident that this ſuppoſition is very natural, that the reſiſtance ariſing from the defect of ſmoothneſs in the parts, is (other things being alike) proportional to the Velocity whereby the parts are ſeparated from one another; that is, if the contiguous Surfaces are equal, that the reſiſtance and the impreſſion of the Bodies upon one another ariſing from thence, is proportional to the relative Velocity, or to what is in a given Time proportional to it, the tranſlation of the Bodies. But if the Tranſlations (or Velocities wherewith the are ſeparated from one another) are he Reſiſtances or Impreſſions, are as the es wherein the Bodies two by two e another.

therefore, ſince neither the Superficies, nſlations are equal; tho' all the reſt are, Force preſſing the Bodies towards one the want of Smoothneſs,*&c.*)the Impreſ- ich the Bodies two by two make upon ther, are in a ratio compounded of the the contiguous Surfaces, and of the the tranſlations from one another.

PROPOSITION LXXV.

lid Sphere revolves in an uniform and in- Fluid, about an Axe given by poſition, with n Motion, and by the ſole impulſe of it a noved circularly, and each part of this Flu- re uniformly in its motion; I ſay, the Pe- mes of the parts of the Fluid, are as the of the Diſtances from the Center of the

E H [*Fig.*61.] repreſent a Sphere revol- formly about the Axis *S*. Let the infi- id of the Sphere ſpreading about every onceived to be divided into innumera- entric Orbs of the ſame thickneſs,every which conſidered alone is Solid; let expreſſed by the Circles *B F K*, *C G L*, *&c.* Becauſe the Fluid thus divided is to be homogeneous; (that is, all its ally compreſs'd;)the impreſſions of the ous Orbs upon one another (by the pre- mma) are as their tranſlations from one and the contiguous Superficies in e impreſſions are made conjunctly. If eſſion upon any Orb is greater or leſs oncave part, than on the Convex part, the ſtronger Impreſſion will prevail, and either accelerate or retard the velocity of the Orb, according as it is directed towards the ſame way with its Motion, or a contrary. Wherefore, ſince (by ſuppoſition) a Fluid perſeveres uniformly in its motion, the Impreſſions made on both parts of each Orbit towards the contrary ways, are equal; and therefore the ratio which is equal to the ratio of the Impreſſions, (namely the ratio compounded of the ratio's of the tranſlations and contiguous Surfaces) is alſo a ratio of Equality. Conſequently the Tranſlations are reciprocally as the contiguous Superficies; that is, reciprocally as the Squares of the diſtances from the Center: But (by *Prop.* 72.) the differences of the Angular Motions are as the Tranſlations directly, and the diſtances from the Center of Motion reciprocally: And therefore the ſaid differences of the Angular Motions of theſe concentric Orbs, are reciprocally as the Squares of the diſtances from the Center, and reciprocally as the diſtances from the Center; that is, reciprocally as the Cubes of the diſtances from the Center conjunctly. If, therefore, upon all the points of the Right line *S Z* drawn from the Center, where the contiguous Superficics of the concentric Orbs cut that Line (namely, *A*, *B*, *C*, *D*, *&c.*) perpendiculars to *S Z* be erected, *A a*, *B b*, *C c*, *Dd*, *&c.* reciprocally proportional to the Cubes of *S A*, *S B*, *S C*, *S D*, *&c.* theſe Ordinates will be reſpectively as the ſaid differences of the Angular Motions of the reſpective concentric Orbs, and the ſum of theſe Ordinates, as the ſum of the ſaid differences; that is, (ſince the outmoſt or infinitely diſtant Orb is not moved at all,) as the whole Angular Motions. If now the thickneſs of thoſe Orbs be leſſen'd *in infinitum* (by which means the Medium ſpread round

out becomes ſo far uniformly fluid,) the
the foreſaid Ordinates will become
AZa, *BZb*, *CZc*, *DZd*, &c. and
ire Angular Motions of the Orbs *BF*
, *DMN*, &c. are as the Areas *BZb*,
Zd, &c. reſpectively. But in the Fi-
a, the Ordinates *Bb*, *Cc*, *Dd*, &c.
ocally as the Cubes of the Right lines
SD, &c. from whence (by *Prop.* 73.)
BZb, *CZc*, *DZd*, &c. are reci-
s the Squares of the Right lines *SB*,
&c. And therefore the Angular mo-
e Orbs, *BFK*, *CGL*, *DMN*, &c.
ocally as the Squares of their Semidi-
B, *SC*, *SD*, &c. But the Periodic
reciprocally proportional to the An-
ons: Therefore the Periodic Times
s *BFK*, *CGL*, *DMN*, &c. are in
(becauſe the reciprocal of the inverſe)
e Squares of their diſtances from their
namely, of the Right lines *SB*, *SC*,
And therefore the Propoſition is evi-
e medium of the Sphere circumfuſed
d to conſiſt of innumerable thin con-
d Orbs. But if there be ſuppoſed to
a vaſt number of infinite Right lines
h the Axis Angles mutually exceed-
nother by equal exceſſes, and theſe
being revolved about the Axis, and de-
nio Superficies, Orbs be imagined cut
erable Rings; each of theſe Rings will
thers contiguous to it, one internal,
ernal, and two lateral, cut out of
rb. By reaſon of the attrition of the
d exterior, the intermediate can't
ny otherwiſe than formerly, before
ere cut: otherwiſe the parts of the
not perſevere in their motion uni-

formly; but the middle one would be accelerated and retarded in their motion, (as was ſhewn before in the intire Orbs,) contrary to the ſuppoſition. And therefore any ſeries of Rings going on directly from the ſolid central Sphere *in infinitum*, and comprehended between the two next Conic Superficies, will be moved after the ſame manner as theſe were moved before the diviſion of the Orbs into Rings; bating what hindrance each Ring in this Series meets with from the attrition of the Rings laterally. But here the attrition is nothing at all, becauſe all the Rings equally diſtant from the middle ſolid Sphere, (that is, cut off from the ſame Orb) are revolved in the ſame Time: For if it were not ſo, but thoſe which are towards the Poles, finiſhed their revolution quicker or ſlower than thoſe which are towards the Circle that lies in the middle between the Poles; the ſlower, by reaſon of the mutual attrition, would be quickened in their motion, and the quicker would be retarded, contrary to the ſuppoſition, which was, that the Fluid perſevered in its motion uniformly. Since therefore the Periodic Time of all the Rings, at the ſame diſtance from the Center, is the ſame, they will revolve after the ſame manner as they would do, were they not as yet cut off from the ſolid Orb; that is without any attrition: And therefore the ſame Law holds in this caſe alſo of Orbs cut into Rings, as held before any diviſion into Orbs; that is, the Periodic Times of each of the Rings will be as the Squares of their diſtances from the Center of the middle ſolid Sphere.

Let each Ring be divided by tranſverſe Sections, into innumerable Particles conſtituting a Fluid abſolutely and uniformly ſuch: And becauſe theſe Sections have no reference to the

ircular Motion, but conduce only to
tution of a Fluid; the circular Motion
vere as before. By theſe diviſions all
ely ſmall rings will either not change
ity and Force of mutual Attrition at
ll change them equally: and the pro-
the Cauſes continuing the ſame, the
of the Effects will likewiſe remain the
is, the proportion of the Motions and
imes as before. And therefore the Pe-
es of each of the parts, in the above-
ortex, will be as the Squares of the
rom the Center of the Vortex. *Q.E.D.*

COROLLARY 1.

iſtance of the Parts of the Vortex be
a greater diſtance from the Center
leſs, either on the account of the
ckneſs or leſs fluidity of the Particles
poſe the Fluid, or on the account of
Cauſe; then the more remote parts
x will move ſlower than in the ratio
ve in the Propoſition: that is, the
ime of the parts of the Vortex more
m the Center, will be to the Periodic
e parts nearer the Center, in a greater
duplicate ratio of the Diſtances of
les from the Center.

COROLLARY 2.

ortex be not extended *in infinitum*,
id moved after the manner of a Vor-
tained in a rigid Veſſel held by force,
nt Figure from a ſpherical one con-
the central Globe; the Particles of
will be moved, not in Peripheries of
centric with the central Globe, but
moſt conformed to the Figure of the
d the Periodic Times will be as the
the Mean Diſtances from the Center

PRO-

PROPOSITION LXXVI.

TO *point out and refute, in few words, the Errors committed in the above-deſcribed* Carteſian *Phyſics.*

1. Since from the preceding *Prop.* 'tis evident, that every Vortex made by a ſolid Sphere, revolved uniformly about an Axis given by Poſition, will be propagated *in infinitum*, if not hindred; if the World was made up of ſuch kind of Vortices, (as it is according to *Cartes's* Syſtem,) there would be as many Vortices going forwards *in infinitum*, as there are Fix'd Stars, every one of which, (according to *Cartes*) like our Sun, makes a Vortex. Nor would theſe Vortices be bounded by certain limits, but creep on, and by little and little run into one another. And, by this means, each part of the infinite Fluid making up the Univerſe, would be agitated by that Motion which reſults from the actions of all the Central Spheres or Suns. But how foreign this uncertainty is, from that certain Order, Situation, and Motion of the Fixed Stars, which are firmly connected by a very ſimple Calculus, let ſuch as are acquainted with Aſtronomy judge.

2. Further, becauſe the parts of the Vortex, generated in the manner above-deſcribed, that are nearer the Center, being (by the preced. *Prop.*) moved quicker, urges the exterior, and conſtantly communicate Motion to them by that action; and theſe exterior Particles transfer at the ſame time the ſame quantity of Motion into others that are ſtill more exterior; becauſe (by Hypotheſis) it is a condition of the Vortex, that each part of the Fluid perſevere in its motion uniformly, and is not incited, ſometimes more, ſometimes leſs, in the ſame diſtance from the Center:

'tis evident that Motion is perpetually
d from the Center to the Circumfe-
the Vortex, and abſorbed by the infi-
he Circumference. And therefore to
rvation of a Vortex in the ſame ſtate,
required ſome active Principle, from
e Central Sphere (or Sun of that Vor-
always receive the ſame quantity of
as it impreſſes on the matter of the
But from whence the *Carteſians* can
n active Principle reſiding in each
noves the whole Vortex, I cannot ſee;
es himſelf, in *Parag.* 146. *third Part*
boldly ſays, *There is nothing to be met*
Phænomena of the Planets, that can't be
r from what has been already ſaid. The
g *Kepler* ſaw well enough the neceſſity
active Principle in the central Spheres
ortices: *And therefore leſt the Motion*
uiſh by degrees, which Creating Om-
impreſſed upon them in the begin-
Things, he would have it *maintained*
e perpetual care of the Creator, or by the
a Spirit deſtin'd to that imployment.
hout ſuch an active Principle, the
heres and the interior parts of the
lways propagating their Motion into
r, and receiving no new Motion,
ſarily move ſlower by degrees, and
ove circularly. And conſequently
of *Des Cartes,* without ſuch an active
ical Principle naturally tends towards
diſſolution.

rtes and the *Carteſians* acknowledge,
dies which are carried in this Vortex
again, are of the ſame denſity with
f the Vortex, where they are placed,
by the ſame Law with them, as to

r from that which is obſerved in the
namely, that the Periodic Times of the
the Vortex are as the Squares of the
from the Center, if the matter of the
e homogeneal; or as ſome higher Power
iſtances, if the matter of the Vortices
er at a greater diſtance; whereas in
while (by *Prop.* 22.) in the Planets,
es of the Periodic Times are as the
the Diſtances; (that is, the Periodic
the Planets are as thoſe Powers of the
, whoſe Exponents are $\frac{3}{2}$, which are
wer than thoſe whoſe Exponents are 2,
ber greater than two;) 'tis evident
, that the Planets are not carried by
corporeal Vortices.

the Vortex were extended *in infinitum*,
nt (from the preced. *Prop.*) that each
the Fluid ought to deſcribe a perfect
nd (by *Corol.* 2.) that the aberration
rcular Path proceeded from the rigid
r what is equivalent to it,) in which
er of the Vortex is contain'd. From
it evidently follows, that the greater
ex is in reſpect of the Path deſcribed
d part, (or by the Planet carried toge-
it,) the more ought the Path of the
come up to a perfect Circle; that is,
ntricity of a Planet nearer to the Sun
eſs than that of one more diſtant from
; whereas Aſtronomical Obſervations
d aſſure us, that the Orbit of Mercury
more excentric than that of Saturn.
ecauſe (by *Corol.* 2. preceding *Prop.*)
icles of the Vortex, and the Planets
long with them, will be moved in lines
onformed to the figure of the Veſſel, in
e fluid is contain'd, the Aphelia of all

t is, in its return round, if it be not o-
hindered, deſcribes a Circle; to whoſe
ie Axis of the central Sphere, whereby
d is turn'd about in the form of a Vortex,
ndicular: and if the Path of the Body
ned to this Plane, that inclination will
inually leſſen'd, and they at laſt will
. For it is evident from the demon-
of the preced. *Prop.* that each part of
d compoſing the Vortex, deſcribes ſuch
; wherefore even the Planet, which is
nly by theſe Particles being relatively
etween them, will alſo deſcribe ſuch
and if by any external Force impreſſed
a little from hence, the inclination
eſſened by degrees by the encounter of
cles of the Fluid deſcribing the above-
ed Circles, and the Planet it ſelf will
ſuch a Circle, being thus at laſt rela-
reſt in the fluid of the Vortex, and car-
it together with it. But there is not
et to be found having the Axis of the
endicular to the Plane of the Orbit;
ig theſe two thouſand Years, wherein
ions have been made, has the Inclina-
he Orbit of any Planet to that Plane
id leſſened; nor are there any fibres of
aſſigned by theſe Philoſophers in the
or the preſerving the ſame Inclination.
pology that *Cartes* brings for the va-
entricity of the Planets, and the vari-
nation of the Orbits to one another,
arious ſituation of the Aphelias, which
rag. 34. *Part* 3. *Princip.* (namely this:
ve ſee it happens in all other natural
hat they are only nearly ſo, and in ſuc-
ges are continually changed,) ſhews
at he was unacquainted with the Cal-

culatory part of Aſtronomy. For tho' it can't be denied that Phyſical Cauſes have a place in the motion of the Stars, yet no Perſon uſed to theſe matters will judge that ſuch caſual and almoſt irregular inequalities, (ſuch as *Cartes* ſeems to inſinuate,) are to be admitted in the motions of the Stars; but will rather ſay, with that moſt acute and beſt fitted for Aſtronomical Speculations *Jer. Horrox,* who labour'd to defend the contrary Opinion, *We may juſtly wonder at ſome Slingers or Enginiers, that can throw Bodies out of Slings or Cannons with ſuch incredible certainty of aim. But if we ſee ſo great exactneſs in thoſe momentaneous things, that are moved thro' the thick unſettled Air; ſhall we not expect a much greater in thoſe perpetual Bodies as are revolved in a quiet and pure Æther by an eternal Law?*

6. Since the Globules compoſing the fluid of the Solar Vortex are leſſer near the Sun, and conſequently their Maſs, contained in a given Space and filling it, is leſs denſe than in a greater diſtance from the Sun; it follows, that even the Planets which are of the ſame denſity with the particles of the Vortex, in which they ſwim, are leſs ſolid the nearer they are to the Sun; which *Cartes* alſo acknowledges *in Parag.* 147. *Part* 3. *of Princip.* But the contrary to this we ſhall demonſtrate in its proper place hereafter; namely that each Planet, the nearer it is to the Sun, the ſolider or more denſe it is; that is, it contains more Matter under an equal bulk. Which alſo agrees much better with the Archi-typical Ratio's, Geometrical Concinnities, and Final Cauſes: From the conſideration of which, 'tis evident, that the Divine Being has placed the Planets at different diſtances from the Sun, ſo that each, according to its degree of denſity, may enjoy a greater or leſs

degree

of Heat; and conſequently the more
lanet is nearer the Sun, ſince all more
ort of Matter require a greater heat for
formance of natural Operations. And
ratio of the denſity of the Planet be
ſame with what *Kepler* makes it, name-
reciprocal of the Roots of the diſtances
e Sun; but the reciprocal of the diſtances
e Sun drawn into the roots of the appar-
ances: Yet here (where he was moſt of
aken,) *Kepler* came at this general Propo-
by his Architypical ratio's, that the den-
ners are nearer the Sun; whereas *Cartes*,
as ſince him, from a neglect, (or rather
pt) of the Architypical ratio's and final
, cheriſhed opinions entirely contrary to
inal Cauſes, and true Phyſics or Mecha-
which generally lead us to the ſame thing;)
ongſt others, this opinion, that the ſo-
f the Planets near the Sun, is leſs than
thoſe farther off.

ycho formerly, from the motion of the
s, demonſtrated, that the Planets were
ved in ſolid Orbs; becauſe the Comets
thro' them. The ſame Comets do now
qual evidence ſhew, that the Planets are
rried by corporeal Vortices; ſeeing the
s move in Paths very oblique, and ſome-
cutting the Plane of the Zodiac at Right
s, and ſometimes directly contrary to the
of the Planets, and they preſerve theſe
ns with the greateſt freedom and length
e, being ſubject to this univerſal Law,
a radius drawn to the Sun, they deſcribe
proportional to the Times; which would
gether impoſſible, if the intire matter of
rtex were carried about the Sun with a
ſufficient to carry along with them the
vaſt

vaſt Bodies of the Planets. Nor does it help *Cartes* at all, that he has baniſhed the Comets beyond Saturn, contrary to the credit of all Obſervations, (which *Tycho* and the Aſtronomers never forc'd to ſerve their Hypotheſes, as *Cartes* would inſinuate ;) ſince the matter of the Solar Vortex beyond Saturn (according to himſelf) has a greater moment to carry them along with it, than near Venus and Mercury; both becauſe the Globules compoſing the Vortex are greater there than here; and eſpecially becauſe they are moved much ſwifter, ſince they run a Circle immenſly greater *within a few Weeks*, as *Cartes* has it, *Parag.* 62. *Part* 3. *Prin.* From all together, and each of theſe in particular 'tis evident, that this Hypotheſis of Vortices is entirely diſagreeable with the Phænomena of Aſtronomy, and conduces not ſo much to the explaining as the diſordering of the Celeſtial Motions.

PROPOSITION LXXVII.

TO *enumerate and explain the Cauſes of the Celeſtial Motions brought by Mr.* Leibnitz.

'Tis ſo natural in enquiring after the Cauſes of the Celeſtial Motions, to ſeek them in the ambient fluid Æther, that the Philoſophers have looked upon the Planets to be carried about the Sun, after the ſame manner as ſo many Straws ſwimming in Water, are carried about by the Water put into the motion of a Vortex, by a Stick whirled about its Axis in the middle of a Veſſel at reſt. This ancient opinion lay neglected for a long time: But in the laſt Age was revived by ſome great Men; for before *Cartes*, (of whom above,) it was approved of by *Torricellius* and *Gallilæo*; for the reaſons without doubt, ſummarily propoſed about the beginning of *Prop.* 71.

This

opinion pleas'd ſo well, that that moſt
hiloſopher Mr. *Leibnitz*, (even after the
tion of *Newton's Mathemat. Princip. of Nat.*
adopts it; and *judges that there remains*
more, than to ſhew how the Cauſes of the Ce-
otions ariſe from the motions of the Æther,
king Aſtronomically) from deferent Orbs, but
h as are not ſolid, but fluid. A name ſo
ted among Geometricians certainly de-
o have his *Eſſay upon the Cauſes of the Cele-*
otions (which he communicated to the
d World in the *Acts of the Learned*, Pub-
Lipſic in *February* 1689.) exactly con-

Mr. *Leibnitz* (if any Man) is able to ac-
date Vortices to the Celeſtial Motions,
n reaſons for the Laws of the Heavenly Bo-
vered by Kepler. Eſpecially ſince *he has*
ſome peculiar Light in that matter, and the
he made, ſeemed to himſelf, who is ſo
ting a Perſon, *to ſucceed very well and*
to Nature, ſo that hereupon he had good
having arrived at the true Cauſes of theſe
We will therefore lay down the Me-
is Philoſopher uſes in explaining theſe
orks of Nature, in as few Words as poſ-
d, as near as we can, in his own.
Leibnitz takes for granted, that all Bodies
n a fluid deſcribe a Curve line, (and con-
ly that the Planets themſelves,) are acted
the motion of the Fluid itſelf. For all
deſcribing a Curve, endeavour to recede
along a Right line, which is a Tangent,
e nature of Motion;) there muſt there-
ſomething that keeps them in. But
nothing contiguous except the Fluid,
poſition,) and no endeavour to move
n be reſtrained but by a contiguous

Body likewiſe in Motion, (from the nature of Matter;) the fluid itſelf therefore muſt neceſſarily be in Motion.

Since from the moſt exact Obſervations that have been made, every Planet deſcribes an Orbit about the Sun, according to this Law, that by radii drawn from the Sun to the Planet, Areas are always cut off proportional to the Times; 'tis reaſonable enough that the Æther or Fluid Orb of each Planet ſhould move about the Sun after ſuch a manner as may agree with the ſaid Law of Motion. And this will be done, if each of the innumerable circular concentric Orbs of ſmalleſt thickneſs, into which the fluid Orb of the Planet is divided by the Mind, have its proper Circulation, ſo much the ſwifter in proportion, as each of them is nearer to the Sun; that is, if the Velocities of circulating, in the parts of the fluid, are reciprocally proportional to the radii or diſtances from the Sun. By which means it will happen, that whether the Planet be more or leſs diſtant from the Sun, the Sectors that are deſcribed in equal Times by the Radius, are equal. For the ſaid Sectors are in a ratio compounded of the direct ratio of the radii or diſtances from the Sun, and of the reciprocal of the Arcs or Circulations, and therefore (from the nature of the Circulation) in the ratio of equality. The Circulation above deſcrib'd (where the Velocities of Circulating are reciprocally proportional to the diſtances from the Center) he calls *Harmonic*, becauſe ſuppoſing the Diſtances from the Center to increaſe equably or Arithmetically, the Circulations decreaſe in an Harmonic Progreſſion; being reciprocally proportional to the Quantites that are in an Arithmetic Progreſſion, which is the known property of Harmonic Proportionals.

This

Great Man therefore ſuppoſes the Pla-
e moved by a double Motion, or one
nded of the Harmonic Circulation of
rent fluid Orb, and of a *Paracentric* ac-
nd receſs from the Sun. The Circula-
the Æther therefore makes the Planet
e Harmonically, not by any Motion of
, but as it were by a perfectly quiet
ng in a deferent fluid, whoſe Motion
. Conſequently neither retains an im-
circulating ſwifter, than it had in an
or nearer Orb, but one that languiſhes
ens, as it paſſes through the ſuperior (re-
greater Velocity than their own) till
it inſenſibly accomodates itſelf to the
pproaches; and on the contrary, while
s from the ſuperior to the inferior Orbs,
ves their Impetus. Conſequently the
nic Proportion holds not only in Arcs of
but in deſcribing any other Curve.
the Area deſcribed in an Element of
cut off by radii drawn from the Sun to
net, moved in any other Curve, doth
nparably differ from the Sector of a
whoſe Angle at the Center is the ſame,
ius any one of the preceding.
other Motion compounding with the
the intire Motion of the Planet (name-
aracentric,) ariſes from a double Curve,
, the *excuſſory impreſſion of Circulation and*
attraction (or what is equivalent) com-
d together. For ſince every Body en-
s to recede from the Curve line, it de-
along a Tangent, by this Motion alone,
net carried by the Vortex circulating
nically, will recede from the Sun. And
entrifugal Forces are ſhewn by the Au-
be reciprocally as the Cubes of the
radii.

radii. The other component part of the Paracentric Motion is the Sun's Attraction, or what ſupplies the place of it, the Planets Gravity: And tho' he call this Force Attraction, he does not queſtion but it is derived from the impulſe of the ambient Fluid, like the actions of the Magnet.

Becauſe it is certain by Obſervation, that each primary Planet deſcribes an Elliptic Orbit, in one of whoſe Foci is the Sun, ſo as that by radii drawn from the Sun to the Planet Areas are always cut off proportional to the Times; and there is no Law of Circulation in a Vortex that anſwers this condition, but the Harmonic: It remains that a Law of Gravity be ſought after, that join'd with the Centrifugal conatus of the Body circulating Harmonically, may compoſe a paracentric Motion, which in conjunction with an harmonic Circulation makes the Body to move in the Perimeter of an Ellipſe, which (after the premiſing of a Lemma ſhewing the ratio of a Circulation in a Conic Section to the paracentric Velocity) he defines, by demonſtrating the following Theorem: *If a heavy Body be carried in an Ellipſe (or any other Section of a Cone) with an harmonic Circulation, and the Center both of Attraction and Circulation be in the Focus of the Ellipſe; the Attractions or Sollicitations of Gravity will be as the Squares of the Circulations directly, or as the Squares of the radii or diſtances from the Focus reciprocally.*

He has found therefore that there is ſuch a thing as Gravity in the Planets, tending to the Center of the Sun, the Law of which is, that it is reciprocally proportional to the Squares of their diſtances from the Center. And he farther acknowledges that this Law was known before to Mr. *Newton*; his *Principles of Philoſophy* being pubbefore that time, and an account of n in thoſe *Leipſic* Acts.

aving annexed ſome Corollaries ha-wing from his chief Theorem, he clo-ſico-Aſtronomical *Eſſay*, with a con-it there remained two things to be his Argument: The one is to explain ion of the Æther it is, that makes the avy, or impels them towards the Sun, eciprocally as the ſquares of the Di-(as he expreſſes it) as the ſquares of ies: The other is, what is the cauſe mpariſon of the Motions between nt Planets of the ſame Syſtem, that dic Times ſhould be in the ſeſqui-of the mean Diſtances. And ſince to the Author) they both ought to on the contiguous Bodies, all which the name of the Æther, theſe two plain more diſtinctly the motion of or Æther that conſtitutes each Sy-ut theſe things requiring a deeper and therefore not to be contain'd in compaſs as a Sheet, he ſays, 'tis more him to publiſh his Thoughts upon r ſeparately.

ROPOSITION LXXVIII.

it at ſuch Things as are not agreeable to ral Philoſophy, in the above recited Cau-leſtial Motions, aſſigned by Mr. Leibnitz

re ſome of the Reaſons againſt Cor-tices carrying Planets, that likewiſe iſt the above deſcribed *Eſſay of* Leib-*he Cauſes of the Celeſtial Motions*; name-Paths of ſome Comets are very ob-e Zodiac, ſometimes they cut this

Plane at Right Angles, nay, they are even directly contrary ſometimes to the courſe of the Planets. And ſince the Comets (all the while they are obſerved by us) deſcribe Areas about the Sun proportional to the Times, by parity of reaſon, an Harmonically circulating Vortex ought to be ſuppoſed, ſuch an one being as neceſſary for the motion of a Comet as of a Planet; that is, there ought to be one Vortex contrary to another.

In conſidering indeed but one Planet carried about the Sun (or any other Center) if it be to be carried by a Vortex, without any proper motion of its own, only ſwimming quietly along in a deferent Fluid, circulating Harmonically, and obeying its Motion, (which the Author ſuppoſes;) the Vortex ought to circulate Harmonically, ſo as that the Areas cut off by Radii drawn from the Planet to the Sun, be proportional to the Times. No other conſtitution of a Vortex can make the ratio between the Areas and Times to be the ſame. If therefore the Vortex of our Author will not ſolve the Motions of the Planets, certainly no other will. And this being once ſuppoſed, all the reſt, which our Author draws from thence by Geometry, proceeds very well and juſtly, as things uſually do under his Hand. To which I add, that ſuch a Vortex as this (whoſe Fluid circulating Harmonically in a Plane thro' the Center, to which the Axe is perpendicular,) may be produced Mechanically, by a ſolid Sphere, if it be revolved in an uniform and infinite Fluid, about an Axe given by poſition, with an uniform Motion, and the Fluid be made to move round by the ſole impulſe of it, and every part of the Fluid perſevere in its motion uniformly. And this is a third thing, which remains for the Author to explain

concerning the motion of the Solar
or Æther; namely, how it might be im-
ind put into an Harmonic circulation.
ice univerſally the Times of deſcribing
ing any Space, are directly as the Spa-
, and reciprocally as the Velocities; and
cular motion the ſpaces run in one revo-
re as the radii, and in an Harmonic cir-
the Velocities or Circulations are reci-
as the radii; 'tis evident, that the Periodic
f the parts of a Vortex circulating Har-
ly, are as the Squares of the radii: But
demonſtrated, *Prop.* 75. that this is the
of a Circulation in a Plane thro' the
o which the Axis is perpendicular in the
put into its motion by a central Sphere.
f the motions of two or more Planets
pared together, it will appear, that the
tter of the Solar Vortex is not moved in
ionic Circulation. For we have alrea-
onſtrated, that the Periodic Times of
of a Fluid circulating Harmonically
ſequently of Bodies that are carried by
ſwimming in the Fluid) are as the
of the radii. But the Periodic Times
anets are not as the Squares, or in the
, but in the ſeſquiplicate ratio of the
from the Center. And therefore dif-
anets are not moved by an Harmonic
on.

uthor no doubt will ſay, that he did
oſe that the whole Fluid of the Solar
was moved by a continual Harmonic
on, no where interrupted, from Mer-
aturn incluſively; but that the Fluid
about Mercury moved in an Harmo-
ulation from its Perihelium to its A-
then (this Harmony being interrupted,)

again Circularly harmonical from the Perihelium of Venus to its Aphelium, but not in an Harmony continued with the former circulation near Mercury; that is, ſo as that the circulation of the Fluid carrying Venus, is to the circulation of the Fluid carrying Mercury, as the diſtance of Mercury from the Sun, is to the diſtance of Venus from the ſame: And again, (the former Harmony being interrupted,) the Fluid carrying the Earth is moved Harmonically from the Perihelium of the Earth to the Aphelium, if this Harmony be conſidered ſolitarily, and not compared with the Harmony of the Fluid to Venus or Mercury; and ſo on in the other Planets: that is, that by reaſon of the thickneſs of each Orb, there holds a particular Harmony ſeparate and diſtinct from the reſt; and in like manner in the Satellites of Jupiter and Saturn, aſcending from the Center of Jupiter and Saturn. But it muſt be confeſſed, that the Author does ſeem likewiſe to extend his Harmonic circulation alſo to different Planets, in *Parag.* 17. where he ſays: *In double the diſtance, only a fourth part of the Angle ſeen from the Sun, is finiſhed in the ſame Element of Time; in triple the diſtance only the ninth part*: For certainly theſe numbers have no place in the ſame Planet. But if, (as he ought principally to do,) he ſpeaks expreſly concerning the interrupted Harmony of the circulating parts of the Fluid, he will hardly perſwade Philoſophers that things are thus; namely, that by reaſon of the thickneſs of the Orb of Mercury there is an Harmonical circulation from the Perihelium to the Aphelium, or the Velocities of the Fluid are as the diſtances from the Center reciprocally: But comparing this Circulation with the Circulation of the Æther at any other Planet, the one is to the other reciprocally

ly as the Squares of the Radii: For we
monſtrated (in *Prop.* 27 and 28.) inde-
ly from all Phyſical Cauſes, that the Ce-
of the Planets are reciprocally as the
Roots of their diſtances from the Sun.
ly the Vortex of the Sun would be mon-
nd theſe deferent fluid Orbs more ab-
n the deferent ſolid ones, if by reaſon
ickneſs of each of the ſix deferent Orbs
monic circulation has a place, and
Harmony in the mean while is five
errupted; namely, at the interſtices of
rent. And it makes contrary to this
ion, that a Comet carried thro' the
in conſequentia, and paſſing thro' theſe
s of the Orbs carrying the Planets, is
uſt as if it were carried by an Æther
ng Harmonically; that is, ſo as that
s deſcribed by the radius that carries
roportional to the Times. It may be
likewiſe, how difformly difform the
rtex would be, in which the thickneſs
rb of Saturn, where the Harmonic cir-
of the Æther holds, is greater than the
of Mercury from Venus, or of Venus
Earth, or of the Earth from Mars, in
e of which the Harmony is interrupt-
where, having laid aſide the recipro-
of the radii, which they keep in the
y, the circulation of any two of theſe
l together, are reciprocally as the ſquare
he Radii, being about to reſume the
armony. The famous Author himſelf
have diſcovered this fault of his Har-
Circulating Vortex, when he con-
at this likewiſe was one thing remain-
ely, *to explain what ſhould be the cauſe*
pariſon of the motions between different

Planets of the ſame Syſtem, ſo as that the Periodic Times may be in the ſeſquiplicate ratio of the Mean Diſtances: For he ſaw plain enough, that this could not be the effect of an Harmonic Circulation.

Perhaps ſome Perſons may wonder, that Mr. *Leibnitz*, who deduced the Motions of the Planets from Cauſes (at firſt ſight) ſo different from thoſe eſtabliſhed before, ſhould notwithſtanding light upon the ſame Law of Gravity, whereby every Planet is kept in its Elliptic Orbit. The reaſon of that coincidence is thus: Tho' *Leibnitz* makes uſe of the Matter of a Vortex Circulating Harmonically for carrying the Planet about; yet becauſe in the Planet thus carried about, and deſcribing any Curve, he confeſſes an Excuſſory Conatus, whereby a Planet would be made to move in a Right line touching the Curve, if not hindred. He does the ſame as if he had made the Planet to move equably in a Right line, if not hindered: For theſe two Motions together make it ſo; that is, all one as if it were carried in free Spaces, acted upon by no external force at all. For in both caſes it will deſcribe a Right line with an equable Motion: For the Body deſcribing the Right line with an equable Motion, runs in it after ſuch a manner as (by *Prop.* 1. *Elem.* 6.) that the Areas cut off by radii drawn to any given Point, are proportional to the Times. Since therefore the Harmonic Circulation and Excuſſory Conatus together perform the ſame as an equable Motion in a Right line does, he could not but find the Law of Gravity the ſame with that found out before; namely, that which would force the Planet from the ſame Right lined Tangent which it would run after the ſame manner, into the ſame Conic Section towards the Sun, poſited after the ſame manner.

SECTION XI.

er Syſtemes of the World, and Forces neceſſary to preſerve them.

ierto we have been explaining the old
hilolaic Syſtem of the World, by ſhew-
it Phænomena would happen in it, and
y agree with Obſervation. We largely
ilſo concerning the Direction and Law
Forces, whereby the Syſtem of the Sun
n that ſtate; and after what manner the
phers, who were Maintainers of it,
it was done. It remains that we diſ-
riefly the Order and Motion which other
phers aſcribed to the Bodies of this Sy-
hich Opinions are now-a-days called the
of their Aſſertors; and we will at the
me ſhew the Direction and Law of the
whereby thoſe Syſtems might ſubſiſt.

PROPOSITION LXXIX.

deſcribe the Semi-tichonic *Syſtem of the*
orld.

maintainers of this Opinion, place the
which is not moved progreſſively or out
lace, in the Center of the Sphere, in
uperficies *ABCD*, [*Fig.*62.] they will have
ed Stars to be placed. But grant that the
s moved about its own Axis from Weſt
by a Diurnal Motion. Beſides, in this
the Sun is moved about the Earth in
e of a Year, at a Diſtance equal to that
h the Earth is about the Sun in the *Philo-*
tem: And about the Sun, as a Center,

the five Planets, *viz.* Mercury, Venus, Mars, Jupiter and Saturn, are revolved towards the ſame way, and at the ſame Diſtances as they were in that. Moreover, the Moon revolves about the Earth, and about Jupiter and Saturn, their proper Satellites, in the ſame Periods as was ſaid before, at *Prop.* 15.

This Order, and theſe Motions of the great Bodies of the World pleaſe moſt of thoſe Philoſophers, that the other Syſtem explain'd at large before diſpleaſes, and which, from the reviver of it, *Copernicus*, is commonly call'd the *Copernican* Syſtem, as by *Longomontanus* and others. For in this the apparent Diurnal Revolution of all the Stars from Eaſt to Weſt, and the proper or ſecond Motions of the Planets are intirely the ſame as in the *Copernican* Syſtem, and the Earth continuing in the mean while in the Center of the Sphere of the Fixed Stars, is only moved about its own Axis. And tho' they place the Fixed Stars in the Superficies of a Sphere, whoſe Center is the Earth, that is to be looked upon rather as the Aſtronomical than the Phyſical one.

For the Fixed Stars ſcattered as it were thro' the whole Mundane Space, may be placed at different Diſtances, without any prejudice to this Syſtem; and their Motion *in conſequentia*, explained by aſcribing a very ſlow Motion to the Axis of the Earth, as we have explained it in *Prop.* 64.

PROPOSITION LXXX.

TO *explain the Direction and Law of the Forces whereby the* Semi-tychonic *Syſtem may be preſerved.*

'Tis evident firſt of all, that the ſame Forces and the ſame Direction are neceſſary here for the

ing the five Primary Planets, Mercury,
Mars, Jupiter and Saturn, in their Or-
ut the Sun, and the Attendants of Ju-
d Saturn in their Orbits about Jupiter
urn, as in the *Copernican* Syſtem. The
refore of the Centripetal Force, where-
this Syſtem, the Secondary Planets
er tend to Jupiter, the Satellites of Sa-
Saturn; and the Primaries Mercury,
Mars, Jupiter, and Saturn, and their
s, towards the Sun, is known: For
Force which, in *Prop.* 47. we demon-
o be the ſame with our Gravity, which
ocally proportional to the ſquare of the
. 'Tis farther evident, that this Syſtem
ppoſed, the Moon is kept in its Orbit
ity propagated towards the Earth by the
w: For *Prop.* 46. where this was de-
ted, holds as much, whether the Earth
e in the ſame place unmoved, or be car-
an Annual motion about the Sun. But
difference between the *Copernican* and
onic Syſtem is, that in the latter, the
it were, with its five attending Primary
nd their Secondaries,) is moved about
h in an Orbit ſimilar and contrarily po-
hat, in which the Earth with its atten-
carried about the Sun in the former;
he Force of Gravity neceſſary in the
n Syſtem, that the Earth and its At-
might be carried about the Sun,
occaſion for another Force equal to it,
a contrary Direction, that the Sun,
with the attending Planets, may be
about the Earth at reſt in the *Semi-*
yſtem: For theſe two equal and con-
rces impreſſed upon the Earth, keep it
Nay, if one contrary impetus and not
to

to be repeated, do but put a ſtop to the progreſſive motion of the Earth and its attendant the Moon, along the Tangent of the *Orbis Magnus*, and afterwards that other Force now deſcribed, act upon the intire Solar Syſtem, the *Copernican* Syſtem will be turned into the *Semi-tychonic*. Therefore to preſerve the *Semi-tychonic* Syſtem, there is occaſion for both the above explained accelerative Gravity of all Bodies towards all others, which is, in a given diſtance, as the Body towards which it tends, and the Body being given towards which it tends, is reciprocally as the ſquare of the diſtance; and alſo of another certain Gravity, equal to that whereby the Earth tends towards the Sun, acting equally upon the Sun, Mercury, Venus, the Earth, Mars, Jupiter and Saturn, and their Attendants, along lines parallel to a Right line connecting the Sun and Earth, and with a direction from the Sun towards the Earth. For I do not reckon among theſe Forces, that Force whereby the Earth is revolved upon its own Axe; becauſe only once to be impreſſed upon the Earth, as alſo upon the other great Bodies of the World revolving about their own Axes: Nor do I take in (for the ſame reaſon) that Force, whereby the Planets would deſcribe the Tangents of their Orbits: For we are now treating of thoſe Forces and their Law, neceſſary to preſerve the Syſtem, and not of the Forces whereby it may be produced.

PROPOSITION LXXXI.

TO *deſcribe the* Tychonic *Syſtem of the World.*

This differs from the Syſtem deſcribed in *Prop.* 79. only in this, that the Earth being deprived of all Motion, is abſolutely at reſt; and there-

re all the Stars, as well Fixed as Wan-
re not only carried by their ſecond Mo-
ut alſo by that common or firſt Motion
aſt to Weſt, in the ſpace of a Natural

ſaid, that the great *Tycho* liked this Sy-
ſt; tho' others deny, that *Tycho* allowed
a Diurnal Motion to the Stars. But it
confeſſed, that he diſputes againſt the
Motion of the Earth as well as the An-
nd conſequently it may be ſafely con-
that he aſcribed a Diurnal Motion to
vens and all the Stars.

PROPOSITION LXXXII.

explain the Direction and Law of the For-
s, whereby the Tychonic *Syſtem may be*
l.

ſame Forces that are required in the
chonic Syſtem, are required alſo in the
, ſince the ſame motion of the Sun and
are ſuppoſed in both. And beſides theſe,
her Forces, whereby all the Stars, both
nd Wandring, are drawn from their
eal motions and retained in their Or-
ich they deſcribe by their Diurnal mo-
ut the Earth. But theſe Forces, ſince
ods in all are the ſame, are (by *Corol.* 1.
.) as the Semi-diameters of the Circles
ſcribe. And therefore, beſides the For-
ained in *Prop.* 80. there will be need of
Force tending towards the Earth, that
s in the receſs from the Earth as the di-
increaſe; contrary to what happens in
thro' the whole Solar Syſtem, and in all
Virtues propagated from a Center, or
nter, in Right lines, every way thro'
ions round about, which (by *Prop.* 48.)
in

in its receſs from the Center, is leſſen'd in the duplicate ratio of the diſtance increaſed. And that this Force, whereby the Stars are retarded in their Orbits, deſcribed by their Diurnal Motions, may be the more eaſily eſtimated, let it be compared at the Moon with the Force of Gravity, whereby (by *Prop.* 46.) the Moon is retained in the Orbit, ſhe deſcribes in her Monthly Courſe round the Earth. And ſince the Circles deſcribed by the Moon in her Monthly and Diurnal Courſe, are equal, the Forces whereby the Moon is retarded in them, (by *Corol.* 3. *Prop.* 26.) are reciprocally as the ſquares of the Times; that is, the Force whereby the Moon, (when ſhe is carried in the Equator by a Diurnal Motion about the Sun) is kept from flying off in a Right line, which is a Tangent to the Orbit, is to the Force, whereby it is kept in the Orbit that it deſcribes about the Earth in a Periodic Month, as at 27 × 27 to 1; or as 729 to 1. But this Force tending towards the Earth, holds only in thoſe Stars, which are placed in the Equator: For the Force, whereby any Star declining on one ſide of the Equator, towards either of the Poles, is retracted from the rectilineal Motion, and kept in an Orbit parallel to the Equator, (by *Prop.* 12.) tends to the Center of that Circle, and not to the Earth, which is ſituated without the Plane of it; and it is, (by *Corol.* 1. *Prop.* 26.) as the Semidiameter of that Circle; that is, in a given diſtance from the Axe, equal or the ſame, and in a given diſtance from the Earth, as the Coſine of Declination.

To preſerve therefore, the *Tychonic* Syſtem, there is required 1*ſt*. That accelerative Gravity of all Bodies towards all others, which in a given diſtance, is as the Body towards which it tends; and

Body being given towards which it reciprocally as the Square of the Di- 2*dly*. Another accelerative Force e- that whereby the Earth, by a ſuperior nds towards the Sun, and acting equal- the Sûn, Mercury, Venus, the Earth, ıpiter, Saturn, and their Attendants, ıes parallel to a Right line connecting and Earth, and with a direction from to the Earth. 3*dly*. An accelerative ending towards the neareſt Point of the he World, proportional to its diſtance t Point, and reaching to the fartheſt ars. 4*thly*. Another Force like it tend- ıe neareſt Point of the Axis of the E- and proportional to the diſtance from nt. This is the Force, whereby the tars are retain'd in the Orbits, which cribe in their proper motion from Weſt upon the Poles of the Ecliptic, and act at all in the Places that lie be- ıe Center and the Fixed Stars, but be- act at the Fixed Stars themſelves; and *Corol.* 3. *Prop.* 26.) to the like Force in of the World, in the duplicate ratio of ural Day, to the Period of the Fixed on the Poles of the Ecliptic. And theſe ll ſuffice.

PROPOSITION LXXXIII.

deſcribe the Ptolemaic *Syſtem*.

is Syſtem the Earth continues abſolutely in the Center: Next to the Earth, the then Mercury, afterwards Venus; a- enus the Sun, next Mars, then Jupiter, turn ſurrounding Jupiter juſt as in the , [*Fig.* 63.] finiſhing their Revolutions under

der the Zodiac from Weſt to Eaſt. Above Saturn the intire Sphere of the Fixed Stars, (whoſe Center is the Earth,) is alſo revolved very ſlowly from Weſt to Eaſt, upon the Poles of the Ecliptic. But both the Fixed Stars and all the Planets, revolve upon the Poles of the Equator from Eaſt to Weſt, in the ſpace of a Natural Day. We don't reckon up the ſeveral Orbs, which the aſſertors of the *Ptolemaic* Syſtem, place above the Fixed Stars; ſince they are only of uſe in the *Theory*, and don't belong to the Aſpectable World. And for the ſame reaſon, we omit likewiſe the Epicycles of each Planet: For in the Orbits here delineated, the Centers of the Epicycles themſelves are carried about, in whoſe Circumferences the Bodies of the Planets themſelves are moved.

In this Scheme of the Planets, we have not delineated the Orbits according to the proportion they have in the Heavens; as well becauſe *Ptolemy* himſelf has not declared it in any, but the Sun and Moon; and becauſe that tho' the Circles, that carry about the Centers of the Epicycles, are at ſome diſtance, yet the Heavens of the Planets are, in this Scheme contiguous, the inferior ſometimes in the Apogæum, aſcending from the Earth, to a diſtance equal to that, to which the next ſuperior deſcends in its Perigæum: as becauſe the Orbits of Mercury and Venus, (not to mention the Moon) taken from the Theories of *Ptolemy*, do almoſt vaniſh in reſpect of the Orbit of Saturn, and therefore are hardly to be deſcribed in this Scheme: For in this Syſtem, the diſtance of Saturn from the Earth, is an hundred and ſixty times greater than the diſtance of Mercury from the Earth: But in the *Copernican* Syſtem, the diſtance of Mercury from the Sun, is only four and twenty times ſs than the diſtance of Saturn from the

the Order of the Planets deſcribed above
the generality of Philoſophers and A-
ers, yet there were ſome, who placed
next above the Moon, then Mercury,
n Venus; and this Opinion is uſually a-
to *Plato:* But *Porphyrius* placed Venus im-
ly above the Sun, and Mercury above
the three others (Mars, Jupiter and Sa-
ontinuing as before: Perhaps induced by
ſon, that it was better for the two Lumi-
o be neareſt the Earth; whereas the *Pto-*
minding their motions more, placed the
the middle of the Planets, being ſatisfi-
the motions of the other Planets in their
les depended upon the motion of the

PROPOSITION LXXXIV.

explain the Direction and Law of the Forces
whereby the Ptolemaic *Syſtem may be pre-*

, for the keeping the Stars within the
they deſcribe, by their Diurnal Motion
he Earth; the ſame Forces are neceſſary
s were in the *Tychonic* Syſtem, for the
ing the ſame Effect; namely, ſuch as
nd from any Point of the World to the
t Point of the Axis, and be in the ratio
diſtances from it. Secondly, to contain
x'd Stars within the Orbits which they
e, by their proper motion from Weſt to
Forces like thoſe made uſe of in the *Ty-*
Syſtem, are neceſſary. Then to keep
minaries (the Sun and Moon) in the Or-
hich they deſcribe by their ſecond Moti-
ince theſe Orbits are ſimilar to their Or-
bits

bits in the *Tychonic* Syſtem, the like Forces will be neceſſary in this Syſtem, to retain them in the ſaid Orbits, as in that; namely (by *Prop.* 80.) the accelerative Gravity of the three Bodies, Sun, Earth and Moon, towards one another; (that may not affect any other Mundane Bodies, tho' they lie between them,) which, in a given diſtance is as the Body towards which it tends, and towards a given Body, is reciprocally as the Square of the diſtance from it; and beſides, another ſort of Gravity equal to that, whereby the Earth would tend towards the Sun, acting equally upon the Sun, Earth, and Moon, by Lines parallel to a Right line, connecting the Sun and the Earth, and with a direction from the Sun to the Earth, but ſuch as affects no other Planet. For that Force, whereby the Sun is retained in its Annual Orbit, is not the ſame with that whereby the Moon is retained in its Menſtrual Orbit, propagated unto the Sun: For the Force, whereby the Moon is kept in its Orbit, is the ſame with Gravity, with us, (by *Prop.* 46.) But if the Sun were retained by the ſame Force, propagated ſo far as itſelf, the Cubes of the Diſtances of the Sun and Moon (by *Prop.* 27.) would have the ſame ratio as the Squares of their Periodic Times; and therefore ſuppoſing the diſtance of the Moon from the Earth to be 60 Semidiameters of the Earth, the Sun would not be above 340 Semidiameters of the Earth diſtant; and its Horizontal Parallax would be above 9 minutes, contrary to all Obſervations; and beſides, there would be no room for the Epicycles of Mercury and Venus, between the Heavens of the Moon and the Sun, where they are placed according to this Syſtem.

of this Sect of Philoſophers are not
to put any more than a mental diſtin-
tween the Motions of the Sun and
nder the Zodiac, and the Diurnal
upon the Poles of the Æquator, ſo
motion of each is really in the ſame
iral Line, ariſing from the compoſition
bove-mentioned motions, into which
leſtial Motions are imagined to be di-
Aſtronomers, for the greater eaſe; and
ſimple Motion in a Right line, may
ounded of two or more Motions. But
that Spiral is not a Line deſcribed in a
Body can't (by *Prop.* 11.) be retained
ſingle Force tending to an immovable
Either therefore, there is need of the
ifferent Forces deſcribed above, or of
Force equivalent to them, tending to
moved in the mean while forward and
d, and that in a very compounded

s, to keep the other Planets (Mercury,
ars, Jupiter, and Saturn) within the Or-
deſcribe by their proper Motion, the
nding to the Earth are not ſufficient,
lemonſtrated, *Prop.* 13;) nor are the For-
ng to the Sun, ſince the Orbits of Ve-
Mercury do not ſurround the Sun in
em, and for that reaſon they are not
toward him in all their Parts: Where-
m *Prop.* 11. the thing propoſed is cer-
nd there is no Point in the Syſtem,
ſs a Body worthy to be taken notice of,)
be a Center, to which the Forces may
hereby the Planets are kept from ge-
in the Tangents; one of the above-
ed reaſons contradicting it.

But if this *Ptolemaic* Syſtem be underſtood literally, that is, if the motions deſcribed by *Ptolemy*, are not look'd upon as motions only ſeparated by the Mind, but as ſuch as are really in the Planets; then there will be need of a Force in the center of the Epicycle of every Planet, (Saturn, Jupiter, Mars, Venus and Mercury,) tending to the center of the deferent; that is, in an imaginary Point, (or where there is no Body at all,) to an imaginary Point: And beſides, there is need of a Force in the Planet itſelf, that may tend to the center of the Epicycle; that is again, a Force in a Body, that may tend to an imaginary Point. But even thoſe Forces whereby each Planet, and the center of its Epicycle are urged, are not propagated beyond its proper Point and Planet, nor do they affect any other Planets; and there is ſtill need of a peculiar Force for the keeping every center of an Epicycle, and every Planet in its proper Orbit; ſo that there is no Law which reſpects all the Planets: And the ordinary reſpective Gravity of the Satellites of Jupiter and Saturn, towards the centers of Jupiter and Saturn, continue in the mean time untouched.

After that the Phaſes of Venus and Mercury were obſerved by the help of Teleſcopes, the *Ptolemean* Syſtem of the World was ſoon thrown aſide by Philoſophers: becauſe 'tis evident from them, that the Orbits of Mercury and Venus, muſt encompaſs the Sun; as was ſhewn in *Schol. Prop.* 6. Such therefore as ſtill maintained, that the Earth was Immovable, made Mercury and Venus move in Epicycles round the Sun as a Center; thus far embracing the *Philolaic* Syſtem of the World, and bringing in again the old *Egyptian* one; that *Vitruvius* in his *Book of Architecture* and Venerable *Bede* in his *of the Nature of* approved of.

iolus in his *Almageſt*, adds Mars to the Atts of the Sun; ſo that ſuch as have no tes of their own, are made Satellites to n, and only Saturn and Jupiter that have tes, beſides the Luminaries, revolve about rth, as their immediate Center; and this , which is almoſt the ſame with the *Ty*he calls his own.

re is no need of explaining the Forces eir direction, that are requiſite to prehe *Ægyptian* Syſtem, or that other, which different from it, the *Ricciolic*: For that s very eaſy, from what has been ſaid a- As they are compoſed of the Syſtems deſcribed, ſo the Forces neceſſary to e them, are parts of the reſpective Forces ed before.

SCHOLIUM.

Section was ſubjoin'd, to the end, that ly Aſtronomers, (who are no ways afraid ultiplicity of the Celeſtial Motions, and t a Poſtulate, that any Star may be moth any motion,) weighing the ſeveral difSyſtems, might embrace that which agrees ith the Phænomena: But that the Philos may likewiſe chuſe ſuch a Syſtem as rees with Nature, namely, either the Old *c* or *Copernican* Syſtem, which agrees with ænomena, and may be preſerved by Graone, propagated in the ſame tenor thro' iverſe; or by an accelerative force of all towards all others, that is, in a given e, as the Body towards which it tends, ciprocally as the ſquare of the diſtance, he Body is given towards which it tends:

Or the *Semi-tychonic*, deſcribed in *Prop.* 79. that performs the ſame, as the *Copernican*, in regard of the Phænomena; but then beſides the ſame Gravity, as is requir'd in the Copernican Syſtem, (by *Prop.* 80.) it ſtands in need of another Force, whoſe Magnitude and Direction are perpetually changed; for this Force is equal to that, whereby the Earth would tend towards the Sun, and acts equally on all the Bodies of the Solar Syſtem, according to Right lines parallel to a Right line that connects the Sun and Earth, and with a direction from the Sun towards the Earth: Or (if it be reſolved upon, that the Earth ſhall be or is intirely at reſt) the *Tychonic Syſtem* deſcribed in *Prop.* 81. doing the ſame as to the Phænomena, in which (by *Prop.* 82.) beſides the two aforeſaid Forces, there is need of two more; namely, one equal in every point of the Axis of the World, that encreaſes as the Diſtances, and is at the Moon to the Force of Gravity in the duplicate ratio of the Periodic Month, to the Natural Day; and another Force like it, and ſimilarly propagated in every point of the Axis of the Ecliptic, that acts only upon the Fix'd Stars, and is at the Fix'd Stars to the former Force in the Axis of the World, in the duplicate ratio of a Natural Day, to the great Year, or period of the Fix'd Stars upon the Poles of the Ecliptic: Or the *Ptolemaic* Syſtem itſelf deſcrib'd in *Prop.* 83. But this does not agree with the Phænomena of the Heavenly Bodies, as has been ſhewn in *Prop.* 84. But if, ſetting aſide the Phænomena of the Phaſes of Mercury and Venus, we only attend to ſuch Phænomena as were known by *Ptolemy*, and his Syſtem be taken for true; (by *Prop.* 84.) beſides the two ſorts of Attractive Forces, equally diffuſed thro' the Axis of the World, and the Axis of the Ecliptic, there is

of an accelerative Gravity of the Sun,
nd Moon towards one another, without
g any of the other Planets, which if the
e be given, is as the Body towards which
, and towards the given Body, recipro-
; the ſquare of the diſtance from it; and
there is occaſion for another Gravity,
to that whereby the Earth would tend
the Sun,) acting equally upon the Sun,
nd Moon, along lines parallel to a Right
nnecting the Sun and Earth, and with a
n from the Sun towards the Earth.
d above, it ſtands in need of a particu-
e in the center of the Epicycle of each
Planets, Saturn, Jupiter, Mars, Venus,
ercury, that tends to the center of its
t; and ſtill of another particular Force
lanet itſelf, tending to the centre of its
Epicycle, without any prejudice in the
hile to the ordinary Gravity of the Sa-
f Jupiter and Saturn. But if the *Ægyp-*
r *Ricciolic*) Syſtem be moſt fancied,
indeed may be made to ſolve the Phæ-
,) then, (beſides the Forces reſiding in
; of the World and the Ecliptic,) there
of a mutual Gravity of the Sun, Mer-
enus, the Earth with the Moon, (and
as in the *Copernican* and *Tychonic*; but
one as does not reach ſo far as Jupiter
urn; and beſides, of a Force equal to
hereby the Earth would tend towards
, which acts equally upon the Sun,
y, Venus, the Earth together with the
and Mars,) along Right lines parallel
ht line connecting the Sun and Earth,
h a direction from the Sun towards the
and beſides of a particular Force in

the Center of the Epicycle of both Planets, Jupiter and Saturn, that tends to the Center of its Deferent, and of a particular Force in Jupiter and Saturn, tending to the Center of their proper Epicycle,

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THE

Plate 1 Book 1.

Fig: 1.
p: 2

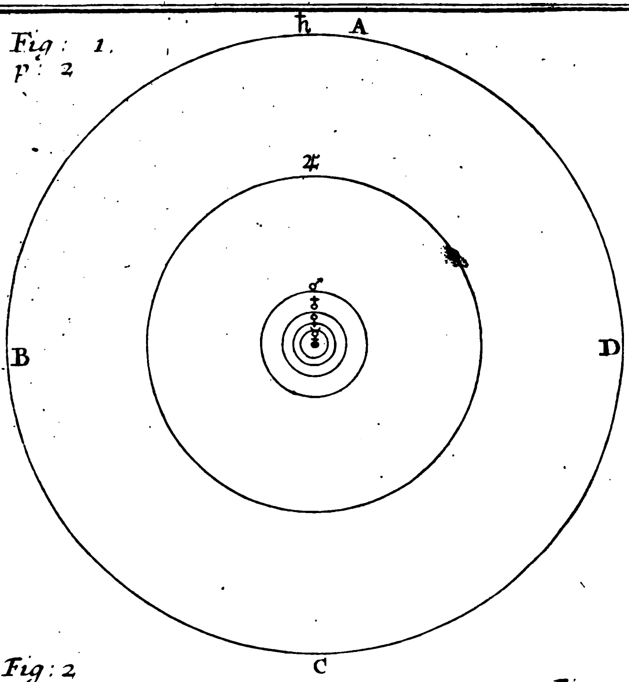
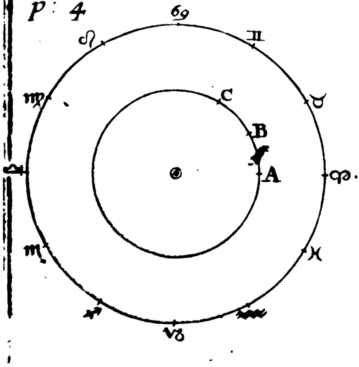


Fig: 2
p: 4



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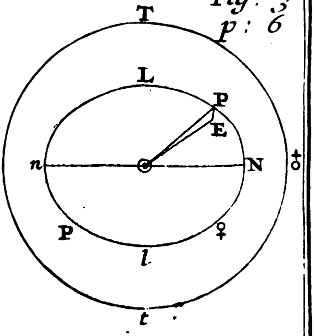

Plate 2. Book 1.

g:4.
8

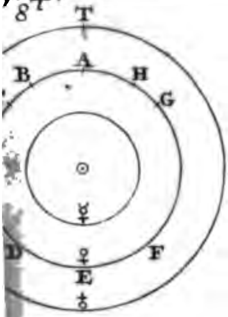

Fig: 5
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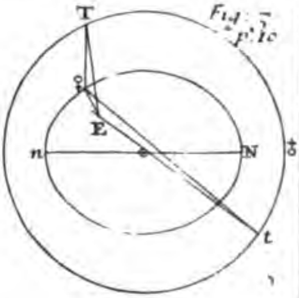

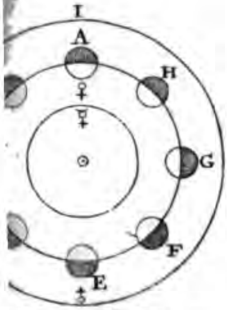

Fig: 7
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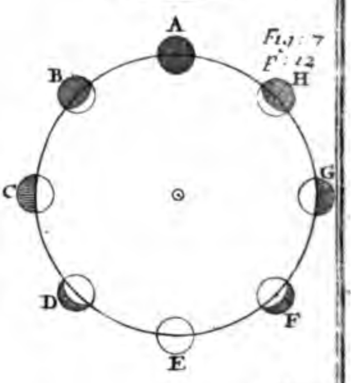

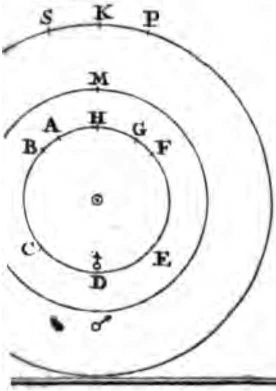

Fig: 9
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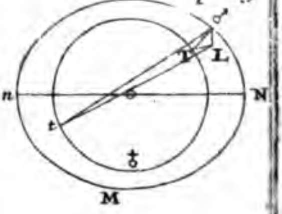

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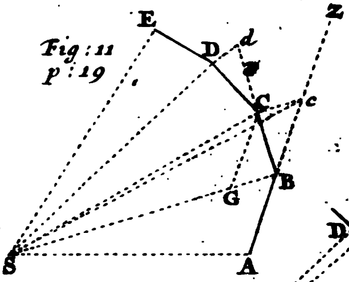

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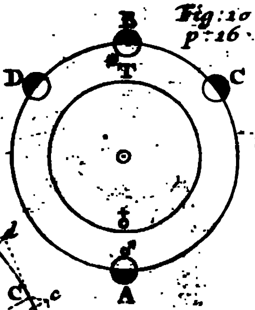

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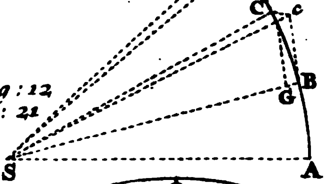

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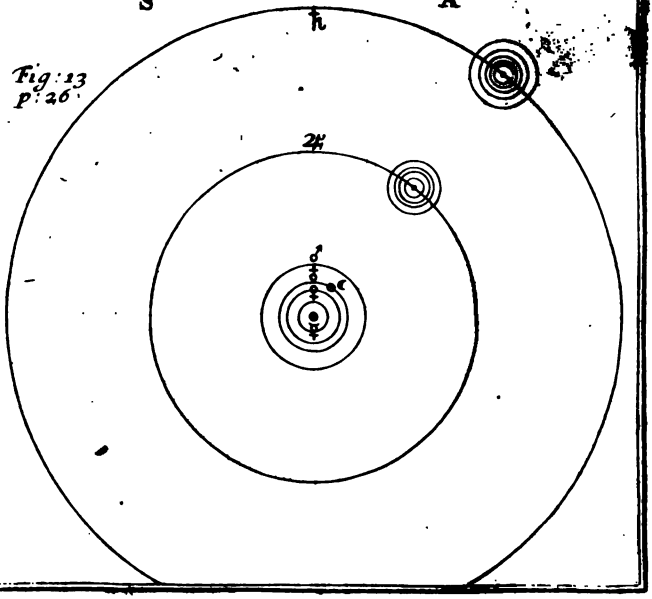


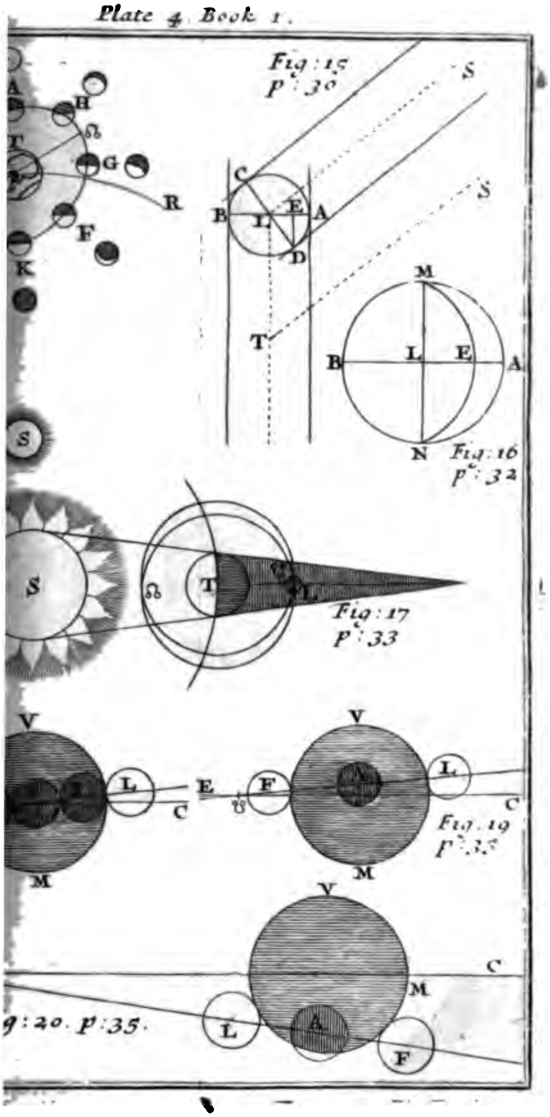


Fig: 15
P: 30

Fig: 16
P: 32

Fig: 17
P: 33

Fig. 19
P: 35

g: 20. p: 35.

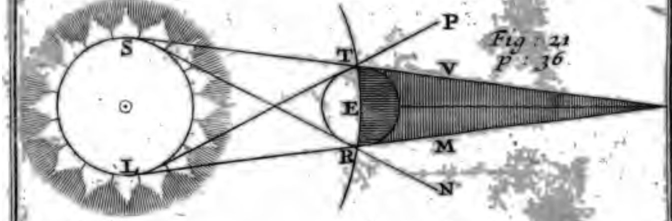
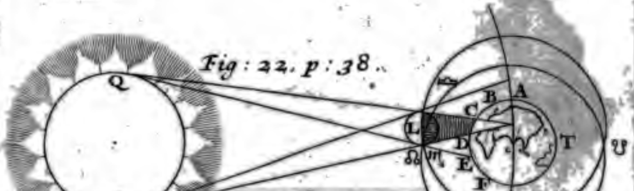
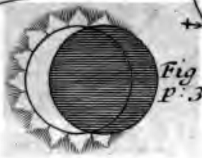


Fig: 21
p: 36

Fig: 22. p: 38.





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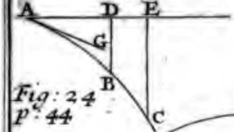

Fig: 24
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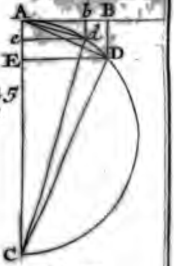

Fig: 25
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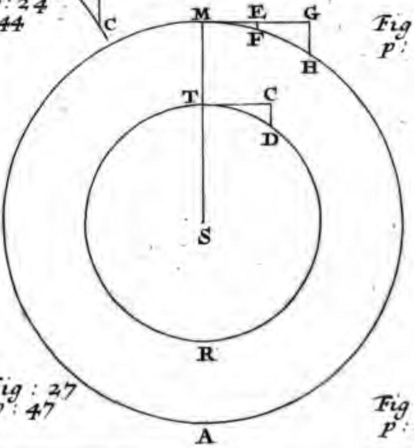

Fig: 27
p: 47

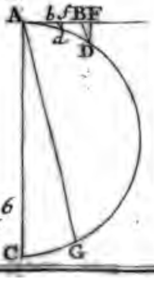

Fig: 26
p: 46

Plate 6 Book 1.

: 28

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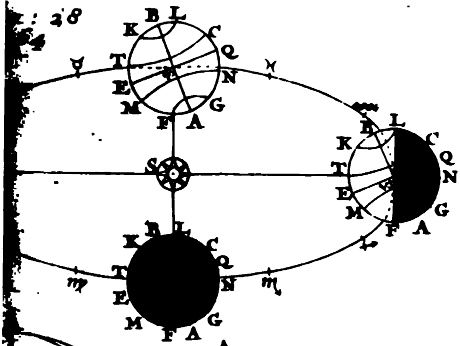


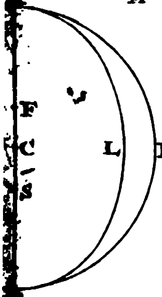
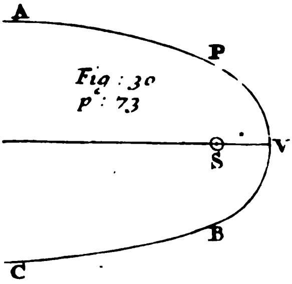


Fig: 30
p: 73



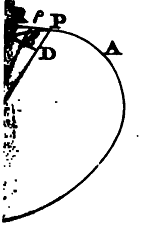


Fig: 32
p: 78

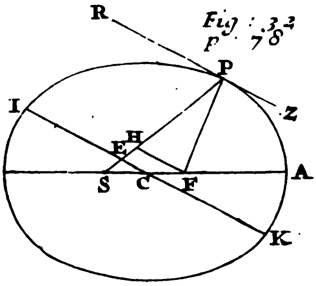


Plate 7 Book 1.

Fig: 33
p: 79

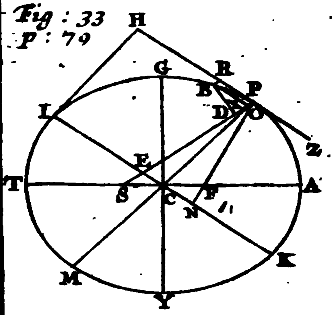


Fig: 36
p: 83

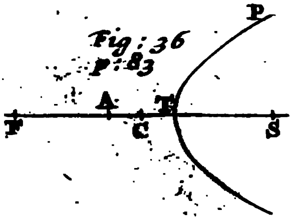


Fig: 34
p: 82

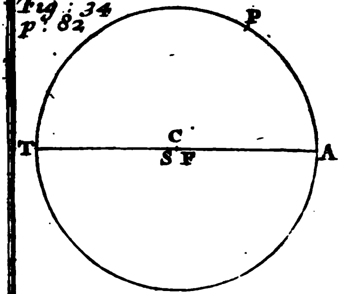


Fig: 37
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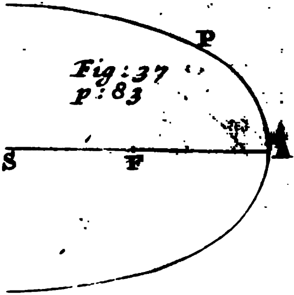

Fig: 35
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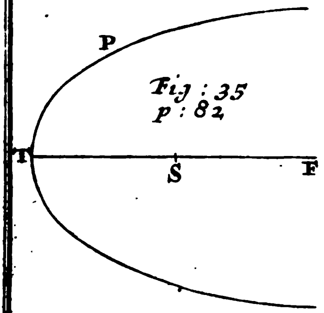

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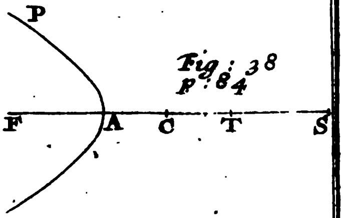

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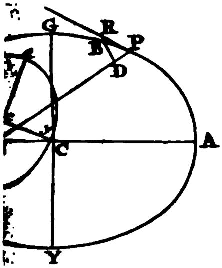

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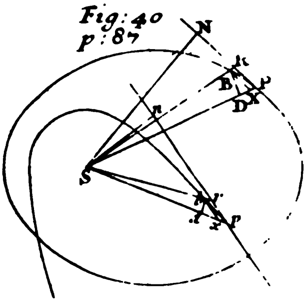

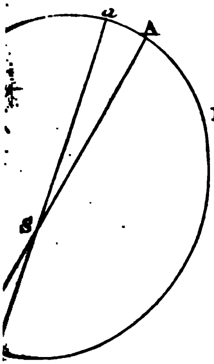

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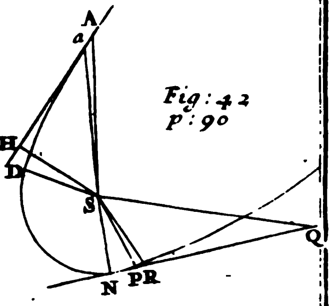

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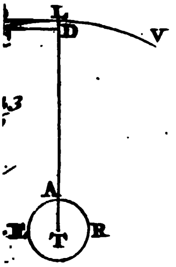
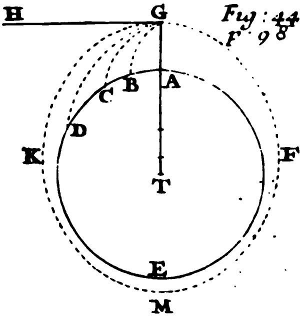


Fig: 44
p: 98



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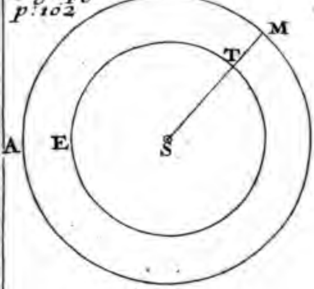

Fig: 45
p: 102

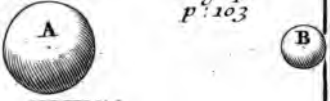

Fig: 46
p: 103

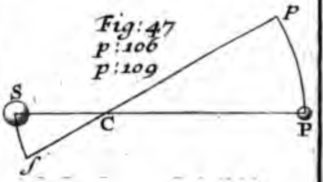

Fig: 47
p: 106
p: 109

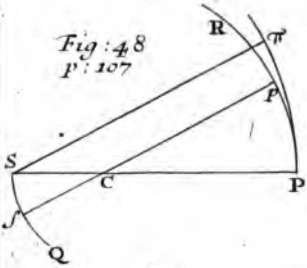

Fig: 48
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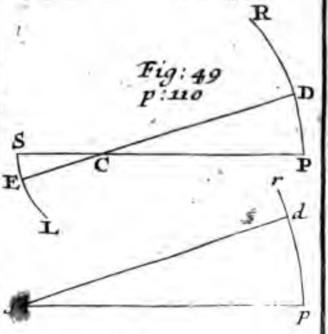

Fig: 49
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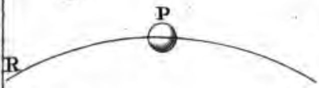

Fig: 50
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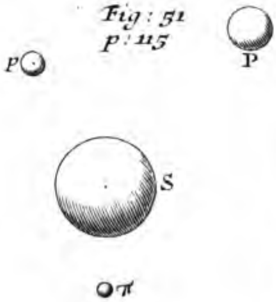

Fig: 51
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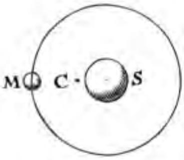
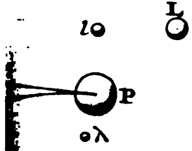


Plate 10 Book 1.



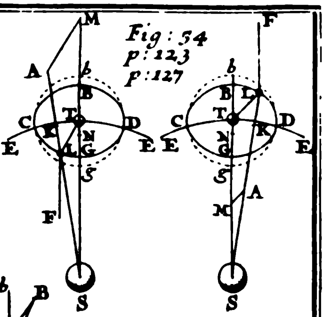


Fig: 54
p: 123
p: 127

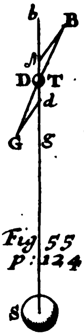


Fig 55
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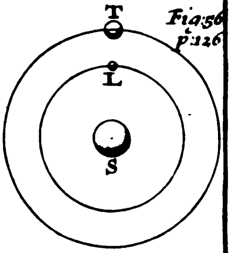

Fig: 56
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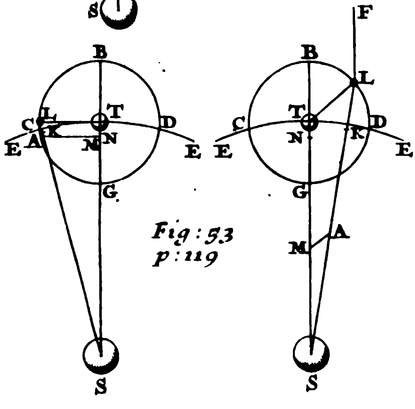
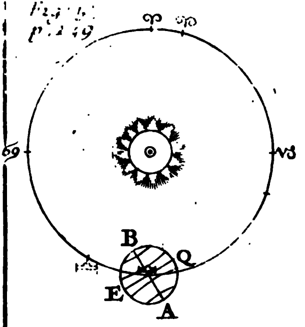


Fig: 53
p: 119

Plate 11 Book 1.

Fig: [illegible]
p: 149



Fig
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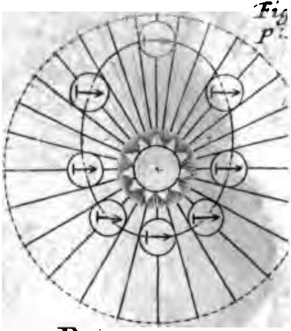

Fig: 59
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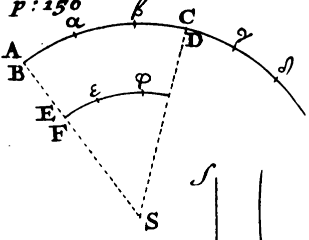

Fig: 60
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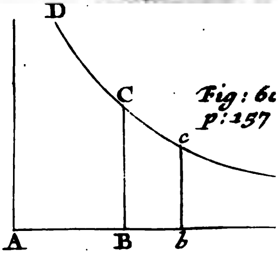

Fig: 61
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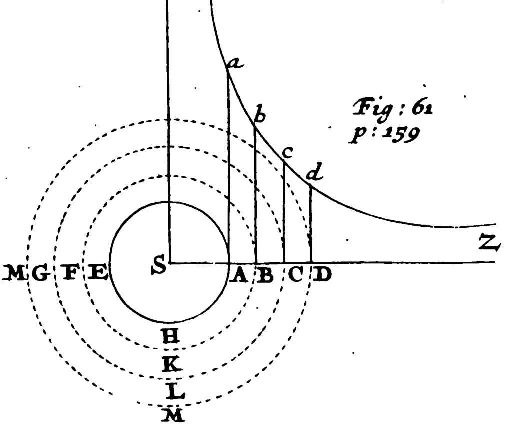

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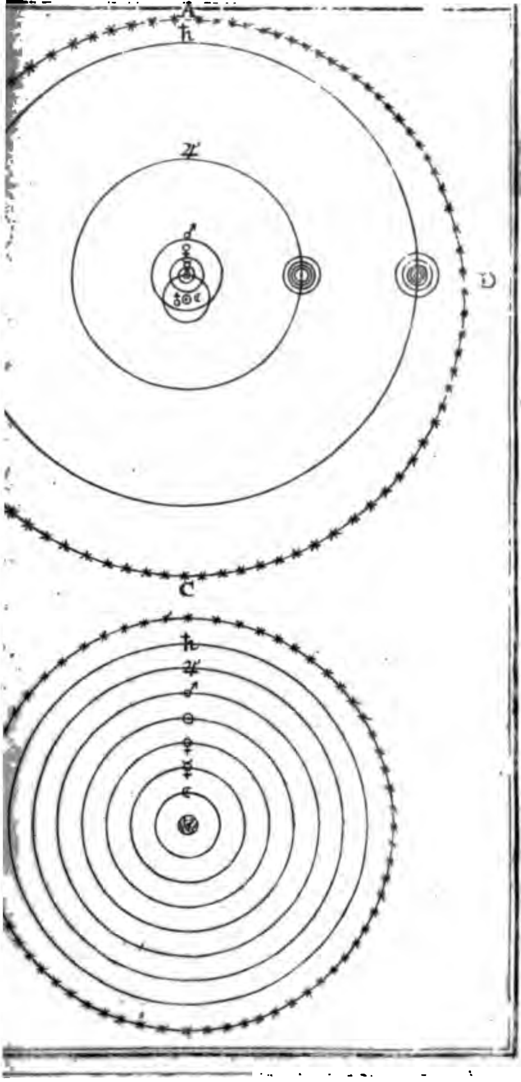

THE
LEMENTS
OF
ſtronomy,
ſical and Geometrical.

The SECOND BOOK.

Of the Firſt Motion.

the preceding Book, we have laid down
: Principles of a True and Phyſical Aſtro-
my; that is, we have explained, in gene-
: motions of the Stars, as are really in na-
ut ſome notice muſt be taken of the Ap-
e of them; that is, the Elements of an
my fitted to them, are to be formed.
t is out of all doubt to any Philoſopher
nſiders the Reaſons of Things, that the
carried about among the Planets, ſo 'tis
certain to every one that enjoys his
hat the Earth appears at reſt, in the

middle of the Univerſe, and that the Heavens, and all contained by them, are moved as to Senſe, round about it. Now not only the reaſon of this Phænomenon is to be aſſigned, (which has already been done, in the foregoing Book,) but the Methods muſt be explained, that the Aſtronomers have uſed, to define the places of the Stars, ſeen from the Earth, who either thought, with the common People, that things were really ſo as they appear, or who thought that this Syſtem of Appearances ought to be retained, tho' they knew for certain, that things were not after this manner; and that 'tis according to this Syſtem that we muſt ſpeak with the common People; and not make our Reaſon and Philoſophy perpetually offer violence to our Sight and other Senſes.

To this end is this Second Book deſigned; namely, the explaining the Words us'd in Aſtronomy, and the giving the deſcription and uſe of Spheres, Globes, and other Inſtruments, made to ſet before our Eyes principally the Diurnal Motion, and in general to ſearch out in this Phyſical Syſtem, and to handle according to the apparent Syſtem, and Calculate all thoſe matters as are uſually ſaid by Aſtronomers to belong to the *Doctrine of the Sphere*: And tho' the Doctrine of the Sphere be uſually firſt treated of, by all Authors, we have not been afraid to invert the Order; ſince by this method better care is taken againſt young Perſons in learning Aſtronomy, their taking the Syſtem where the Earth is at reſt for true, to which they are firſt accuſtomed, and to which they are more inclined by their Senſes; and that the Doctrine of the Sphere, in *Kepler*'s judgment, *ſtands in need of ſeveral things, that are borrowed of the Theoric part of Aſtronomy* (or Doctrine of the proper Motion

of

lanets, a faint Draught of which you
e preceding Book,) *by anticipation, where-*
orie Part may be deliver'd alone, without
e aſſiſtance of the Diurnal or Firſt Motion,
Doctrine of the Sphere, which treats of it.

SECTION I.

Generation of the Circles of the
e, and of the Terms uſed in Aſtro-
, depending thereupon.

PROPOSITION I.

explain the generation and nature of the
liptic and Zodiac, and the diviſion of them,
e Secondaries of the Ecliptic, and their uſes,
Terms uſed in Aſtronomy depending upon

bſerver placed upon the Earth, by ex-
his Sight every way equally, if not
d, bounds the World by a Spherical Su-
concentric with himſelf, (that is, with
h) and judges all remote Objects, whoſe
is unknown, tho' really very unequally
d from one another, to be placed as it
the ſame concave ſpherical Superficies;
inſtance, he does the Fixed Stars, Sun,
on itſelf; as was ſhewn in *Prop.* 32. *B.* 1.
here, concentric to the Earth and ſur-
g it, defined by the Sight, and beſpan-
th the twinkling Fixed Stars, ſupplying
e of an abſolute Space, is conſider'd as
while we abſtract from the Diurnal
whereby this intire Sphere, or rather
le Heavens, are revolved from Eaſt to
the ſame Space of 24 Hours. But ſince
n of the Annual Motion of the Earth
about

about the Sun, the Sun ſeen from the Earth ſeems to move forward daily towards the more Eaſtern Stars (as was ſhewn in *Prop.* 2. *B.* 1.) the Sun will ſeem to move in this immovable Sphere or Mundane Space; and its Path marked among the Fixed Stars is call'd the *Ecliptic*, becauſe the Eclipſes of the Luminaries happen only in this Line; as was ſhewn above.

Having once ſettled the Ecliptic, 'tis evident that as all the Orbits of the Planets about the Sun, are not in one Plane, but each of the Planets ſeen from the Sun, makes an excurſion, ſometimes on this ſide, ſometimes on that ſide of the Way of the Earth, mark'd among the Fix'd Stars; ſo the ſame Planets ſeen from the Earth, make an excurſion ſometimes on this ſide, ſometimes on that ſide of the Ecliptic, and even in the ſame points of their Orbits ſeem to run off variouſly, according to the different ſituation of the Earth, (as was ſhewn in *Prop.* 5. and 8. *B.* 1.) But their greateſt excurſion is contained within the boundaries of ten Degrees; the Aſtronomers therefore have called that tract which the Sun, Moon, and Planets ſeem to adorn with their motions, and which is a Zone or Belt, twenty Degrees broad, (ten on each ſide of the Ecliptic,) by the name of the *Zodiac*, from the images of Animals, which the Fixed Stars, in that tract, ſeem to repreſent. The number of the images, pitched upon by them, is twelve; either becauſe this number ſeems the fitteſt, for its being diviſible into parts without a remainder, as two, three, four, ſix, and twelve; or becauſe, while the Sun ſeems to run thro' the Zodiac, in the ſpace of a Year, there happen near upon twelve Lunations, or the Moon runs thro' all her Phaſes twelve times. Theſe parts of the Zodiac are called *Signs*, and their

arks are, Aries ♈, Taurus ♉, Gemini ♊,
r ♋, Leo ♌, Virgo ♍, Libra ♎, Scor-
Sagitarius ♐, Capricornus ♑, Aquarius
ces ♓. Every one of theſe Signs is again
l into 30 equal parts, ſo that the whole
(like all others) is divided into 360. Tho'
diac or any other Circle has no begin-
or ending, properly ſpeaking; yet Aſtro-
begin their reckoning from the firſt
f Aries, which the Sun is ſeen in when
ys and Nights are equal, and Spring be-
the Inhabitants of the Northern Hemi-
which alſo is the point of the common
tion of the Ecliptic and Æquator; as
wn in *Prop.* 33. *B.* 1. The reckoning is
om ♈ to ♉, ♊, and ſo on, till a return
to ♈, the beginning again towards the
ay that the Sun ſeems to move about the
And the motion or progreſs this way
o be *in conſequentia ſignorum*, or according
rder of the Signs; namely, that accor-
which the Sun, Moon, and almoſt the
he Planets ſeem to move: But that Phæ-
n, which is carried in the contrary or-
from ♉ to ♈, or from ♈ to ♓, is ſaid
oved contrary to the order of the Signs,
ecedentia.

uſe the Axis of the Earth after ſeveral
ons about the Sun, goes a little off
Site parallel to that it formerly had, de-
the Superficies of a Cone, the images
Stars have removed from the Signs of
diac, to which they originally gave
And this happens not upon the account
notion of the Stars or the Zodiac, but
the Æquator of the Earth is moved to-
with the Axis of the Earth, ſo as that
rſections of the Celeſtial Æquator, with
the

the Ecliptic, or Equinoctial points, (and conſequently all the points of the Zodiac, as they are expreſſible by numbers) remove *in antecedentia*; and the images or the Fixed Stars ſeem to be transferred, in reſpect of them, *in conſequentia*, as was ſhewn in *Prop.* 64. *B.* 1.

If innumerable Circles be ſuppoſed to be drawn thro' the Poles of the Ecliptic, cutting the Ecliptic at right Angles, which are called its *Secondaries*, any point in the Heavens may be referred to the Ecliptic, by the help of them; that is, any Phænomenon is underſtood to be in that point of the Zodiac or Ecliptic, in which ſuch a Semicircle, paſſing thro' the Phænomenon, cuts the Ecliptic. And the Phænomena that are after this manner referred to the ſame point of the Zodiac, are ſaid to be conjunct, or in *Conjunction*; but ſuch as are referred to the oppoſite points, are in *Oppoſition*. If a Quarter of the Zodiac lie between the points to which the appearances are referred, they are ſaid to be in a *Quartile* Aſpect, but if a third part of the Zodiac, in a *Trine* Aſpect; laſtly, if a ſixth part of the Zodiac lie between them, they are ſaid to have a *Sextile* Aſpect.

Beſides, an Arc of the Ecliptic intercepted between the beginning of Aries and the ſaid point of interſection, and reckoned according to the order of the Signs, is called, the *Longitude* of that Phænomenon or Point, as the Arc of a Secondary, intercepted between the Ecliptic and the ſaid Phænomenon or point of the Heavens, is called its *Latitude*; on which account theſe Secondary Circles are called *Circles of Latitude*. The Latitude is either *Northern* or *Southern*; for the Heavens are divided according to Aſtronomers, by the Ecliptic, (a Celeſtial Circle, becauſe originally conſider'd in the Hea-

ns) into the Northern and Southern He-
re.

PROPOSITION II.

*give an account of the geneſis, nature, and
uſes of the Celeſtial Equinoctial and its Se-
es and Parallels, and to explain the Terms uſed
onomy, depending thereon.*

e Plane of the Earth's Æquator or Cir-
ng exactly betwixt the Poles, upon which
th is revolved by its Diurnal Motion, be
d every way, it will interſect the Sphe-
rface of the Heavens, concentric to the
in a correſponding Circle in the Hea-
nd the Axis of the Earth produced will
t the Poles in the ſame Sphere. And
Earth, with its Axe and Æquator, be
about the Sun in the Annual Motion,
Circle deſcribed by the Earth, is ſo ſmall,
d of the Fixed Stars, that the Axis and
of the Earth (every where parallel to
es) being produced, falls upon the ſame
ars, as to Senſe, and therefore all the
xed Stars, that retain the ſame ſituation
l of one another, retain alſo the ſame
in regard of the Æquator and Poles of
ens, excepting ſo far as the Axis and
on the Earth do not continue parallel
another; concerning which we ſhall
the following Propoſition. And be-
e Earth revolves upon its own Axis,
ce of a natural Day, from Weſt to Eaſt,
ſeparate from the Earth will ſeem to
Circle in the Concave Sphere, paral-
Celeſtial Æquator in the ſame time
to Weſt, the Eye being placed in the
d judging its own Habitation to be un-
s was explained at large in *Prop.* 32. *B.* 1.

But becauſe the Earth is carried about the Sun in its Annual motion, the Earth's Æquator continuing always parallel to itſelf, applies itſelf only twice a Year to the Sun, ſo as that the Sun may be found in the Plane of it produced, and conſequently that the light of the Sun may reach to both the Poles of the Earth, and every point of the Earth turn'd about in the Diurnal motion, may be as long in the Light as in the Dark, (as was ſhewn in *Prop.* 33. *B.* 1.) that is, the Sun is ſeen only twice a Year to deſcribe the Celeſtial Equinoctial Circle in its Diurnal motion, in which caſe the Days will be equal to the Nights to all the Inhabitants of the Earth: For the Equinoctial interſects the Ecliptic in the Point in which the Sun appears among the Fixed Stars, when its Light reaches to both the Poles of the Earth, and cauſe the Equinox through all the Globe of the Earth. For in every other caſe, the Sun, in its Diurnal motion (like the other Celeſtial Bodies) ſeems to deſcribe Circles parallel to the Equinoctial, among which they are the moſt conſiderable, that the Sun ſeems to deſcribe in the Heavens, when the Poles of the Earth do moſt incline towards the Sun; that is, when the Sun is ſeen from the Earth in the beginning of the Signs of Cancer and Capricorn among the Fixed Stars; which therefore are called the *Tropics of Cancer* and *Capricorn*, becauſe the Sun returns immediately from thence towards the Equator. There are two other Circles alſo parallel to the Equinoctial Circle (namely thoſe which the Poles of the Ecliptic ſeem to deſcribe in their Diurnal motion; and by conſequence are as far diſtant from the Poles of the Equinoctial as the Tropics are from the Equinoctial,) called the *Polar Circles*, either becauſe they

are deſcribed by the Poles of the Ecliptic
e firſt or Diurnal motion, or becauſe they
iear the Poles of the Equinoctial, which
alled the *Poles of the World*; becauſe all the
d beſides the Earth, ſeems to be moved a-
them in the ſpace of 24 Hours. The Equi-
al Circle, as uſual, is divided into 360
ees,and theſe Degrees are reckoned towards
ame parts with the Degrees of the Ecliptic
odiac,namely, from Weſt to Eaſt, according
e order of the Signs, and from the ſame be-
ng; namely, the firſt Point of Aries. As
Point in the Heavens is referred to the
ac by the Secondaries of the Ecliptic, ſo
e Secondaries of the Equator, it is referred
Equinoctial. And the Arc of the Secon-
comprehended between the Point and the
tor, is called the *Declination* of that Point;
or *South*, according as it is towards this
at Pole. For a Point is ſaid to decline juſt
ich as it is diſtant from the Primary Circle,
the middle and principal one of the Diur-
iotion which moſt affects our Sight. This
nce or Declination, belongs to the Se-
aries of the Equator, which are therefore
, the *Circles of Declination*: The chief of
are the two *Colures*, one of which paſſing
the interſections of the Ecliptic and Equa-
r points of the Equinoxes, is called the
octial Colure; and the other at right angles
the former, is called the *Solſtitial Colure*, be-
it meets the Ecliptic in the remoteſt Points
the Equator, where the Solſtices are ce-
ed.

PRO-

PROPOSITION III.

TO *give an account of the nature and uſes of the Terreſtrial Equator, and its Secondaries and Parallels, and explain the Terms uſed in Aſtronomy and Geography depending thereon.*

The Equator is properly a Terreſtrial Circle, becauſe exiſting originally in the Earth, and transferred to the Heavens only upon the account of the Firſt Motion, (which is only an apparent one in the Heavens.) Its four principal Parallels, the two Tropics, and as many Polar Circles, may be underſtood to be on the Earth, either as ſo many Circles lying directly under the Circles in the Heavens of the ſame name, the generation of which we have ſhewn in the preceding Propoſition; or (which is more natural) as originally generated in the Earth itſelf. And then the Tropics will be Circles on the Earth, lying directly under the apparent Courſe of the Sun, when the Pole of the Earth, which is neareſt, inclines moſt to the Sun: But the Polars, ſuch as bound thoſe tracts of the Earth, as have at that time perpetual Day or Night, as was explain'd in *Prop.* 33. *B* 1.

Theſe four Circles parallel to the Equator, divide the Globe of the Earth into five *Zones*; of which, that is the *Torrid*, which is contained between both Tropics, becauſe lying directly under the courſe of the Sun, and receiving its direct and, by conſequence, moſt powerful Rays. The Inhabitants of this Zone are called *Amphiſcians*, becauſe the Shadow of a Perſon ſtanding upright, moves as well towards the right as the left hand of him that obſerves it, and becauſe the Noon Shadow at certain different times of the Year, is projected towards both the Poles. The tracts of Land included within the two ircles, make two other *Zones*; both of *igid*, becauſe they receive only the oblique nſequently the weakeſt Rays of the at are not therefore able to thaw or diſ- e cohæſion of the Ice, that froze during nter and long Night-time, in the follow- nmer, or as long a Day-time. The Inha- of theſe Zones, in regard of the Shadow, ed *Periſcians*, becauſe the ſhadow (the Sun ing) moves round about them. Between rrid, which is the middle, and the two which are the extreme Zones, lie two call'd the *Temperate Zones*, becauſe par- of the affections of both the adjacent s; the *Northern* of which is bounded Tropic of Cancer and the Arctic Polar and the *Southern*, by the Tropic of Ca- and the Antarctic Polar Circle. And Ancients ſaid that theſe only were ate and habitable, yet experience now us, that both the Frigid and Torrid re fit for the nouriſhment of Animals, getables; ſeveral other Cauſes, partly , and partly Aſtronomical, making com- n for the too direct or oblique inci- f the Rays. The Inhabitants of the ate Zones are called *Heteroſcians*, be- eir Noon ſhadow is thrown only to- ne Pole, *viz.* the neareſt, and carried a- y towards one quarter, as it is with us, bit the Northern Temperate Zone, and wn ſhadow projected towards the right

, ſuch as live upon the Equator, have ys and Nights always equal, becauſe as Equator as the Circle diſtinguiſhing htened part of the Earth from the ob- great Circles of the Sphere; and , (by *Prop.* 11. *Book* 1. *Theodoſ.*)

they biſect one another; conſequently, each Point in the Equator carried round about equably in its Diurnal motion, is as long in the light as in the dark. But to an Inhabitant living without the Equator towards either Pole, the Days are longer than the Nights, when the Pole of the Earth, that is neareſt inclines towards the Sun, and are then the longeſt, when that Pole inclines moſt towards the Sun: And the farther an Inhabitant lives from the Equator, the longer are the longeſt Days to him, till at laſt at the Polar Circle in Summer, there is no Night at all; as has been ſhewn at large in *Prop.* 33. *B.* 1. This is the reaſon why *Ptolemy* and other Geographers, divided the Earth by Circles *Parallel* to the Equator, diſtant from one another, and their beginning (*viz.* the Equator,) ſo much as that the longeſt Day is a quarter of an Hour longer in one *Parallel* than in the other next *Parallel*. They take no notice of any diviſion into leſſer parts, becauſe they would be ſcarce ſenſible; and they looked upon this ſpace (tho' properly a little Zone) as indiviſible in regard of Latitude, and therefore they call'd it a *Parallel*, Circle being underſtood. Making the Equator then to be the firſt Parallel; the ſecond paſſes thorough thoſe parts of the Earth, where the longeſt Day is 12$\frac{1}{4}$ Hours; the third throrough them, where the Day is 12$\frac{1}{2}$ Hours, and ſo on; and in the other Hemiſphere after the ſame manner. Two ſuch ſpaces make a *Climate*, which therefore differ from one another the length of half an Hour. Each Climate receiv'd a name from ſome conſiderable Place lying in it near its middle: and therefore there muſt be 24 of them reckon'd from the Equator to either Polar Circle, ſince the greateſt or longeſt Day is 12 Hours longer. But the Ancients

did not begin this reckoning from the
or itſelf, but made the firſt Climate to
iro' the Iſland *Meroe*, where the longeſt
was found to be an whole Hour longer
12 Hours, either becauſe they left this Re-
near the Equator for a right Sphere, in
l of which, the other Regions on the ſides
are called Κλίματα; or becauſe perhaps they
l the interior parts to be uninhabited:
Ptolemy ſays, there were ſeveral in his
who maintain'd, that there were habita-
s under the Equinoctial itſelf, juſt as it
in a temperate Region; but the People
Age, had not to that Day penetrated thi-
Ptolemy has not given us an exact compu-
of the Parallels, ſo far as to the Polar
itſelf, they being more numerous there-
s, and he not thinking it worth his while
over nice in his enquiries in the more
ern Parts. As for the Climates on the o-
de of the Equator towards the Antarctic
he has given each of them ſuch a name,
ifies that this unknown Climate is as far
t from the Equator towards the South, as
ain known one is from the ſame Equa-
wards the North; as the Climate Αντι-δια-
Αντι-δια-Συήνης, &c.

the Places of the Stars, or any Points in
leavens, are computed as to their Longi-
and Latitude, by the means of the Eclip-
which is properly a Celeſtial Circle) and
ondaries; ſo the Longitudes and Latitudes
ces on Earth, are computed in the Equa-
which is properly a Circle on the Earth,)
s Secondaries paſſing thro' the Poles of the
. Any ſuch Secondaries drawn thro' any
upon the Earth, is called the *Meridian* of
Place, becauſe when the Earth has in its

Diurnal revolution arrived to ſuch a ſituation, as that the Sun is in the Plane of that Circle produced, it is Noon-Day; as ſhall be more largely explain'd hereafter. The *Latitude* of any Place is an Arc of the Secondary of the Equator, or of a Meridian, intercepted between the Equator and the Place: And it is either *North* or *South*, as the Place is diſtant from the Equator towards the North or South Pole. But the Longitude of a Place is computed from the Weſt towards the Eaſt in the Equator itſelf, becauſe the boundary of the Land towards the Weſt is better known than that towards the Eaſt. And becauſe the Equator has no beginning, nor any Cardinal Point, as there is in the Heavens, marked out by its Interſection with the Ecliptic, the firſt Meridian, from whoſe Interſection with the Equator the beginning of the reckoning is to be made, was left to the liberty of the Geographers to be ſettled. Now, they allow of no dry and habitable Land beſides the Contingent they themſelves inhabited, together with the adjacent Iſles; they feigned therefore a firſt Meridian paſſing thro' the moſt Weſtern Place of the Earth, that was then known, (as the *Azores* or *Fortunate Iſlands*; and from thence towards the Eaſt, they diſpoſed of all Places on the whole Surface of the Earth, in regard of Longitude; and called the *Longitude* of the Place, that Arc of the Equator which was intercepted between the *Firſt Meridian* and the *Meridian* of the Place. But when they found the Globe of the Earth was really Inhabited all about, and there was no ſuch thing as a moſt Weſtern Place, the above-mention'd way of reckoning the Longitude of Places was by degrees neglected, which would have been of great uſe in underſtanding the Monuments of the Ancients; (tho' it might

made uſe of;) and every one made the
an of his own City to be the chief, and
otice how the Meridians of other Places
l towards the Eaſt or Weſt ſtand in regard
, or how many Hours there are between
re Eaſtern Meridians leaving the Sun,
e more Weſtern Meridians overtaking of
etween the Sun's ſeeming to leave the
Eaſtern Meridian, and arrival at the more
n.

in, the Inhabitants of the Earth being
red together in regard of the Meridians
rallels, ſome of them are called by Geo-
rs the *Periæci*, and they inhabit in the
arallel of the Earth, but in oppoſite Me-
(The oppoſite Meridians are the oppoſite
of the ſame Circle computed from the
becauſe ſuch Places as are in ſuch oppo-
micircles are referred to oppoſite points
Equator of the Earth.) On this account
æci enjoy the ſame Seaſons of the Year,
on of the inclination towards the Sun of
le of the Earth which is neareſt to both,
declination from the Sun; or ſpeaking
ing to the apparent reſt of the Earth, be-
he Sun approaches after the ſame man-
wards the Parallel of both Places, (becauſe
ne) or recedes from it: But they have
te changes of Noon and Midnight, ac-
g as the Meridian of the one or the o-
turned to the Sun by the diurnal revolu-
the Earth, or (which is all one,) as the
rried about us in its apparent diurnal
tion, approaches to the Meridian of the
the other of them; unleſs they live in a
Zone, where they enjoy the Day toge-
Again, other Inhabitants of the Earth are
Antæci, and they live in the ſame Meri-

dian, but in oppoſite Parallels, ſo that they have the ſame Noon and the ſame Midnight; becauſe they have the ſame Longitude. But the Summer of the one is the other's Winter, according as the Earth in its annual motion turns ſometimes its North Pole, and afterwards its South Pole, more towards the Sun; or as the Sun declines from the Equator to this or the other Pole. And laſtly, others are call'd *Antipodes*, becauſe they live in oppoſite Meridians and Parallels, and walk with their Feet diametrically oppoſite to ours. Theſe have exactly the contrary things happening to them: The one has Summer and long Days or no Nights, at the ſame time as the other has Winter and ſhort Days or perpetual Nights; 'tis Night here when 'tis Day there, and Night there when 'tis Day here. 'Tis evident an Inhabitant of the Equator, is an Antœcian to himſelf; and that the ſame Perſon is both Periœcian and Antipode to him; but that an Inhbitant of both the Poles is a Perœcian to himſelf, and that his Antœcian is the ſame as his Antipode.

POPOSITION IV.

TO give an account of the generation and nature of the Horizon and its Secondaries and Parallels, and to explain the Terms in Aſtronomy depending thereon.

Beſides the lately mention'd Circles of the Equator and Ecliptic, in reſpect to which, Aſtronomers have determined the Places of the Stars viewed from the Earth, there is another called the *Horizon*, which is the great Circle, that every one ſtanding in an open Plain, defines by turning of his Sight round about, and that divides the viſible from the inviſible part of the Heavens. This Circle is the moſt conſiderable

f all in a Sphere to appearance; becauſe
iately taken notice of and determined by
bſerver, tho' the moſt unskilful. Its ge-
n the true Syſtem, has been deliver'd a-
n *Prop.* 32. *B.* 1. But becauſe the Spheri-
perficies, to which we referred all Cele-
hænomena, is ſuppoſed to be concentric
he Earth and not the Eye, a Plane that
thro' the Eye and touches the Earth, will
ivide it into equal Segments; therefore
rizon there deſcribed, generated by the
n of the ſaid Plane and Sphere, is proper-
led the *Senſible* Horizon; becauſe defined
ſe: And the *Rational* Horizon of this Spe-
is that which is made by the Section of
d ſpherical Superficies, by a Plane paral-
he Senſible Horizon, and paſſing thro' the
of the Earth. Theſe two parallel Planes
ced, will mark out the ſame Circle in the
icies of that very great Sphere, wherein
xed Stars are ſeen, becauſe the Earth com-
to the Sphere of the Fixed Stars, is but
Point. From whence 'tis evident, that
rizon conſidered among the Fixed Stars
eat Circle in a Sphere, every way equi-
from the Point, exactly over the Head of
rſon (whoſe Horizon it is,) called the *Ze-*
d from the oppoſite to it the *Nadir*: Which
therefore, namely the Zenith and Nadir
Poles of the Horizon, in which the Secon-
of the Horizon, drawn thro' all the points of
avens, croſs one another, which are there-
alled *Verticals*, and ſometimes *Azimuths*;
e Parallels of the Horizon, whether ſup-
to be above or below, towards the Ze-
r Nadir are called *Almicanters*. There are
f theſe Vertical Circles, that are moſt con-
le, the one paſſing thro' the Zenith (the com-

mon Node of all the Verticals) and the Poles of the World, call'd the *Meridian*; namely the *Celeſtial*, becauſe being directly over the Terreſtrial Meridian: This Circle interſects the Horizon in the Cardinal Points of South and North, and marks them out. The other chief Vertical is at Right angles to this, and interſects the Horizon in the poins of Eaſt and Weſt; and becauſe the former, tho' a Vertical, is called likewiſe by another name, *viz.* a Meridian, this latter has the name of the *Primary Vertical* left intirely to itſelf. By the help of the Horizon and its Secondaries, any point of the Heavens whatever is diſpoſed of according to its Altitude above or Depreſſion below the Horizon, and Azimuths. That is, the *Altitude* or *Depreſſion* of any Point is an Arc of a Vertical Circle intercepted between the ſame Point and the Horizon: And the *Azimuth* is an Arc of the Horizon intercepted between the North or South Cardinal Point and that Point wherein the Vertical drawn thro' the Phænomenon, meets the Horizon; which is *Eaſtern* or *Weſtern*, according as you reckon from the Meridian towards the Eaſtern or Weſtern part of the Heavens. Sometimes the Azimuth is reckoned from the Eaſtern or Weſtern Cardinal Point towards the North or South. But the *Eaſtern* or *Weſtern Amplitude* is always reckoned from theſe Points, which is an Arc of the Horizon, reckoned from the Eaſt or Weſt to the Riſing or Setting point, and therefore both may be *Northern* or *Southern*; tho' this way of naming them ſeems to have flown firſt from intire Conſtellations; and the queſtion was, how large a ſpace of the Horizon any Conſtellation took up in Riſing or Setting.

at portion of the Convex Surface of the n, that the Spectator ſtands upon, being as a Plane parallel to the Horizon; the al Quarters of the Winds are conſider'd in nd the Cardinal ones are the North, South, and Weſt; lying under the Meridian and e Vertical, deſcribed above, and are the ons of the Horizontal Plane with the es of the ſaid Circles. The Motion of the vens will point out to us theſe four Quar- namely the Eaſt, where the Sun riſes in Equinoxes; the Weſt, where the Sun ſets Day: The North, where the Pole of the d is ſeen by us the Inhabitants of the Nor- Hemiſphere of the Earth, and the Stars the *Septem Triones*, always appear: And , the South, whence the Sun ſhines upon Noon. For the Terms in Aſtronomy are mmodated to ſuch tracts of the Earth as ituated in the Northern Temperate Zone, uſe Aſtronomy was firſt cultivated in thoſe . There are as many lying exactly betwen our Cardinals, that have names (in the *En-* and other Tongues a-kin to it) made up of Cardinals that are on their ſides, ſo as that name of the chief Cardinal Point is ſet Betwixt theſe eight are placed eight more, ng Names made up of the Names of the t preceding, ſo as that each of them is made f the two laterals next it, and the princi- Cardinal ſet firſt: Hence it comes to paſs, the name of the chief Cardinals is preſent- oubled, and the names of the other Cardi- in the beginning and end of the Word thus pounded. Between the ſixteen Points na- , there are ſixteen others having compound es, each from one of the eight firſt, to h the name of the Cardinal Point towards which it declines, is connected by a Propoſition; and thus we have two and thirty Wind's Ways, or Points, which are the common interſections of as many Verticals with the Plane of the Horizon, and ſerve to diſtinguiſh the Winds accurately enough. But we uſe the Degrees of the Horizon, in reckoning the Azimuths of the Celeſtial Phænomena, beginning from one of the four Cardinal Points, as was ſaid above. Beſides theſe Points, (while they are conſidered in the Plane of the Horizon) are looked upon as Right lines; but if the ſurface of the Earth be looked upon as Spherical (as it really is,) none of them produced upon the ſurface of the Earth are Right lines,and only the Cardinals are Circles; namely, one pointing out the North and South, being a great Circle, and the ſame with the Meridian of the Place, from whence it takes its beginning; the other ſhewing the Eaſt and Weſt, being a leſſer Circle, paſſing thro' the Place, and parallel to the Equator; excepting when the Place is in the Equator itſelf, in which caſe it is a great Circle; becauſe the Equator itſelf. The other Points of the Compaſs being produced upon the ſurface of the Earth are Spirals *ſui generis*, cutting all the Meridians at given Angles,and are called *Rhumbs* or *Loxodromics.*

PROPOSITION V.

TO *explain the generation and nature of the Celeſtial Meridian and other Hour Circles.*

In *Prop.* 3, we explain'd what ſort of a Circle the Meridian of each Place conſidered upon the Earth was; namely, a Circle compaſſing the Earth and paſſing thro' the ſaid Place and both the Poles; and conſequently a great Circle, becauſe paſſing thro' the oppoſite Points. And the Celeſtial Meridian of the ſame Place is that

e Heavens, which lies directly over and
nſwers it. And becauſe the Obſerver con-
ng in the ſame place, looks upon the
, and therefore the place where he ſtands,
the point over his Head as unmoved, he
iſe conceives of the Celeſtial Meridian
g through the Poles and the Vertex of
Place, which are unmoved, as a Celeſti-
rcle unmoved: And ſeeing the Heavens
ll the Stars ſeem to move round in the Di-
motion, he conceives of the Meridian of
lace where he is, as what has no ſhare of
notion, and as it were, without the mo-
Heavens, and of the Heavens, as if they
revolved within it. And it is always
a at that place, when the Sun, by the Di-
revolution of the Heavens, ſeems to have
ed at the above-mentioned immovable Ce-
l Meridian above the Horizon; and Mid-
t, when the Sun has reached the other
of it below the Horizon. For ſince this
ſtial Meridian paſſes thro' the Poles of
Iorizon and Equator, and (by *Prop.* 15. *B.*
bæric. of Theod.) interſects the Horizon and
Equator together with its parallels at right
es; 'tis evident that the Meridian divides
egments of all the Circles parallel to the
tor made by the Horizon, into equal parts;
therefore ſince the Sun deſcribes in its Di-
l motion, one of theſe Parallels, (the Arc
e the Horizon in the Day-time, and the
or Portion of it below the Horizon in the
nt,) 'tis evident that Mid-day or Noon is
made, when the Sun comes to the Meri-
above the Horizon, and Mid-night when
rives at its oppoſite part, lying concealed
w the Horizon: from whence it has its
e. And for the ſame reaſons the middle
t of the continuance of any Star above or
below

below the Horizon happens, when that Star comes to the Meridian: Where, at the ſame time, it has its greateſt elevation, call'd, its Meridian Altitude.

Again, ſince the ſpace of time between the two neareſt Noons is ſuppos'd to be divided into 24 equal parts, (called *Hours*,) and by reaſon of the equable Revolution of the Earth about its own Axis, the Sun ſeems to deſcribe equably the Equator, or ſome parallel to it upon the Poles of the Celeſtial Equator; beſides the Meridian, there are eleven other *Hour Circles* to be conceived of, paſſing thro' the Poles of the Equator, and together with the Meridian, dividing the Equator into 24 equal parts; and theſe are look'd upon (in regard of the ſame place) as immovable, and like the Meridian, placed without the Sphere, the Heavens in the mean while revolving equably under them. From whence it is evident, that an Arc of the Equator intercepted between any two next of theſe Circles is 15 Degrees, that is, 24th part of an intire Circle. But when the Terreſtrial Meridian is changed, the Obſerver changes all his Hour Circles with it. They are called Hour Circles, becauſe the Sun, ſeen to have arrived at any of them, by its apparent Diurnal Motion, cauſes the Hour of the Day to be that before or after Noon, which this Circle is in order, more Eaſtern, or more Weſtern than the Meridian. An infinite number more of theſe ſort of Circles may be imagined, according as we ſuppoſe the Hour to be divided into 60 Minutes, each of which is again divided into 60 Seconds, and ſo on. Theſe Hour Circles are the ſame in poſition, with the Circles of Declination; (of which in *Prop.* 2.) becauſe they are Secondaries to the Equator: But they differ

in

is, that the Circles of Declinations re-
, together with the Stars and Points of
eavens, whoſe Declination they meaſure;
heſe Hour Circles (as was ſaid before)
ok'd upon as immovable. Hour Circles alſo
e Heavens, anſwer to Meridians on the
; and indeed the Meridians on the Earth,
e real Hour Circles, and are only appa-
ones in the Heavens. For as 'tis Noon in
n Place, when the Earth, revolving by its
al Rotation, arrives at ſuch a ſituation,
the Plane of the Meridian produced fal-
pon the Sun: ſo it is ſuch or ſuch an Hour,
or after Noon, as that Terreſtrial Me-
, in order from the Meridian of the Place,
whoſe Plane produced the Sun is then
.

PROPOSITION VI.

O explain the various appellations of the Sphere
of the World, and other Terms uſed in Aſtro-
depending upon the various inclination of the
m to the Equator.

ce according to the diverſity of Places
the Earth, the Horizon, and conſequent-
Face of the Heavens, (that is, the Sphere
e World,) is different; Aſtronomers make
iverſity threefold, according to the three
ent kinds of Poſition of the Horizon to
quator. For either the Horizon of the
is right to the Equator, cutting it at right
s, or it is oblique, or elſe does not cut it
but coincides with it. Such as have the
'oſition of the Sphere, are ſaid to inhabit
ht Sphere; ſuch as have the ſecond, *an Ob-*
and ſuch as have the third, a *Parallel*
, becauſe each Star ſeems to deſcribe a
e, parallel to the Horizon in its Diurnal
on.

In a right Sphere, (namely, where the Equator *E* ♋ is erect or perpendicular to the Horizon *HO*, as in the 1ſt Figure,) the Equator (by *Prop.* 15. *B.* 1. *Theodoſius*) will paſs thro' the Poles of the Horizon, the Zenith, and Nadir; the Place therefore itſelf, on the Surface of the Earth, will be in the Terreſtrial Equator; for ſuch only have the Zenith and Nadir, in the Celeſtial Equator; all ſuch therefore, inhabit a Right Sphere, and only ſuch as are ſpread along the length of the Terreſtrial Equator. Their Horizon paſſes thro' the Poles of the World *A* and *B* (by the abovecited Propoſition of *Theodoſius*) and every Point in the Heavens will ſeem to aſcend right or perpendicularly above the Horizon; becauſe it deſcribes a Circle parallel to the Equator, (which is right or perpendicular to the Horizon,) in its apparent Diurnal Motion. And the Points, that riſe together, come to the Meridian alſo together, and ſet together; becauſe the Poles, about which the Diurnal Motion is made, by which the Stars ſeem to riſe, come to the Meridian, and ſet, are as well in the Horizon as in the Meridian of this Sphere. And from this ſituation of the Sphere, the way of reducing the points of the Heaven to the Equator, derives its original. That is, the *Right Aſcenſion* of any point in the Heavens is an Arc of the Equator, reckoned towards the Eaſt, intercepted between the beginning of Aries, and the point of the Equator that riſes, together with the ſaid point in the Heavens, in a Right Sphere: After the ſame manner we are to underſtand what the *Right Deſcenſion* is: and inſtead of the Horizon of a Right Sphere, the Meridian of any Place may be aſſumed. Now, becauſe only the Inhabitants of the Terreſtrial Equator have a Right Sphere, e, what the affections of this Sphere are, lent from what has been ſaid above; (e-lly in *Prop.* 33. *B.* 1.) namely, that the s are always equal to the Days, and every of the Heavens is as long above as below orizon.

an Oblique Sphere, where the Horizon (*Fig.* 2.) cuts the Equator *E* ♋ at oblique es; neither of them paſſes thro' the Poles e other, and therefore one Pole of the E-r or World is above the Horizon, the below it; the former therefore is always e, but the latter always inviſible: Nor will quator paſs thro' the Vertex *Z*, but the x will lie between the Equator and the e Pole. There are two kinds of this re; for either the North Pole *B* is elevated e the Horizon *HO*, and the South lies un-; or on the contrary, the South Pole is e-ed, and the North depreſſed. All ſuch have phere of the former kind, as live between erreſtrial Equator and the Arctic Pole, and as live between the Equator and Antarctic the latter. The chief Phænomena of both em, as to Summer and Winter, may be already explained, in *Prop.* 33. *B.* 1. ac-ing to the true Syſtem of the World. And e apparent Syſtem, ſince each point in the vens, by its equable diurnal Motion de-es either the Equator or a Parallel to it; only the Equator, but not one of the Pa-ls is divided into two equal parts, by an que Horizon; the Sun, and all the Stars, decline towards the viſible Pole, continue er above the Horizon than below; and on contrary, ſuch as decline towards the invi-Pole, continue hid longer below the Ho-n, than they appear above the Horizon.

And this holds to a certain limit towards both Poles. For if the declination be increaſed, till the diſtance of the Star from the viſible Pole, is leſs than the elevation of the Pole above the Horizon, the Star does not ſet at all: But if its Diſtance from the inviſible Pole be leſs, it will not aſcend above the Horizon; that is, it will not riſe at all. And the Parallels *OV*, *HI*, bounding the always viſible and the inviſible Stars in a given Place, and touching the Horizon, are called, by ſome of the Ancients, (as *Euclid* in the *Phænomena*, and *Manilius* in his *Aſtronomy*) the *Polars* of that Place, which are therefore greater and more remote from the Celeſtial Pole, according as the Place, whoſe Polars they are, is farther off from the Terreſtrial Equator; that is, as the Sphere is more oblique. All the Stars that lie between theſe Polars have the viciſſitudes of Riſing and Setting, Aſcenſion and Deſcenſion. The *Oblique Aſcenſion* of a Star or any Celeſtial Point, is an Arc of the Equator, reckoned towards the Eaſt, intercepted between the beginning of Aries, and that point of the Equator that riſes together with the Star, in a given oblique Sphere: and this is various according to the variety of obliquity of the Sphere. And the *Aſcenſional Difference* is the difference betwixt the right and oblique Aſcenſion. And the ſame things are to be underſtood concerning the *Oblique Deſcenſion.*

In a Parallel Sphere, where the Horizon *HO*, [*Fig.* 3.] coincides with the Equator *E*♒, and makes one of the Parallels of the Diurnal motion, their Poles alſo (namely, the Zenith and Pole of the World) will coincide; and therefore this poſition of the Sphere agrees only with the Inhabitants of the two Poles of the Earth; and all the Celeſtial Bodies or Points, that eſcribe Circles parallel to the Equator by urnal motion, will likewiſe deſcribe Cir- parallel to the Horizon. Upon this ac- , there will be no riſing or ſetting at all diurnal motion, and therefore no Aſcen- r Deſcenſion in this Sphere, nor any Me- , ſince it paſſes thro' the Pole and Vertex e Place; and ſince theſe two coincide in phere, they can't determine the Circle, Vertical having an equal right to claim me of a Meridian. The Phænomena of n ariſing from the Annual motion of the , viſible in a parallel Sphere, are explain- *Prop.* 33. *B.* 1; namely, ſuch as agree to habitants of either Pole. In the apparent n 'tis evident, that the Sun, during its de- g from the Equator towards that Pole, is in the Zenith, is always ſeen, and the Day half a Year long; as on the ry 'tis half a Year Night, while the Sun es towards the other part of the Equator, nſequently of the Horizon; and that the ing of the Day and Night falls upon the ngreſs into the Equinoctial Points.

PROPOSITION VII.

HE Latitude of the Place is an Arc ſimilar to the Arc of the Elevation of the Pole above the n.

$aebq$ [*Fig.* 4.] be the Earth, its Center es a and b, Equator eq. Let any Place Surface be taken, as l, whoſe Latitude is Arc of the Meridian intercepted between the Equator. Let the Celeſtial Sphere centric, and its Poles, Equator and Me- B, A, EQ, and $BEAQ$, anſwer to les, Equator and Meridian of the Earth. be the Zenith, and N the Nadir of the

Place *l*; namely in the interſections of the Right line *T l* produced, with the Superficies of the Celeſtial Sphere. Therefore the great Circle *HO*, in the Sphere, deſcribed upon the Poles *Z* and *N*, is the Horizon of the Place; and the Arc of the Meridian *O B*, is the height of the viſible Pole *B* above the Horizon; and this, I ſay, is ſimilar to the Arc *l e*, namely, to the Latitude of the Place. For, ſince *A* and *B* are the Poles of the Circle *E Q*, the Angle *B T E* is right; (by *Prop*. 10. *B*. 1. of *Theod. Sphær.*) and for the ſame reaſon, the Angle *Z T O* is right. If therefore from the equal Angles *ETB*, *ZTO*, you take the common Angle *Z T B*, the remaining Angles *E T Z*, *B T O*, will be equal. And therefore (by *Prop*. 33. *Elem*. 6.) the Arcs *e l*, *B O*, they ſtand upon at the Center, are ſimilar.

PROPOSITION VIII.

TO *explain the Cauſes of the Crepuſculum or Twilight, and to define its Limits.*

The Twilight is that dubious light, which we have before the Sun riſes and after the Sun ſets. If there were no Atmoſphere about the the Earth, nor any brightneſs from the *Aura Ætherea* that is near the Sun, ſo ſoon as by the diurnal motion of the Earth any place upon its ſurface comes within the ſhadow of the Earth, by the diurnal motion of the Earth, or ſo ſoon as the Sun deſcends below the Horizon of that Place, there would be nothing but mere Darkneſs; becauſe the Spectator being forſaken by the Rays of the Sun, is left deſtitute of Light: But not only the Sun, but the Ethereal *Aura* that is ſpread about the Sun very nearly to it (and its inflamed Atmoſphere as it were) do alſo ſhine and give ſome light: Now, this ſpend-

more time than the Sun does in riſing
tting, before Sun riſe the Aurora ſhines
n a manifeſtly Circular figure, riſing into
rightneſs of the ſame Figure with that of
egment of the Circle, that is already riſen
: Atmoſphere of the Sun, and intirely dif-
t from that, which proceeds from the il-
ıation of the Atmoſphere of the Earth,
by the Sun: Which is to be underſtood in
manner concerning the Twilight after
et. Becauſe the matter that thus ſhines,
aſon of the nearneſs of the Sun, ſhines
imes more, ſometimes leſs, the bounda-
f the Twilight, which ariſes from thence
ot ſo certain, eſpecially when it acts in
nction with the other and more powerful
of this dubious Light. For after that the
itant of the Earth ſtanding upon its ſur-
omething beyond the bounds of Light, is
ed into the ſhadow of the Earth, the At-
ere that ſurrounds the Earth and is ex-
d to a conſiderable diſtance above the
is ſtill inlighten'd, and does alſo inligh-
e Place where the Obſerver ſtands with
lected Rays. This cauſe is often upon
ange, according as more or fewer Par-
fit for reflecting or other ways con-
the rays of the Sun to us, are found
ded in the Air; and according as theſe
les get up to a greater or leſs height,
depends upon the gravity of the Air (it
the Fluid in which they ſwim,) that is
by the Barometer. For if they hang
nd very near the ſurface of the Earth,
heſe alſo ſoon ceaſe to receive any of the
of light, being revolved preſently after-
together with the Place over which they
into the Earth's ſhadow. If they are ei-

Q 2

ther very rare as they float in the expanded ſpace, or unfit to reflect the Light, they will return to us either ſuch a Light from the Sun as is nothing to ſpeak of, or at leaſt ſo weak and thin as not capable of affecting our Sight, as we really experience in all the expanded ſpace, that lies without the Earth's ſhadow; for tho' it lies perpetually open to the Sun's Rays, yet it ſends back to us ſo weak and faint a Light, that it ſcarce deſerves that name.

Tho' the duration of the Twilight depending upon both theſe cauſes is various, yet 'tis certain, that the beginning of the Morning Twilight happens generally about the time that the Sun is not above 18 Degrees below the Horizon; and that the Evening Twilight ends about the ſame time, or when the Sun is got to the like degree of depreſſion below the Horizon. *Tycho* would have this depreſſion of the Sun that bounds the Twilight to be 16 Degrees: Others extend it to 19; that is, till the leaſt Fix'd Stars become viſible. *Caſſini* from his own Obſervation, extends it only to 17 Degrees. *Ricciolus* finds by his Obſervations, that 'tis not the ſame in the Morning as it is in the Evening, and that it is different in different Seaſons of the Year.

SCHOLIUM.

There is a Light that ſeems very near a-kin to the Twilight, which was firſt obſerved by the Quick-ſighted Mr. *Caſſini*, in the Year 1683; a little before the Vernal Equinox in the Evening, and extended along the Ecliptic from the Sun towards the Eaſt. The Obſervations made afterwards by Mr. *Caſſini* and *Fatio*, evince, that this Light is diffuſed from both ſides of the Sun almoſt along the Ecliptic, but ordinarily deflects a little from it towards the North rather

than

the South. Its form is pointed on both
; its two points being ſometimes more,
times leſs diſtant from the Sun, at firſt a-
two Signs or a little more; but three
s after, its diſtance from the Sun encreaſed
ree Signs or 100 Degrees: Its breadth is
e 30 Deg. near the Horizon. But it can't
en where it is broadeſt; namely, juſt at
un it itſelf along the Circle of its breadth.
des are ſtreight, bating its being gibbous
le ſometimes about the middle, between
un and either Point, and they are inclin'd
e another, ſometimes at a greater and
imes a leſs Angle. This Angle in its
bigneſs is about 21 Degrees. From the
, it is evident, that this Luminous Phæ-
non moves together with the Sun, juſt as
oes, thro' the Ecliptic; which is alſo true
th the Points, allowing for their greater
ſ diſtance from the Sun during the increaſe
reaſe of the whole. The brightneſs of it
h like that of the Milky Way, or the Tail
omet, and is pellucid like the latter; 'tis
r in the middle, leſs towards the extremes,
enſibly decreaſing, till it vaniſhes in the
nding Blue of the Heavens: From
it appears to different Obſervers, accord-
the ſharpneſs of their Sight, at the ſame
and Place, of a different magnitude, and
ding as the Heavens are clear or cloudy,
e true Twilight and brighter Stars are pre-
r abſent, it is more or leſs extended and
ly terminated, and appears fainter always
Morning than in the Evening. This
by reaſon of the Twilight laſting all
long, can't be ſeen in the midſt of Sum-
the Regions about either Pole; but in
rning and Evening of the the ſame Day

it may be ſeen about the middle of Winter (if the Moon be away.) In places near the Equator it may be ſeen at any time of the Year. And in any place it is brighter, the more erect it is, in regard of the Horizon; becauſe it is then more out of the Twilight and Vapours that hang about the Horizon: And therefore it is beſt ſeen in theſe Northern Regions in the Morning after the Vernal Equinox, in the beginning of *October*; and in the Evening, at the end of *February*; the Ecliptic along which it is ſpread, being then more erect in regard of the Horizon, (in the beginning and end of the Twilight, which is then ſhorteſt) the Equinoctial Points being at that time in the Horizon.

The Body (or rather collection of Corpuſcles) whoſe appearances theſe are, ſeems to ſurround the Body of the Sun in the form of a Lens; as Mr. *Fatio* conjectures. The Plane thro' the edge of the Lens is in the Plane of the Ecliptic, or at leaſt not far from it: The Edge itſelf is between the Orbits of Venus and the Earth, but nearer to the latter. The Particles which fill up this lentiform ſpace (like the particles of the Sun's Atmoſphere, of which we ſpoke but juſt now) ſeem to produce the abovemention'd appearance, by reflecting the light of the Sun.

For a Ring whoſe oppoſite Faces are plane and parallel, would appear in the form of an Elllipſe, more obtuſe towards the Points than this Phænomenon. Mr. *Caſſini* takes theſe particles reflecting the Sun's light, contained in this ſpace, for an infinite number of Planets, ſince being ſeparated, they would, like the Planets, exert their motions about the Sun; by which method, this morning and evening light owes its original to innumerable Planets, juſt as the Milky Way does to innumerable Fix'd Stars.

Ir.*Fatio* ſuſpects that this light is coeval with
World: *Caſſini* on the contrary, that it was
luced a little before the firſt Obſervation he
made of it, and that it was not in being
Years before, ſince he had often viewed a Co-
very intenſely with his Eyes, in the ſame
e as this Phænomenon ought to poſſeſs.
t it has appear'd formerly, and afterwards
ppeared, he thinks is very probable from
e ancient Hiſtories that ſeem to give a de-
ition of it; but eſpecially from the Obſer-
n of Mr. *J. Childrey*, who no doubt ſaw
s is evident from his deſcription, *pag.* 183,
in the Advertiſement at the end of a Book,
iſhed *Anno* 1661. intitled *Britannia Baconica*;
uſe it appears even now in the ſame place,
near the ſame Conſtellations, as it did at
time of the Year. His Words are theſe:
e is ſomething more, that we would recommend to
)bſervation of the Mathematicians; namely, that
e Month of February, *and a little before and*
it (as I have obſerved for ſeveral Years) about
Clock in the Evening, when the Twilight has
ly left the Horizon, a Path of Light tending from
wilight towards the Pleiades, and touching them
were, preſented itſelf very plainly to my Sight.
Path is to be ſeen whenever the weather is clear;
eſt of all in a Night when the Moon does not
. And a little after: *I am apt to believe that*
Phænomenon has been formerly, and will hereaf-
ppear always at that above-mentioned time of
ear. But the Cauſe and Nature of it, I can't ſo
as gueſs at; and therefore leave it to the enquiry
ſterity.

PROPOSITION IX.

TO *explain what is meant by the Poetic Riſing and Setting of the Stars, and the kinds of it; namely, the Coſmical, Achronical and Heliacal.*

In the foregoing Propoſitions we have treated of the true Riſing and Setting of the Stars; namely of their aſcent above the Horizon of a certain Place, and deſcent below it, or rather of a depreſſion of a given Horizon below a Star, and elevation of the ſame above it; and that without any conſideration of the Sun. But now the Riſing and Setting of the Stars are to be compared with the motion of the Sun along the Ecliptic, and conſequently with the Daylight and Seaſons of the Year. For the ancient Huſbandmen, and from their traditions the Writers of Husbandry, as alſo Phyſicians, Poets and Hiſtorians, have made uſe of theſe marks to expreſs the Seaſons of the Year by, and conſequently none of them can be underſtood without an explication of theſe Terms.

There are uſually reckoned three kinds of the Poetic riſing and ſetting of a Star, The *Coſmical*, *Achronical* and *Heliacal*. A Star is ſaid to riſe *Coſmically*, when it riſes together with the Sun, but to ſet Coſmically, when it ſets when the Sun riſes; ſo that the Coſmical riſing and ſetting is all one with the Morning riſing or ſetting, as if the beginning of the Artificial Day, or the Riſing of the Sun, were the ſame with that of the World. A Star is ſaid to riſe or ſet *Achronically*, that riſes or ſets when the Sun ſets; and conſequently the Achronical riſing or ſetting is all one with the Evening ones. *Kepler* maintains that theſe words are to be taken in another ſenſe; namely, ſo as that to riſe and ſet *Coſmically*, ſignifies the ſame as to aſcend above

or deſcend below the Horizon; but that
or ſet *Achronically*, is the ſame as to riſe
in the Sun's oppoſite, or in the other A-
or extreme of the Night: In which ſenſe
y, and to this day moſt Aſtronomers ſay,
et is Achronical, when it is oppoſite to the
nd bright all the Night: So that to riſe
onically is, as uſual, the ſame as to riſe
the Sun is ſetting; but to ſet Achroni-
is to ſet when the Sun is riſing; which is
only called the Coſmical ſetting.

tar riſes *Heliacally*, when it before lay hid
the Rays of the Sun, ſo as that it did riſe and
ether with the Sun, but now gets ſo much
the Rays of the Sun, as when the Sun is
the Horizon the Star becomes viſible,
to ſet ſoon after under the Horizon, or to
ear by the ſupervention of the Daylight.

Star is ſaid to ſet *Heliacally*, which was
ſeen above the Horizon when the Sun
ot much below the Horizon, but now has
elf ſo much among the Rays of the Sun,
he next Day being riſen above the Hori-
or juſt ſetting below it, it can no more be

If there were no Twilight, this Helia-
ing and ſetting of the ſame Star (or rather
ition and Occultation,) would not be a-
one Day diſtant from one another at moſt.
that caſe a Star would be ſeen either before
iſe, if its Oblique Aſcenſion in a given
on were never ſo little leſs than that of
n, or it would be viſible after Sun-ſet, if
ever ſo little greater. Nay, if there
no Atmoſphere at all, the ſmalleſt Stars
be viſible even in the Daytime when the
ines. For the reaſon why they don't ap-
s this: The particles of the Atmoſphere
tened ſtrongly by the Sun, affect the Eye
of

of the Spectator with ſo vivid a light, that the Retina (or whatever it be that is the Senſor of Sight) is ſcarce moved by the very weak image of the Star, and conſequently can't take notice of it or ſee. But if there were no Atmoſphare ſpread about the Earth, nor any ſurrounding Terreſtrial Bodies that reflect Light; if the direct Rays of the Sun were turn'd off from the Eye, the Eye thus free from the Rays of a Body either very bright in itſelf, or ſtrongly enlightened, would ſee clearly (as it does now in the Night) even the ſmalleſt Stars, unleſs they are hid under the Atmoſphere of the Sun: For this together with the Body of the Sun, make up one lucid Body. Since proportionable to the leſſer brightneſs of a Star, there is need of a greater depreſſion of the Sun below the Horizon, to make it viſible; 'tis evident that the Heliacal ſetting of the Sun is ſooner, and the riſing later. That the leaſt Stars may be ſeen, the Twi-light muſt be quite over, or the Sun muſt be full 18 Deg. below the Horizon: For the ſeeing of a Star of the ſixth Magnitude, the Sun muſt be 17°; and ſoon, 'till you come to the Stars of the firſt Magnitude, which are to be ſeen in that part of the Horizon which is towards the Sun, when the Sun is 12°. The Planets ſhine with a brighter and fuller light, and therefore have no need of ſo great a depreſſion of the Sun below the Horizon, ſo that 11 Degrees are enough for the ſeeing of Mars and Saturn; about 10 for Jupiter and Mercury; and 5 deg. are commonly look'd upon as neceſſary for Venus, tho' it has been frequently ſeen while the Sun ſhone, and not at an Heliacal Riſing or Setting. But theſe things in the Planets depend upon their various diſtance from the Earth, and the more or leſs fullneſs of the Orb of the Inferior.

l the Fix'd Stars under the Zodiac, and ſuperior Planets, Saturn, Jupiter and , the Sun in its Annual Motion towards Eaſt getting before them, riſe Heliacal- the morning, a little before Sun-riſe; is, a few days after their Coſmical ri- They ſet Heliacally in the evening; ly, a little before their Achronical ſetting. he Moon, which always gets before the riſes Heliacally in the Evening; when ry old and haſtens to a Conjunction with un. The inferior Planets Venus and Mars, times getting before the Sun, and at other being left behind by him, (as has been at large in *B.* 1.) ſometimes riſe Helia- in the morning, namely, when they are grade; ſometimes in the evening, when are Direct; which is to be underſtood in manner of the Heliacal ſetting of them. he Fix'd Stars, plac'd a great way from the ac towards the elevated Pole, may in the Day both riſe and ſet Heliacally together, ndergo other changes in regard of this of Riſing or Setting; as will appear to any n that ſhall conſider it.

e *Grecians* and *Romans* anciently uſed a not exactly fitted to the motion of the and by that means ſometimes they were e the Sun, and ſometimes behind him. But eaſons of the Year not returning with their eous Calendar, but with the Sun and the ces; that their Country, Domeſtic, or ary Affairs might be diſpatched in their r Seaſons, the Ancients propoſed the ri- nd ſetting of the Stars inſtead of a Calen- For the Fix'd Stars moving from the E- ctial Points but very ſlowly, and by a n not then taken notice of, they made no queſtion, but that the Sun returning to the ſame place in reſpect of the ſame Fix'd Star, did alike come again to the ſame Station in reſpect of the Equinoxes or Solſtices; that is, when the ſame Star riſes or ſets Coſmically, Achronically, or Heliacally, (which they chiefly regarded,) that the ſame ſeaſon of the Year was reſtored to the Globe, and conſequently that the ſame labours or affairs were to be returned to; from hence they called the ſtated Seaſons of the Year *Sydera*, or the Stars. But it is found that the Fixed Stars, after a long ſeries of Years, remove out of their places; and therefore, if the Day of the Riſing or Setting of the Fix'd Star given among the Ancients is to be reduced to our Calendar, an account is to be taken of the motion of the Fix'd Stars in the intermediate time. And now-a-days Calendars may eaſily be had, and the *Roman* Calendar (which we uſe,) comes nearer to the Motion of the Sun, upon which the Seaſons of the Years depend and are ordered, than the Riſing and Setting of the Fix'd Stars; the obſerving their Riſing and Setting came to be neglected by degrees, and is not uſed by any but the Poets, who are wont to deſcribe and paint out as it were the ſeveral Seaſons by the circumſtances of the various Riſing and Setting of ſo many Stars, than which nothing is more beautiful and affecting; tho' it be generally done by them very erroneouſly, becauſe they deſcribe at this time the Day of our Calendar by that Riſing of the ſame Star by which it would have been juſtly deſcribed in the time of *Cæſar*; whereas the difference between theſe two times is nearly 14 Days.

SECTION II.

he Diviſion of Time, and ſeveral other things depending thereon.

PROPOSITION X.

O explain the Diviſion of Time into Hours,
Days and Weeks, and the Terms uſed in Aſtro-
lepending thereon.

have thus far been explaining the gene-
of the Circles of the Sphere in the appa-
yſtem of the World, and the more uſual
s in Aſtronomy depending thereon: But
there are no terms in Aſtronomy that
oftener than thoſe, whereby Time or any
of it is expreſſed; 'tis neceſſary to treat a
concerning them,and to ſhew, by the way,
ivil diſpoſition of Time, ſo far as it is
n an Aſtronomical Calculation.

e moſt conſiderable parts of Time are, a
n *Hour*, *Month* and *Year*. The firſt of them
ay, (becauſe a ſpace of Time moſt obvi-
nd known by us;) and it is either Natural
tificial. A *Natural* Day the duration of an
apparent revolution of the Sun about the
. The *Artificial* is that part of the Natural,
g which the Sun is above the Horizon;
is oppoſed to the *Night*, which is taken
e Sun's continuance below the ſame: But
latural Day takes in both. The Natural
s either the Aſtronomical or Civil; and
iffer from one another only as to their be-
ng, according to the cuſtom of the People,
ttlement of the Aſtronomers. The Aſtro-
al Day is the ſpace of time which flows

between the Sun's leaving a given Celeſtial Meridian, and next return to the ſame; that is, the ſpace wherein a revolution of the whole Celeſtial Equator is performed, and of that part of it beſides which anſwers to a portion of the Ecliptic, which the Sun deſcribes in the mean while by its Annual motion towards the Eaſt. Becauſe this portion of the Equator to be added to the intire Equator is not every where equal, (tho' its mean Quantity is nearly that of one Degree,) as well on the account of the Obliquity of the Ecliptic, as that the apparent Annaual motion of the Sun about the Earth is not equable, the Natural or Aſtronomical Days are not preciſely equal: But of this inequality of the Natural Days, I ſhall treat in its proper place afterwards; taking no notice at preſent of this nicety. Moſt Aſtronomers begin the Day at noon: But *Copernicus* following *Hipparchus* begins it at Night, and this beginning is retained in the *Prutenic* Tables. Differing Nations make different beginnings of the Day. The *Babylonians* begin their Day with the Riſing of the Sun; The *Jews* and *Athenians* with the Setting of the Sun, as the *Italians* do ſtill. The *Egyptians* at Midnight; which is the cuſtom of the *Britains*, *French*, *Germans*, and ſeveral other *European* nations. And anciently the ſame was done by the *Jews*. For we naturally refer the Night, which we ſpend in ſleep and ſilence, partly to the preceding, and partly to the followlowing Day; that is, we begin the Νυχθήμερον, at Midnight. The *Umbri* anciently made the Noon to be the beginning of their Day, and this the *Arabians* obſerve at this time.

Tho' the Artificial Day was hardly divided anciently any other way than into the Morning and the Evening, and the Night into

Watches;

es; yet afterwards the Natural Day was d more accurately into 24 parts, called Hours are either equal or unequal. An Hour is the four and twentieth part of a al Day. This ſort of Hour was always y Aſtronomers, and is now in uſe among all Nations. The *Italians* reckon the from One to 24, from the beginf the Night; (that being the beginning Natural Day,) and their Clocks are made hat manner. But We, the *French*, *&c.* not 24, but twice 12 Hours; perhaps e number of the Strokes ſtruck by the might not be tedious. So that we divide y into the Hours before Noon and the after Noon. An Hour is commonly dinly into four Quarters (which are called by ſome,) a more minute diviſion being t buſineſſes unneceſſary: But the Aſtros (and now all the Politer ſort of Peoubdivide it into 60 Minutes, and each e into 60 Seconds. An *unequal Hour* is the part of an Artificial Day, as alſo the part of the Night. 'Tis call'd Tempoecauſe of a various length in different or Seaſons of the Year: Thus the Sumiurnal hour is longer than the Winter, e Nocturnal ſhorter; and the Diurnal n Summer is longer than the Nocturnal, Winter ſhorter: At the time of either ox, the Diurnal hours are equal to the rnal, and then theſe unequal hours behe ſame with the equal ones deſcribed aupon which account theſe equal hours are by Authors *Equinoctial* Hours. The *Jews*, *s*, and *Romans* made uſe of theſe Tempours. Such as begin the Civil Day, at rning or Evening, do it becauſe the Sun is then in the Horizon, the moſt ſenſible of all the Circles of the Sphere. They have this advantage, that by the number of their equal Hours they know how much of the natural Day, that is, how much time is gone ſince Sun riſe; but they have this diſadvantage, that the times of the Sun-ſet, Noon and Midnight, in different Seaſons of the Year, are marked with different numbers of the Hours, and they can't be known but by Calculation: They likewiſe have this advantage, they immediately know how much time remains till Sun-ſet, and ſo know how to accommodate their Labours or Travail to it; and this diſadvantage again, that they can't reckon the Hour of Sun riſe, Noon, or Midnight, without computing. Such as begin the Civil Day from Noon or Midnight, do it, becauſe the Sun is then at his greateſt Elevation or Depreſſion; but the name of the Hour of Sun riſe or Sun ſet, they get only by Computing. Laſtly, ſuch as ſhou'd uſe the unequal Hours, juſt now deſcribed, could always tell Sun riſe or Sun ſet by the twelfth hour; Noon and Midnight always by their ſixth; but could know the Quantity of the Hour only by Calculation.

A *Week* is the ancienteſt collection of Days as ever was; as is evident from the Sacred Writings. The *Jews* uſed this collection of old, and ſince the reception of Chriſtianity, other Nations too: Every Day of it has a name given to it from ſome of the Planets, after the following manner: The Hours of the natural Day are four and twenty, the Planets but ſeven, and in this order in the common Syſtem, Saturn, Jupiter, Mars, Sol, Venus, Mercury, the Moon; they began at the firſt Day of the *Jewiſh* Week, aſcribing to the Sun (the Author of the Day) the

irſt Hour of it, to Venus the ſecond, to
ury the third, to the Moon the fourth;
beginning from Saturn, they aſſign'd the
Hour to him, and ſo on; by this means the
Iour of the following Day happens to the
n, and ſhe therefore beſtows a name upon
Day of the Week from her own; which
od is continued in the following Days to
nd of the Week.

POPOSITION XI.

O explain the diviſion of Time into Months,
Years, and the various ſorts and collections of

o' a Month be properly that ſpace of time
in the Moon goes thro' the Zodiac; yet
there are about twelve Months that paſs
the Sun runs once thro' the Ecliptic,
pace of time alſo wherein the Sun runs
one Sign of the Zodiac, is call'd a *Solar*
; which is about 30½ Days. Again, the
Month is either *Periodic*, in which the
having departed from any Point of the
c returns to it again, and is ſomething
an 27 Days; or *Synodic*, which is ſome-
more than 29½ Days, wherein the Moon
ro' all its ſeveral Phaſes, and on that ac-
is of principal uſe in the marking of
Both the Solar and Synodic Lunar
i, is either *Aſtronomical* (of which we
poken already) or *Civil*, according to the
nt cuſtom and inſtitution of each Na-
For ſome, as the *Egyptians*, liked the So-
nths, and made each of them to conſiſt
Days; and to compleat the Year, at the
twelve ſuch Months they annexed five
made up of thoſe twelve times ten Hours;
ach Solar Month exceeds 30 Days by

But moſt of the Ancients fancied the Synodic Lunar Month moſt, ſuch as the *Jews* anciently, the *Grecians* and *Romans*, to *Julius Cæſar*'s time; and the *Mahometans* at this time. But that theſe ſort of Lunar Months, that do not conſiſt of whole days, might be fitted to Civil uſe (where pieces of days can't be taken notice of, and conſidered,) they made their Civil Months alternately of 30 and 29 days, (calling the former *Full*, and this *Cave* Month;) ſo that two ſuch Months are equal to two Lunar Months of 29$\frac{1}{2}$ days a piece; and the New Moon would not ſenſibly remove from the firſt day of the Civil Month in the courſe of ſome Years.

A Year is ſometimes taken for the time of the revolution of a Planet through the Zodiac; in which ſenſe a Month is ſometimes called a Year: and ſometimes for the time of an intire apparent revolution of the Fix'd Stars thro' the Zodiac, which they call the *Annus magnus*, or *Great Year*. But a *Year* is properly that time which the Sun takes to run thro' the Zodiac in. And it is of two ſorts, the Aſtronomic and the Civil: The Aſtronomic is alſo twofold, according to the two different bounds of the Sun's revolution; namely, the Sydereal and Tropical. The *Syderial* Year is the ſpace of time, that the Sun having departed from a Fix'd Star returns to the ſame in; and it is 365 Days, 6 Hours, and 10 Minutes nearly. The *Tropical* Year is that wherein the Sun departing from one of the Cardinal points, the Equinoctial or Solſtitial, returns to it again; and is ſomething leſs than the Sydereal, becauſe the Cardinal points of the Ecliptic themſelves go backwards, and as it were meeting the Sun, make the Sun to return to the ſame point of the Ecliptic ſomething ſooner than to the ſame Fix'd Star, where

that

t point of the Ecliptic was, when the Sun
before in the ſaid Point. This Tropical
r is 365 Days, 5 Hours, $49\frac{1}{5}$ Minutes very
; and wants about 21 Minutes of the Sy-
al.

he *Civil* Year is the ſpace of time, that the
ion of the Sun or Moon, or both, points
received by the ſettled Cuſtom of any Na-
. Hence 'tis evident, that there are three
ns of the Civil Year; namely, either purely
r, or purely *Solar*, or *Luni-Solar*, which is
e up of both. The *Lunar* Year conſiſts of
ve Lunations or Synodic Months that are
ned in 354 Days, at the end of which the
begins again. And ſince this Year wants
11 Days of the Tropical Year, that brings
Seaſons over again; it comes to paſs, that
beginning of this Year falling upon the
ng now, in 8 Years time would fall upon
ter, and after as many more upon Autumn,
upon Summer, and laſtly, at the end of 33
s return to the Spring again. And this is
d the *Wandring Lunar Year*; becauſe its be-
ing wanders thro' all the Seaſons of the
, and that in the memory of one Man.
called alſo *Annus Solutus*, becauſe free and
gaged from the motion of the Sun, which
at all conſidered in the ordering of it. This
of Year is in uſe among the *Turks*, and
s to be found out at firſt in thoſe Countries,
e the differences of Summer and Winter
ot very evident and ſenſible; and where,
ant of Aſtronomy, the return of the Sun
e Cardinal points of the Zodiac, that de-
the Solar Year, is not eaſily known: On
h account taking the Signs of the Seaſons
the Moon, they took the beginnings of
Years from the beginnings of her Phaſes

or Appearances; now, there is no number of Lunations, that comes ſo near to the Solar Year (which they had ſome ſmall knowledge of) as the Twelve. I am apt alſo to think, the reaſon of the beginning the Civil Day from the Sun-ſet is to be taken from the Moon: Becauſe it ſeems neceſſary for them to begin the Civil Day with their Year and Month, that is, with the New Moon, which at firſt they knew no otherwiſe than by the help of their Eyes; which was, when they ſaſt the New Moon in the Evening immediately after Sun ſet. And upon this account, the firſt, ſecond, third, *&c.* Day of the Month, was called the firſt, ſecond, third Moon, and the day of the Full Moon was called conſequently, the fourteenth Moon.

Such of the Ancients as were willing to retain the Lunar Year, thought it notwithſtanding proper to adapt it ſo to the Sun, that its beginning might be in a manner fix'd, and that it might keep the Cardinal points of the Year ſhewn by the Seaſons, from receding much from their Months; and to this purpoſe they us'd Intercalary or Embolimean Months. And firſt, in the ſpace of three Years, they made up an intire Month to be intercalated every third Year, out of the eleven odd Days in every Solar Year. But becauſe this artifice was not exact enough, in every eight Years they intercalated three Months, every one of which conſiſted of 30 days, (namely at the end of the third, fifth and eighth Year, tho' ſome others otherwiſe,) and at laſt eight Months in 19 Years. Hence 'tis evident that ſome of theſe Years are ſimple, namely of 354, or 355 days, the number that there is in the Wandring Lunar Year; ſome embolimen or intercalary, of 384 or 384 days, when there are thirteen Lunations

s in the Year. Theſe Luni-ſolar or fix'd Lu-
Years, are uſed by the *Jews* and the Clergy
ıe Roman Communion.

'here are three ſorts of Solar Years, or ſuch
ıre fitted to the motion of the Sun alone
the viciſſitudes of the Seaſons depending
eon; the *Egyptian*, *Julian*, and *Gregorian*.
Egyptian Year conſiſts of 365 Days, which
divided into twelve Months of 30 days a-
e, and five other days (called ἐπαγόμεναί) ad-
at the end of them, as was ſaid above. In
conſtitution of this Year there is no regard
to the motion of the Moon: But becauſe
Year wants 5 Hours 49 Minutes, or almoſt
ours of the Solar Tropical Year; in four
rs time it gets near a whole day before the
r Tropical one, and in 1460 Years its be-
ıing wanders thro' all the Seaſons of the
r; on which account it may in a manner
alled a *Wandring* Year, altho' in the Age of
Man, or 60 Years, it does not anticipate
e 15 Days.

ulius Cæſar finding the *Egyptian* Year to get
re the Tropical, becauſe the ſix Hours,
reby it wants of the Tropical, are intirely
ected in its conſtitution; did therefore add
6 Hours to each *Julian* Year; ſo that a
an Year conſiſts of 365 Days, ſix Hours.
becauſe this piece or quarter of a Day,
be conſidered or taken notice of in Civil
he added the Day made out of them in
Years to every fourth Year, between the
and 24th of *February*, (becauſe the *Romans*
they uſed the Lunar Year before *Cæſar's*
intercalated the Embolimean month;)
fore from that time they writ *bis ſexto Ka-*
Martii, or the ſixth of the Kalends of *March*

twice; from which cuſtom the Year had the name of the *Biſſextile*.

And this *Julian* Year conſiſting of 365 Days, and adding in four Years one Day, ſo that after three ſimple Years there may follow a fourth of 366 Days, is the beſt fitted for Aſtronomical Computations; becauſe it is a mean between the Natural or Tropical Year of 365 Days, 5 Hours and 49 Minutes, and the Sydereal Year of 365 Days, 6 Hours and 10 Minutes, and does, as it were, give an ocular Demonſtration of the natural regreſs of the Equinoxes. This form of the Year held common among all Polite Nations from *Auguſtus* (who reſtor'd it when it was almoſt loſt) unto the Year 1582, when the *Julian Calendar* was reformed by *Gregory* the XIIIth. But it is ſtill in uſe among the *Britains*, *Iriſh*, and others. Yet it muſt be confeſs'd, that the quantity of the *Julian* Year is too big, on which account the beginning of this Year alſo by little and little creeps forwards in regard of the Seaſons, or (which is all one) the Times of the Equinoxes and Solſtices creep backwards in regard of the Days of this Year. And ſince this regreſs is about $10\frac{4}{7}$ Minutes every Year, in about 133 Years it will be a matter of a Day, and conſequently from the Year of Chriſt 325, wherein the *Council of Nice* was held, to the Year 1582, wherein the Pope reform'd the Calendar, namely 1260 Years, this regreſs was a matter of 10 Days. Hence it came to paſs, that whereas in the time of the *Nicene Council* the Vernal Equinox happened near about the 21ſt day of *March*, in the Year 1582 it was found to have crept up to the 11th of *March*. That he might therefore bring the Equinox to its former place, ten days were ſuppreſſed in the Month of *October*, in the Year 1582, and the 5th day

was call'd the fifteenth; by this means what
ld otherwiſe have been the 11th day of the
ch following, and the time of the Equinox,
mes the 21ſt of *March*; that is, the Equi-
happens the ſame day, *viz.* 21ſt of *March*,
did in the time of the *Council of Nice*, which
led the time of keeping *Eaſter*. Still if no
er reformation ſhould be made, but the ta-
g away this ten days, the Cardinal Points
ld again remove after the former manner,
ſome Years time. The Pope therefore to
id this, order'd that once in 133 Years (in
ch time the exceſs of a *Julian* Year above
Tropical riſes to a whole Day) one day
ld be taken away from the Calendar; that
hat three days ſhould be taken out of four
dred Years; and this he appointed to be
e, by making every hundredth Year of the
iſtian Æra common, which according to
Julian Account is always *Biſſextile*; but e-
four hundredth, which was to continue
xtile, as in the *Julian*. The New form of the
r ſettled by the Authority of Pope *Gregory*
Ith, was call'd from him the *Gregorian*, and is
rved in *Italy*, *France*, *Spain*, *Germany*, whereſo-
the authority of the Pope reaches; and
he cloſe of the laſt Age, was admitted, as to
Civil Months, by ſeveral of the Reformed
Germany: As for the finding the *Paſchal*
n, they kept themſelves ſtill to *Kepler's Rudol-*
e Tables, till ſomething more certain could
ix'd upon and ſettled.

s the form of the Year among different
ions is various, ſo likewiſe is its beginning.
Jews begin their Eccleſiaſtical Year with
New Moon of that Month, whoſe Full
on happens next after the Vernal Equinox:
Church of *Rome* begin their Year with

the Sunday that falls upon the ſaid Full Moon, or that happens next after it; or with the *Feaſt of the Reſurrection* of our Lord, which was the cuſtom in *France* before the Year 1564. The *Jews* begin their Civil Year with the New Moon, that has its Full Moon happening next after the Autumnal Equinox. The *Grecians* began their Year with the New Moon that happens next after the Summer Solſtice: And the *Romans* anciently began theirs with the New Moon next after the Winter Solſtice. And this ſeems to be the reaſon why *Julius Cæſar* did not fix the beginning of his Year (tho' a Solar) in the Winter Solſtice it ſelf, but tarried for the following New Moon, where he might place the beginning of his Year or the Calends of *January* according to the cuſtom then receiv'd. Tho' perhaps the ancient opinion was not yet intirely loſt, that the Solſtices were celebrated in the eighth parts of the Signs of Cancer and Capricorn, and not in the very beginning of the Signs: by which no more was meant among the Learned, than that the Sun was really in the eighth parts of the Signs, when he was commonly thought to be in the beginning; which is all one as to ſay that the Equinoxes and Solſtices were anticipated for 6 or 7 Days. And if the Sun had been in the firſt point of Capricorn on the 24th of *December*, he would be exactly enough in the eighth Degree of it on the 1st of *January*. By this Method he brought it about that the Celebration of the Olympic Games ſhould fall upon the firſt Day of the Month *Quintilis*, which he call'd by his own Name, *July*: And perhaps theſe Games themſelves were by ancient Rite inſtituted on the account of the Summer Solſtice.

Tho'

ho' the firſt of January is now look'd upon
ie beginning both of the *Julian* and *Grego-*
Year almoſt thro' all *Europe*; yet others aſ-
other beginnings to it both formerly and
-a-days. We have ſpoken of the moveable
t of *Eaſter* already. The *Venetians, Florentines,*
Inhabitants of *Piſa* in *Italy*, and of *Treves*
rmany, make the Vernal Equinox to be the
nning of their Year. The ancient Clergy
e the 25th of *March*, or the Feaſt of the
inciation of the Bleſſed Virgin, that is,
e Incarnation of Chriſt, to be the begin-
of the Year: Which the *Church of England*
etains as the beginning of the Year. And
ivil Year begins here alſo, tho' the Vul-
egin it at the firſt of *January*, with the
bouring Nations.

e more celebrated collections of Years or
s, meaſured by a repetition of Years, are
ympiad, of 4 Years; a *Luſtrum*, of ſometimes
metimes 5 Years; a *Jubilee*, of 49 or 50
; a *Seculum*, or an Age, of 100 Years; an
Magnus, or Great Year, ſtanding for an
apparent Revolution of the fix'd Stars, till
having departed from a fix'd Point, they
to the ſame again, which is 25000 or
Years, or thereabouts.

PROPOSITION XII.

O enumerate the chief and moſt noted Epochas,
and to explain their relation to each other.

the Aſtronomer in computing the motion
e Celeſtial Bodies, aſſume certain fix'd
to begin at; ſo they do certain moments
me, from which as Roots, their Compu-
may begin and proceed. Theſe Roots,
they alſo call *Epochas* and *Æras*, com-
have a name from ſome famous Action
or

or Event, that happen'd near the beginning of each. The moſt conſiderable, and which we are beſt acquainted withal, is the *Epocha of Chriſt's Nativity*, or the firſt day of the *January* that we ſuppoſe to follow next to the Nativity of our Lord: And 45 years before is the *Epocha of Julius Cæſar*, or the beginning of the *Julian Year*; at which time *Cæſar* rejecting the old *Roman* Year, that had been uſed from *Numa*, as they believed, ordered the *Julian* Year to be obſerved thro' the *Roman* Empire. 'Tis true, this Year was not obſerved always from the beginning, but after various and confuſed intercalations extraordinarily; in about the 10th Year after its beginning, that is, the 5th of the Vulgar Æra, or Birth of our Saviour, it was reſtored by *Auguſtus* in the ſame order as if it had been continued without any interruption from the beginning.

Tho' the Epocha of the Nativity of Chriſt above deſcribed be eſtabliſhed by common uſe, and now almoſt univerſally received, yet *England* and *Ireland*, in affairs both of Church and State, uſe an Epocha a full year after it; and ſuppoſe Chriſt to be born, not on the 25th of *March*, near the ending of the forty fifth *Julian* Year, but of the following Year; namely the forty ſixth *Julian* Year; and they ſuppoſe Chriſt to be conceived on the 25th of *March* of the forty ſixth *Julian* Year; and number the Year not named from the Nativity, but from the *Incarnation* of Chriſt, that is, the Conception; and by conſequence the ſame in number, for the greateſt part of the Year, with the remaining round Number, that ſhews the Year from the Nativity. But in the Months almoſt between the firſt of *January* and the 25th of *March*, they reckon a different year from the reſt, as

they

ought according to their Æra. That this
ence between the Æras that take their
from Chriſt,may be better underſtood, we
o take notice, that *Dionyſius* called *Exiguus*,
he Author of this Æra above five hundred
s after Chriſt, from which time they be-
o reckon from the Nativity or Incarnation
hriſt, whereas before they uſed to number
years by the Conſuls, and from the Build-
f *Rome*, in the *Roman* Empire; and out of
the Years of each King's Reign. This
ſometimes call'd from its Author, the *Dio-*
, is by no means to be looked upon as ex-
d agreeable to Hiſtory. For 'tis not yet
d, whether it was the firſt, ſecond, third,
, or fifth Year, before the Vulgar Æra
hriſt was born, as may be ſeen in *Kepler*'s
of the Year wherein Chriſt was born. But
ppoſed Æra is not too far diſtant from
me, which is ſufficient for an Æra. And
ding to *Dionyſius*, the Author of this Æra,
t was conceived the 25th of *March* of the
ear current of this Æra, and born the fol-
g Winter, at the end of the 46th Year of
Cæſar. This computation of the Year
firſt univerſally received, but is now left
tain'd only in *England*, together with the
ning of the Eccleſiaſtical and Civil Year,
eaſt of the Annunciation,) which obtain-
thoſe days: For in the Chriſtian World
this Epocha was tacitly departed from;
w the common opinion is, that Chriſt
orn the Winter before the *Dionyſian* An-
ation, namely at the end of the 45th of
Cæſar, and therefore they reckon the
from the Nativity; and ſo make Chriſt
ear older than *Dionyſius* the Author of the
id, and the *Engliſh* with him.

Since moſt Aſtronomical Tables are now fitted to the more common Chriſtian Æra, the other more celebrated ones are to be compared and reduced to it. And the firſt of them is the *Epocha of the Creation*, about which there are ſeveral controverſies. Some ſay the World was created 3950 Years before the common Æra of Chriſt, and theſe ſays *Gaſſendus* come neareſt the truth; which is likewiſe approved of by the *French* Aſtronomers. Others 3983 years; and with theſe *Petavius* agrees. *Kepler* 3993 years, in the middle of Summer, from Aſtrology. *Hevelius* in the 3963d Year currrent before Chriſt, the 24th of *October* of the *Julian Year* continued backwards, at 6 a Clock in the Evening, in the Meridian of *Eden*; becauſe he found in his Tables of the Sun, that at that time the Sun was in the beginning of ♎, and its Apogæum in the beginning of ♈. Others make it much more ancient, and place it 5199 Years before Chriſt.

The Epocha of the *Olympiads*, of all Profane ones, is the moſt Ancient and Celebrated: It is placed in the Summer of 777 years before Chriſt, the firſt of *July*, the *Julian* Year being continued backwards. Not much after is the *Epocha of the Building of Rome*, which is generally fix'd in 752 years before Chriſt.

The *Æra of Nabonaſſar*, always famous among the Aſtronomers, began at the 26th day of *February* of a *Julian* Year produced backward, and in the Year 747 before Chriſt. And becauſe this was the firſt of the *Ægyptian* Years, *Ptolemy*, and after him the other Aſtronomers, and *Copernicus* himſelf, compute from thence the motion of the Stars by *Ægyptian* Years. For the *Ægyptian* Years conſiſting of 365 Days, tho' unfit for common Uſe, becauſe its Cardinal Points wander thro' all the Seaſons, yet is very

fit

r Aſtronomical Computations, becauſe it
urbed with no intercalation.
ie *Epocha of the Death of* Alexander *the Great*
vs next in order,on the 12th day of *Novem-*
the year 324 before Chriſt. And this alſo
the firſt of the *Ægyptian* Wandering Years
o the Founders of Æras will have it, tho'
vent happen'd before or after the begin-
of the Year) *Theo*, *Albategnius*, and others
iſed this Epocha, computed the *Ægyptian*
from hence. Between the Æras of *Na-*
ir and *the Death of* Alexander *the Great*,
are 424 years peciſely.
aſs over the *Æra of the Abyſſinians*, call'd
he Æra of *Diocleſian* and *the Martyrs*: As
he *Æra of the Arabians*, that is, the *Hegira*,
ight of Mahomet: And the *Epocha* of the
ns, or as it is call'd *Jeſdegird*; and others,
g ſuch as are of ſmall uſe to us) which
be ſeen at large in Writers upon that ſub-
and in fine, the Epocha of the *Julian* Pe-
f 7980 years, invented by *Scaliger*, begin-
4714 years before the Chriſtian Æra, and
quently before the World was created.

SECTION III.

pheres and other Inſtruments invented
repreſent the firſt Motion of the Hea-
ns, and of their Uſe.

PROPOSITION XIII.

O *deſcribe the common Inſtruments, by which*
the Firſt or Diurnal motion is imitated, and
ſtrated to the Sight, namely the Celeſtial Globe
here:

For the better conception of the aforeſaid Circles in the Heavens, Artiſts preſent us with the Image of the Starry Heaven upon the Celeſtial Globe, which is a Sphere made of Metal, Paper, or any other like ſubſtance, eaſy to be moved upon two Poles. Upon the Convex ſurface you have marked the Places of the chief Fix'd Stars, ſo diſpoſed in regard of one another and the Poles of the Sphere, as the Fixed Stars themſelves, which they repreſent, are diſpoſed in reſpect to one another and to the Poles, about which the Heavens ſeem to revolve in the diurnal motion. The Images alſo of the Aſteriſms are drawn upon it, that theſe Stars ſeem'd to repreſent to the Ancients, drawn to it either by the force of Fancy or Religion.

There are drawn alſo upon this Globe, the Equinoctial Circle exactly between the Poles, and the two Tropics parallel to it, on each ſide of it at $23\frac{1}{2}$ Degrees diſtance, and as many Polar Circles at as great a diſtance from both Poles. The Ecliptic alſo is drawn reaching to both the Tropics, and interſecting the Equinoctial upon the Surface of the Globe, in the points ſo related to the Places of the Stars, as the points of the Celeſtial Equinoctial (where the Sun appears in the time of the Equinoxes) are related to the Stars themſelves, in the Age wherein the Globes are made: As well the Equinoctial as the Ecliptic is divided into Degrees, beginning at the point that repreſents the Vernal Equinox; the Equinoctial into 360, without any break, but the Ecliptic into 12 Signs, diſtinguiſhed by their proper Marks, every one of which is ſubdivided into 30 Degrees: And the Secondaries of the Ecliptic drawn thro' the beginning of each Sign, and meeting in the Poles, divide the whole Globe it ſelf into as many

y Signs. Only two Secondaries of the
ıtor, namely, the Colures, are drawn upon
Celeſtial Globe, one of which, namely,
Solſtitial Colure, is the Secondary alſo of
cliptic. Such a Globe as this repreſenting
Starry Firmament, is made movable upon
aforeſaid Poles, ſo inſerted into the Meridi-
a Brazen immovable Circle placed without
Globe, as to fit that ſide of this Meridian,
h is divided into Degrees; which ſide
fore is to be taken for the true Meridian,
hickneſs of the Circle only ſerving for
gth.

ſides, the Globe incompaſſed with the
dian (as was ſaid) is let into a Wooden
ivance repreſenting the Horizon, ſo as
half of it is above the upper Surface of
Wooden contrivance, which is therefore
looked upon as the Horizon. And that
Globe may come near enough in every part
Horizon, there are notches in the Hori-
for the inſertion of the Meridian, that is,
inent without the Globe. Upon the
th of this artificial Horizon, which is
enough, beſides the inmoſt Circle divided
ts proper Degrees, reckon'd from conveni-
eginnings, there are the *Julian* and *Grego-*
Calendars, with the Degrees of the Signs
e Ecliptic, that the Sun is in nearly upon
Days in that Age: There is alſo a Circle
e Winds or Points of the Compaſs.

e Horizon is ſupported by four or more
Pillars faſten'd to a Foot or Baſe, upon
there is placed a Mariner's Compaſs and
le, to ſet the Globe pretty nearly agreea-
the Quarters of the World. You have
Quardant of Altitude that may be faſten'd
vertical Point of a Meridian by the help
of

of a Screw, its Limb divided into Degrees, it is to betaken for a true Quadrant: and its Degrees are reckon'd from its extremity that touches the Horizon upwards towards the Zenith. In the Globes made now-a-days, there is alſo a Circle called the *Hour* Circle, faſten'd to the Meridian, having the Pole for its Center: This little Circle is divided into twice twelve hours, that it might be fitted to our way of reckoning the hours, beginning at the Meridian. And ſince the Pole carries the Index along with it, the Index will point out the hours according to the revolution of the Globe; if it be at twelve a Clock of the day, at the ſame time as the place where the Sun is at that time is under the Meridian.

What is commonly call'd a *Sphere* differs from a Globe in this, that its ſurface is not united and perfect, as 'tis in a Globe, but the more principal Circles made of Armillæ or Hoops (from whence it has the name of an *Armillary Sphere*) ſhadow out a Spherical ſurface. In ſuch a Sphere, beſides the Horizon and Meridian, (between which being immovable, it is revolved like a Globe,) there are four great Circles; namely the Equator and Ecliptic, or rather Zodiac, (for like a Belt or Swath, it is 20 deg. broad;) and the two Colures and four leſſer Circles, *viz.* the two Tropics, and as many Polar Circles: And thus the number of the Circles of a Sphere came to be reckoned Ten. This Sphere is ſo excavated, that by its tranſparence as it were, it is a repreſentation of the World; and preſents us with the little Globe of the Earth in its Center at reſt, and ſuſpended by the Axe, which could not be done ſo well in a Globe, whoſe Superfices is intire and united. From what has been ſaid, 'tis evident, that in

an

rmillary Sphere, the above deſcribed pla-
of the Fixed Stars can't be marked at all,
pting ſome few, that appear in the Zodiac;
they likewiſe are commonly taken no no-
of; and therefore the Problems relating
he Fix'd Stars can't be ſolv'd by ſuch a
re. Tho' ſince the ſaid Circles of the
re are delineated upon the ſurface of the
e, all the Problems that are done by the
illary Sphere, may be performed very ea-
y the Globe: And therefore Spheres, tho'
eat name formerly, when the ſtudy of A-
omy began to revive, are very ſeldom uſed
but to help the imagination of the young.
theſe Armillary Spheres there are ſome-
annex'd the Planets made to be immova-
different Circles, in the order as they are
ned by the Aſtronomers of the *Ptolemaic*,
nic, or *Copernican* Sect. And therefore in the
aic and *Tychonic* Sphere, the Planets are
d in different Circles about the Earth at
in the Plane of the Zodiac, within the
that repreſent the Starry Heaven. But
there is required a Spherical ſurface (ſha-
d out by the hoops) concentric with the
to repreſent the Firſt or Diurnal motion,
his is movable in the *Copernican Sphere*;
ore the Makers do place in it the Planets
ut the hoops compaſſing the Earth, which
able about the imaginary Sun. Sometimes
es of this kind and Globes are made to go
eight or Spring, ſo as to imitate the riſing,
, and all the Diurnal motion of the Fix'd
Sometimes alſo the Planets perform their
tions in their proper time: But things of
ture don't belong to this place.

PROPOSITION XIV.

TO *deſcribe the Terreſtrial Globe, as it is commonly made.*

The Terreſtrial Globe does not differ from the above deſcribed Celeſtial Globe but in this, that inſtead of the Stars and Conſtellations compoſed of them, Lands and Seas are delineated upon it. And the Tracts, Bays, Promontories, and other things belonging to the Earth, are painted upon the convex ſpherical Surface, in regard of one another and the Poles upon which the Globe is turned in the Meridian, juſt as the Places themſelves are upon the Earth, which they repreſent, in regard of one another and the Poles upon which the Earth really revolves by the diurnal motion. Theſe places of the Earth thus imitated will really have that ſituation, if each of them be transferr'd upon this little Earth agreeable to their Longitude and Latitude by Obſervation. The Equator, and its Secondaries or Meridians paſſing thro' every ten Degrees, are alſo painted upon the Terreſtrial Globe; one of which, namely that which paſſes thro' the *Azores*, or *Fortunate Iſlands*, is the moſt conſiderable, and called the firſt Meridian; it being that Meridian from which the Longitude of a place is reckoned, as was ſaid above; and by conſequence from whence the numbering of the degrees of the Equator towards the Eaſt is continued. The Parallels alſo to the Equator, namely the Polar Circles and Tropics, and ſeveral others at ten deg. diſtance from one another, together with the Rhumbs (in the ſpaces repreſenting the large and vaſt Seas) ſhewing the Courſes of Ships guided by the Mariner's Compaſs and Needle, are all drawn upon it. And that the Terreſtrial Globe, beſides its own uſes, might

it ſupply the place of a Celeſtial Sphere
en ſuch one is wanting,) the Makers have
l an Ecliptic to it. But becauſe this Circle
rely a Celeſtial one, and has no proper
e upon the Earth, therefore they have pla-
its interſection with the Terreſtrial Equa-
(which interſection they might have placed
where) at the point where the firſt Meridian
ſects the Equator.

PROPOSITION XV.

O reſolve ſeveral Problems relating to the Situ-
ation of places of the Earth to one another, and
Diurnal motion of the Stars, by the Globes;
s, to ſhew the chief Uſe of both Globes.

o' the ſolution of the following Problems
e Globes is not accurate and exact, yet it
s very much to form and inſtruct the minds
niors; and we ſhall diſpatch them in one
oſition.

d here the Celeſtial Globe is to be looked
as the great Machine of the World itſelf;
un and all the Stars, and whatever we ſee
e concave Firmament, are to be concei-
s upon the convex ſurface, view'd by an
lac'd without that ſurface: But there is
eed of ſuch a fiction in the Terreſtrial
e. And becauſe the Globes are the In-
ents we make uſe of in this Propoſition,
are to be look'd upon as perfect; as for
anner of making them ſo, we are not now
quire, but to apply them as they are al-
made to the ſolution of the following
ems and others like them.

To find the Longitude and Latitude of any gi-
ven Place upon the Terreſtrial Globe.

ng the given Place to the Meridian, (al-
underſtand that ſide of it as is divided into

Degrees, which, as we ſaid before, is the true Meridian) and the degree of the Equator, then found under the Meridian, ſhews the Longitude of the Place. For the beginning of reckoning thoſe degrees is taken from the firſt Meridian; as was ſaid before. Beſides, reckon from the Equator in the Meridian to the given Place, the degrees of Latitude: For the Meridian is divided into four times 90 degrees, two of which 90 begin to be numbered from the Equator towards both Poles; the reckoning of the two other 90 degrees begins from both Poles, and ends in the Equinoctial under the Horizon. The Latitude is North or South, according as the Place given is to the North or South of the Equator.

2. *The Longitude and Latitude being given, to find the Place itſelf upon the Terreſtrial Globe.*

Look in the Equator for the degree of the given Longitude; 'tis evident that the Place ſought is in the Meridian drawn thro' this point taken in the Equator. Apply this point therefore to the divided Meridian, and reckon the degrees of the Latitude given toward the North Pole, if it be northern, or towards the South Pole, if it be ſouthern, and the point at the end of the reckoning will be the Place ſought.

3. *To rectify both the Globes to a given Latitude, together with the Quadrant of Altitude; and to diſpoſe the Globes agreeable to the four Quarters of the World.*

If the Latitude of the Place given be North, let the North Pole be raiſed above the Horizon, if South, the South Pole, till the Arc of the Meridian intercepted between the Pole and the Horizon, be of as many degrees as the Latitude of the place given is. For by *Prop.* 7. the Eleva-

ition of the Pole is an Arc ſimilar to the
ude of the place ; that is, containing as
y degrees. From whence 'tis evident to
ſight what Stars always appear ; namely,
as upon the turning of the Globe quite
d, don't deſcend below the Horizon : As
what on the contrary, never appear. Then
oute from the Equator in the Meridian up-
ls, the degrees of the Latitude given, and
point where the reckoning ends, ſhall be
ertex of the given Place : (For the height
e Pole is equal to the diſtance of the Equa-
om the Vertex ; ſince the ſame Arc, name-
e diſtance of the Pole from the Vertex is
omplement of both to 90 deg. :) Fix the
rant of Altitude by means of the Screw, to
ertex thus found, ſo as that its divided
(which ſerves inſtead of a Quadrant,)
proceed as it were from thence. Laſtly,
ehelp of the Mariners Compaſs fix'd to
ot of the Globe, let the Right line that
e common Section of the planes of the
on and Meridian be placed in the Meri-
line, ſo as that the elevated Pole of the
may look towards the elevated Pole of
Vorld. For we ſuppoſe the Magnetic
e to be directed along the Meridian line ;
ſt, that its declination (that hinders it) is
n. By this means, the Meridian of the
will coincide with the Meridian of the
; and its other points with the correſpon-
points of the Place. But ſuch an Hori-
plane muſt be pitched upon for the
to ſtand upon, as will make the Horizon
Globe to be parallel to the Horizon of
ce, as it ought to be.

4. *To find the Degree in the Ecliptic that the Sun is in at any given time, by the help of the Calendar and Circle of Signs drawn upon the ſurface of the Horizon, and by that to aſſign its Place in the Ecliptic itſelf.*

Look into the Calendar for the Month and Day propoſed, and oppoſite to it you will find the Sign and degree thereof, that the Sun is then in. Then in the Ecliptic drawn upon the Globe, look for that Sign of the degree of it, and that will be the Place of the Sun ſought. If you would be more exact, you muſt get the Place of the Sun, the help of an Ephemerides, and find that out, in the Ecliptic drawn upon the Globe. After the ſame manner may the Places of the Planets found by ſuch Ephemerides, be marked upon the ſurface of the Globe; by ſeeking firſt the Longitude of the Planet in the Ecliptic, and by protraction its Latitude from the Ecliptic either way, by the help of a pair of Compaſſes. To find the degree in the Ecliptic the Sun is in pretty nearly at a given time, you need only to know what Day of each Month the Sun enters a Sign of the Ecliptic, and compute one degree for every Day from thence. For this purpoſe there are ſome memorial Verſes compoſed, which may be ſeen in the Writers of the Doctrine of the Sphere.

5. *To find the right Aſcenſion and Declination of the Sun, or Star, or any given Point; and conſequently to find the Time of the Culmination of a given Star or Point.*

The right Aſcenſion of the Sun or any point upon the Celeſtial Globe, is found after the ſame manner as the Longitude and Latitude of a place on the Terreſtrial Globe; (by *Probl.* 1.) For the right Aſcenſion and Declination in the Celeſtial, is all one with the Longitude and Lati-

in the Terreſtrial Globe. Beſides, ha-
ught the Place of the Sun to the Meri-
ing the Index to twelve a Clock, (we
t twelve a Clock when the Sun is in
idian,) and turn the Globe round till
or given Point comes to culminate,
Index ſhews the hour, when that will
But becauſe the Hour Circle is too
ie part of an hour can't be eaſily diſtin-
n it: To be more exact, you muſt do
cauſe the intire Equator (together with
n portion of it, correſponding to the
notion of the Sun during that time)paſ-
Meridian in the ſpace of a natural day;
nt that a 24th part of the Equator, or
paſs the Meridian in one hour, and one
our Seconds of time, and ſo on. If there-
n the right Aſcenſion of a Star, you take
e right Aſcenſion of the Sun, you will
t portion of the Equator which paſſes the
n, in the time between one culminati-
e Sun and the next following culmina-
he Star: If inſtead of that, you take the
eſſary for the paſſage of the ſaid Arc of
ator by the Meridian, (allowing an
fifteen degrees, and four minutes for
ree, as was ſaid before; which Opera-
lled the *Converting an Arc of the Equator*
;) you will have the time of the Cul-
n of that Star from Noon. But if the
cenſion of the Star be leſſer than the
ſcenſion of the Sun, ſo that the latter
Subtracted from the former, you muſt
a whole Circle or 360 degrees.

Place of the Sun or any Star being given,
nd its Oblique Aſcenſion or Deſcenſion, as al-
s Eaſtern and Weſtern Amplitude.

the given Place of the Sun or Star to

the Eaſtern part of the Horizon, (having rectified the Globe for the Latitude of the Place, by *Prob.* 3.) and mark the degree of the Equator that aſcends with it: For the Arc of the Equator, intercepted between the beginning of Aries and the Point thus marked (expreſſed by the number annexed to the point) is the Oblique Aſcenſion of the Sun or Star it ſelf. And the Arc intercepted in the Horizon between the Eaſt (that is, the interſection of the Equator and Horizon) and the Place of the Sun or Star is the Eaſtern Amplitude of that heavenly Body. If you bring the ſame Place of the Sun or Star to the Weſtern part of the Horizon, the degree of the Equator then deſcending, will be the Oblique Deſcenſion; and the Arc of the Horizon, intercepted between the Weſtern point of the Horizon, and the ſetting Celeſtial Body will be its Weſtern or ſetting Amplitude.

7. The right and oblique Aſcenſion of the Sun, or a Star being given, to find half the time of its continuance above or below the Horizon; and if it be the Sun, whoſe Aſcenſions are given, to determine the Length of the Day and Night, and the hour of the Riſing and Setting of the Sun; but if it be a Star, to find the hour of its Riſing or Setting, by the time of its Culmination being given: And on the contrary, the Place of the Sun and Length of the Day being given, to determine the Places on the Earth to whom they agree.

If you fix the Index to twelve a Clock when the Sun's place is under the Meridian, and turn the Globe backwards till the Sun's place is at the Eaſtern part of the Horizon, the Index will tell you the Hour of Sun-riſe; and if the Place of the Sun be brought to the Weſtern part of the Horizon, the Index will point out the Hour of its ſetting; the ſame Index will likewiſe ſhew

the

Culminating, Riſing or Setting of the given , when the Star marked on the Globe comes he Meridian, Eaſtern or Weſtern part of the rizon: For in all theſe inſtances Nature itſelf nitated. And the Hour of the riſing or ſet- ; of the Sun or Star being known, the conti- nce of them above or below the Horizon is -evident.

To find the ſame more exactly by computa- n without the Globe, the Aſcenſions being en, do thus: Let the Aſcenſional Difference converted into time, and the addition of it ix hours, if the Celeſtial Body decline from Equator towards the Pole that is elevated, he ſubtraction of it from ſix hours, if it de- e towards the Pole that is depreſſed will half the continuance of that Celeſtial Body ve the Horizon: And this added to the time its Culminating will give the Hour of its ing, ſubtracted from it will give the Hour s Riſing. And conſequently its whole con- ance above the Horizon, (which is the gth of the Day, if the Celeſtial Body be the) will be given, which ſubtracted from 24 rs, gives its continuance below the Horizon.

f the Place be required where the Day is given Length, when the Sun is in a given nt in the Ecliptic; bring the Place of the to the Meridian, and the Index to twelve lock; turn the Globe till the Index ſhow the ur of the Riſing or Setting of the Sun agree- to the given Length of the Day; then ele- ng or depreſſing the Pole, let the Globe be ught to ſuch a ſituation as the Sun's place be in the Horizon when the Index is at ſaid hour; for in this caſe, the Globe is in ſituation, wherein the Sun being in a gi- point of the Ecliptic, makes the Day of the given Length. Therefore all the Places of the Earth, whoſe Latitude is equal to the height of the Pole of the Globe in this poſition, will anſwer the queſtion. And if the given Place of the Sun be in the Tropic towards the viſible Pole; (that is, if the Place be required whoſe longeſt day is a given length,) this Problem will ſerve to diſtinguiſh the Parallels and Climates.

8. *The Latitude and Sun's place being given, and beſides any one of theſe three, namely the Hour of the day, the Altitude of the Celeſtial Body, and the Azimuth of the ſame be given, to find the other two.*

Rectify the Globe for the Latitude of the Place given, by *Prob.* 3; bring the Sun's place to the Meridian, and the Index to twelve a Clock; then if the Hour be given, turn the Globe about, till the Index point at that Hour, or move it accurately, till as many fifteen degrees of the Equator paſs the Meridian forwards or backwards, as there are Hours ſince or before Noon; and if the given Hours have minutes, for each four minutes, make one degree paſs by the Meridian. Fixing the Globe in this poſition, let the Altitude and Azimuth of the Sun or Star, be meaſured by the Quadrant of Altitude firſt fixed to the Zenith: Namely the Altitude in the Quadrant, reckoning upwards from the Horizon; but the Azimuth in the Horizon from South or North point to the interſection of the Quadrant with the Horizon. If the Altitude of the Sun or Star be given, turn the Globe about till the Star arrive at the given Altitude, (which may eaſily be try'd by the Quadrant of Altitude,) and the foot of the Quadrant will ſhew the Azimuth, and the Index the Hour. When the Azimuth is given, bring

ng the foot of the Quadrant to the Azimuth
en, and turn the Globe till the given Star
ne to the edge of the Quadrant, and imme-
tely the Altitude is known, being reckoned
on the Quadrant, and the Hour by the In-
: Unleſs it be better to find it by the num-
of degrees of the Equator, that have paſs'd
Meridian, ſince the Sun was upon it. By
urs we here underſtand Equinoctial ones
kon'd from Noon to Midnight, tho they may
eaſily reduced to *Italic* or *Babylonic* Hours, by
ling (by preced. *Prob.*) the Hour of the ri-
or the ſetting of the Sun, from whence
(being likewiſe Equinoctials) are reckoned:
an eaſy matter alſo to reduce any given
inoctial hour, reckoned from any beginning,
n unequal or *Planetary* hour, by the length
he day. And if the places of the Planets
nd for that time by the help of an Epheme-
s) be mark'd upon the ſurface of the Globe,
Conſtellations and Planets may be eaſily
nd and diſtinguiſhed in the Heavens by the
be, if rectified to the hour of the Night, and
clear ſerene night be compared with the
of the Heavens, and turn'd about gradu-
as the night paſſes, to keep the ſame ſitua-
with the Heavens.

. *To find the time when a Fix'd Star riſes or ſets Coſmically or Achronically.*

ring the given Star to the Eaſtern part of the
izon, and mark the point of the Ecliptic that
with it; the time when the Sun is in that
nt (which you may eaſily find upon the Hori-
of the Globe by the Circle of Signs and the
endar annex'd) is the time of the Coſmical
g of that Star: But if the time of the Coſ-
al ſetting were required, mark in the Globe
point of the Ecliptic riſing when the given
Star

Star comes to the Weſt part of the Horizon, and the time wherein the Sun enters that point, is the Time ſought. To find the Achronical Riſing, mark that point of the Ecliptic which ſets when the Star riſes, and the Time wherein the Sun paſſes that point, is the Time ſought. And if that point of the Ecliptic be taken notice of, that comes to the Weſtern part of the Horizon together with the Star, that day the Sun is in that Degree, the Star ſets Achronically. And this is ſufficient, if by the Coſmical and Achronical Riſing and Setting be underſtood Morning and Evening riſing and ſetting, as it commonly does. But if the Coſmical riſing and ſetting (as *Kepler* would have it) ſignify the ſame as its aſcent above the Horizon of a given place, and deſcent below it, then this happens daily (if at all) to any Fix'd Star, the time of which is found by *Prob.* 7. And the Achronical Riſing of a Star (which according to the ſame Author, ſignifies a Star's riſing when the Sun ſets) will be the ſame, and found after the ſame way as the former: But the Achronical ſetting comes to the ſame thing with the Coſmical Riſing in the common ſenſe.

10. *To find the time, when a given Fix'd Star riſes or ſets Heliacally.*

Since all the Fix'd Stars riſe Heliacally in a Morning, and ſet in an Evening, to find the time when a given Fix'd Star (for inſtance, of the firſt Magnitude) riſes Heliacally, that point of the Ecliptic is to be found, which is 12° below the Eaſtern part of the Horizon when the Star riſes: For when the Sun is in that point of the Ecliptic, he is depreſſed below the Horizon, juſt as much as is required, that ſuch a Star may be viſible when it is juſt come above the Horizon. To find this by the Globe, bring the

Star to the Eaſtern part of the Horizon,
find with the Quadrant of Altitude that
nt of the Ecliptic which is elevated 12 de-
es above the Weſtern part of the Horizon:
e point oppoſite to this is the point ſought,
. one depreſs'd 12 deg. below the Eaſtern
t of the Horizon: That day therefore
erein the Sun poſſeſſes that point of the E-
tic (which may be eaſily found in theCalen-
annex'd to the Circle of Signs in the Hori-
of theGlobe)the ſaid Star of the firſt Magni-
rifes Heliacally. If the Star be of the ſe-
d magnitude, a point of the Ecliptic muſt
ound, that is depreſſed thirteen degrees be-
the Horizon, when the Star is juſt at it: If
e third, 14 degrees,and ſo on; as is evident
Prop. 9. If the time of the Heliacal ſet-
of a given Star of the firſt Magnitude be
ired, bring the Star to the Weſtern part of
Horizon, find as before, that point of the
ptic which at that time is elevated 12 de-
s above the Eaſtern part of the Horizon,
the point oppoſite to it (namely that which
preſſed as many degrees below the Weſtern
of the Horizon) is the Sun's place in the
ptic, when the ſaid Star ſets Heliacally.
r the like manner you may find the Place
e Sun in the Ecliptic for any given time
when a given Fix'd Star did at that time
or ſet Heliacally; and conſequently the
s anciently deſcribed and marked out by
Writers of Husbandry and Medicine, may
ompared with ours: For inſtead of a given
as it is painted upon the Globes, you are
ake uſe of that place of it, in which it was
at Age; leſſening the Longitude of the
one degree and a third part for every Age
the given, but keeping the ſame Latitude.

11. *To determine the beginning of the Morning and end of the Evening Twilight for any time given.*

Rectify the Globe for the Latitude of the Place, (which is ſuppoſed in this and all the preceding Propoſitions to be given,) bring the Place of the Sun at the time given to the Meridian; and the Index to twelve a Clock; then turn the Globe about towards the Eaſt, till the point of the Ecliptic oppoſite to the Sun's place, be elevated 18 degrees above the Weſtern part of the Horizon, in which caſe the Sun itſelf is depreſſed as many degrees below the Eaſtern part of the Horizon; and therefore (by *Prop.* 8.) the Morning Twilight is beginning: and the time of it ſhewn by the Index. But 'tis better to turn the Arc of the Equator, that paſſes the Meridian, (while the Globe is moved about into this ſituation from that which it had when the Sun's place was at the Meridian,) into time, to be ſubtracted from 12 hours, for finding the Time of the beginning of the Morning Twilight. After the ſame manner the end of the Evening Twilight is found, by turning the Globe about till the point oppoſite to the Sun's place be elevated 18 degrees above the Eaſtern part of the Horizon, *&c.*

12. *To find the Diſtance of any two Places on the Earth, that are given, as alſo the Angle of Poſition; or that which a great Circle paſſing thro' the given Places, makes with the Meridian of either of them.*

The neareſt diſtance of any two Places, which is always to be computed in a great Circle, is found by taking their diſtance as they are drawn upon the Globe; (by a pair of Compaſſes form'd for that purpoſe,) and applying that interval to the Equinoctial or firſt Meridian, to know the

num-

ber of degrees between the two Places;
reckoning for every Degree, 60 *Engliſh* or
German Miles. For the magnitude of the
h is ſuch, that a degree of a great Circle
ribed upon its ſurface, is nearly 60 *Engliſh*
5 *German* Miles.

o find the Angle of Poſition, the Globe
t be turn'd about till the firſt of the two
es becomes Vertical,(which it will be, when
rives at the Meridian of theGlobe,being re-
d for the Latitude of that Place;) then
e the Quadrant of Altitude faſten'd to the
ex, and conſequently to the firſt Place, till
ch the ſecond Place; and the Arc of the
zon intercepted between the Meridian and
drant of Altitude, will be the meaſure of
ngle of Poſition between the Meridian of
rſt Place and the Great circle (namely the
Quadrant) paſſing through the two Places,
ſe the Zenith is the Pole of the Horizon.
Arc of the Quadrant intercepted between
two given Places converted into miles,
out the help of the Compaſſes uſed above)
give the diſtance of the Places. But if,
ting the former Praxis,the Angle be found,
he ſame Circle paſſing thro' the two Pla-
nakes with any number of intermediate
dians; (paſſing thro' for inſtance, the ſe-
degrees of the Equator;) the *Rhumbs*
be determined, along which you are to ſail
intermediate Places ſituated in the ſaid
dians, if you would ſail nearly in anArc of
t Circle (that is, the ſhorteſt way) thro'
Places.

Given the Hour of the day according to the
eckoning of one given Place, to find the Hour
t that time in another given Place; and to find
he Place where a given Hour is reckon'd.

Bring the Place to the Meridian, and the Index to the given Hour according to the reckoning of that Place, then turn the Globe about till the other given Place comes to the Meridian, and the Index will point at the Hour ſought. For the ſecond caſe, bring the Meridian and the Index to the given Hour, as before; then turn the Globe till the Index point at the other given Hour; and all the Places then under the Meridian will reckon or have the given Hour of the day.

If you are for calculating it, you will find it more accurately. If the two given Places are under the ſame Meridian, or have the ſame Longitude, they will alſo reckon the ſame Hour from Noon or Midnight; But if their Longitudes differ, the difference of their Longitudes found by the Globe and turn'd into Time, gives the diſtance of the Hours in thoſe Places reckoned at the ſame moment, ſo as that the Place which is towards the Eaſt reckons the greater of thoſe two Hours. In the latter caſe, where the difference of time is given, let it be converted into an Arc of the Equator: And this Arc will be the difference of the Longitude, which being added to the Longitude of the given Place, if it reckons the former Hour, or ſubducted from it,if the latter, will give the Longitude ſought; and all the Places of the Earth under the Meridian having that Longitude, will anſwer the Queſtion. What is here ſaid of our Hours reckoned from Noon or Midnight, may eaſily be applied to *Italic* or *Babylonic*, or even *Planetary* Hours, by reduceing them; as was ſhewn in *Prob.* 8.

4. *To find the Place where the Sun is vertical at a given time; and all the Places where the Sun at that time appears in their Horizon riſing or ſetting.*

aving rectified the Terreſtrial Globe for the
tude of your Place and the Cardinal points
he World, turn the Globe about till that
e become vertical; and let the Globe in
poſition be expoſed to the Beams of the
(by removing the Horizon, that hinders
Rays of the Sun,) and at one view it will ap-
in what places of the Earth it is Day, and
e it is Night; where the Sun is riſing and
e it is ſetting. For ſince the Globe and
it ſelf are ſimilar in Figure and ſimilarly
ed, they will be ſimilarly enlightened by
un, whoſe immenſe diſtance may be look-
on as infinite. That Place, which is the
of the Circle, marked out by the confines
ght and Shade, is the Place where the Sun
ical, and may be determin'd mechanically,
eking the place, where the Shadow of a
erected perpendicularly upon the Surface
Globe vaniſhes, or falls upon the Foot
Style it ſelf. If the ſame things be re-
when the Sun does not ſhine, find (by
Prob.) the Meridian of the Places
it is then Noon, and in this Meridian
at place of the Earth, whoſe Latitude to-
the ſame Pole is the ſame with the Sun's
ation at that time; and that is the place
the Sun is vertical, and if it be made
l, the Horizon will ſhow the confines of
and Shade, or all the places where the
pears in the Horizon; and then 'tis eaſy
nguiſh the Places where he is riſing, from
ces where he is ſetting.

15. *To make an Horizontal Dial for a given Place by the help of a Globe.*

Rectify the Globe (as before) for the Latitude of the Place and the Cardinal points of the World: If beſides the Meridian, there were eleven other Hour Circles (that is, 24 Hour Semicircles) immovable, ſtanding without the Globe, and the whole Globe, beſides the Arc about which it revolves, were pellucid; 'tis evident that the Sun placed in the Meridian would caſt the ſhadow of the Axe upon the other half of the Meridian, and placed in the One a Clock Hour Circle after Noon, would caſt the ſhadow of the opake Axis upon the One a Clock at Night Hour Circle, or the other half of the ſame Meridian beyond the Poles, and ſo on in the other Hour Circles. If beſides all this, you imagine an Horizontal Plane ſhewing the ſhadow of the Axe caſt upon it by Reflexion, when the Sun is in the Meridian, the ſhadow of the Axis projected upon the horizontal Plane, will be in that part of the common ſection of the Meridian and Horizon, which is beyond the Center; that is, in a Right line connecting the Center with the interſection of the Meridian and Horizon: When the Sun is come to the One a Clock Circle, the ſhadow of the opake Axis projected upon the horizontal Plane, will be that part of the common Section of the One a Clock Hour Circle with the Horizontal Plane, that lies beyond the Center, or which connects the Center with the common interſection of the One a Clock at Night Hour Circle; and which does therefore contain with the Meridian or Twelve a Clock line, an Angle, that is meaſured by the Arc of the Horizon, which is intercepted between the Meridian and One a Clock Circle

Noon or Midnight. And after the ſame
er the Sun, when it is at the Two a
Circle after Noon, caſt the ſhadow of
xe upon the Horizontal Plane in a Right
onnecting the Center and interſection of
wo a Clock at Night Hour Circle and
on; which therefore makes with the Me-
line an Angle, that is meaſured by an
f the Horizon intercepted between the
ian and the two a clock Circle: and af-
e ſame manner in the other Hour lines
the Horizontal plane, every one of them
an Angle with the Meridian, which is
ed by an Arc of the Horizon intercept-
ween the Meridian and Hour Circle it
s to. That a Sun-Dial therefore may be
upon an Horizontal Plane, upon any
s a Center, let a Circle be deſcribed re-
ing the Horizon of the Globe, (for the
of it repreſents the Center of the Earth,
e preſent caſe, the Center of the World:)
he Center draw a Meridian line; that
that being produced would reach to the
l Points of North and South, and upon
ter erect an opake Right line repreſent-
Axe of the World, tending to the Pole;
ſo as in the plane of the Meridian to
n Angle with the Meridian line equal
Angle of the Elevation of the Pole above
izon of the place. Then let the one a
ine after Noon be drawn from the Cen-
e Circle on the Eaſtern ſide of the Me-
ne making an Angle with it, meaſured
rc of the Horizon intercepted between
idian and One a Clock Hour Circle. But
the Hour Circles are not to be found
s, let ſome certain Secondary (for in-
ne of the Colures) be pitched upon, and

let

let the Globe be ſo placed, that this aſſumed Secondary may be inſtead of the firſt Hour Circle; that is, that 15 degrees may be intercepted between the fix'd Meridian and this aſſumed one; then let the degrees intercepted between this aſſumed one and the fix'd Meridian, be reckoned in the Horizon. Turn the Globe then, till the aſſumed Secondary of the Equator come to be in the place of the Two a Clock Circle in the Afternoon; that is, till 30 degrees of the Equator be between it and the fix'd Meridian, and the degrees of the Horizon intercepted between the Meridian and the Circle aſſumed thus poſited, being numbered; let lines be drawn to the Eaſtern ſide of it from the Center of the Sundial, making Angles with the Meridian line meaſured by the ſame numbers of degrees, and they will be the Hour lines of One and Two a Clock. And proceeding, after the ſame manner muſt the Lines of the following Hours be drawn, till you come to that Hour that the Sun ſets at in the Longeſt day: Where, 'tis evident, that the Hour line of ſix a Clock is perpendicular to the Meridian line, ſince in any Latitude whatever, there are 90 degrees intercepted between the Meridian and the ſix a clock Hour line. After the ſame manner are the Angles to be determined, that the lines of the half Hours or any other parts of Hours contain with the Meridian line upon the Horizontal Plane. Having done drawing the Hour lines upon one ſide of the Meridian, for inſtance, for the Hours after Noon, the Morning Hour lines are to be drawn ſo, as that the lines repreſenting the equally diſtant Hours from Noon, may make equal Angles with the Meridian, but lying on the other ſide. After the like manner (by the help of the 13th *Prob.*) may a Sun-

dial

e drawn upon any Plane: For any Plane
Horizontal Plane of ſome place.

SECTION IV.

t the determining by Obſervation, the ıation and reſpect, that the Circles of Sphere have to one another.

PROPOSITION XVI.

D determine by Obſervation, the Plane of the
Meridian Circle in any Place.

Place being given, the Horizon is alſo
for that is determined by the Sight.
his being the only one that comes under
tice of our Senſes, the others muſt be de-
ed by the reſpect they have to this; and
e Meridian. Let two planes of Vertical
s be found by Obſervation, that a Cele-
ody, which does not change its Declina-
s upon, when it is at the ſame Altitude
and after its greateſt Elevation above the
n; the Vertical Plane biſecting the An-
tained by theſe two Planes, will be the
of the Meridian, and produced unto the
ns by the ſight will mark out the Meridian
Since (by the ſuppoſition) the Cele-
ody does not change its declination, it
ſcribe by its Diurnal motion a Circle pa-
o the Equator: The higheſt point there-
this Circle (as alſo of the Equator itſelf)
e Meridian Circle, and the points that
equal Heights on either ſide, will be
ſtant from the Meridian. Wherefore
ſly, that Circle from which the Verticals,
ch the Celeſtial Body was at the ſame

Altitude, are equidiſtant on both ſides, is the Meridian; that is, the Circle, whoſe Plane biſects the Angle contained by the abovementioned Verticals is the Meridian itſelf. *Q. E. I.*

As for the Practice, this is a good way enough of ſolving the Problem: Upon a Plane parallel to the Horizon, that is, one to which a Plummet is perpendicular (ſince the Zenith and Nadir, upon which the Plummet falls when produced, are the Poles of the Horizon,) erect a Style, on whoſe lower extremity, as a Center, let a Circle be deſcribed upon the Horizontal Plane, and let a point be marked in the Circumference of it, upon which the ſhadow of the top of the Style falls before Noon; and again that point of it upon which the ſhadow of the ſame falls in the Afternoon: A Right line connecting the center of the Circle with that point which biſects the Arc lying between the two points thus marked, is the common Section of the plane of the Meridian with the Horizontal Plane, which is alſo called the *Meridian line*; becauſe it ſhews the North and South Cardinal points by its direction. A Plane therefore erected upon this line, perpendicular to the Horizontal plane, is the Plane of the Meridian ſought. When the Celeſtial Body made uſe of in the ſolution of this Problem is the Sun, 'tis evident that it muſt be done, when the Sun does not ſenſibly change his declination; (for this is a neceſſary qualification in the Celeſtial Body pitched upon for this purpoſe;) that is, in one of the Solſtices. Becauſe theſe Obſervations are to be made before and after Noon, the Summer Solſtice is to be preferred, ſeeing that in the Winter Solſtice, by reaſon of the Sun's nearneſs to the Horizon, the

uncer-

ertain Refractions will render the operation
ibtful; and beſides, then the Sun aſcends
l deſcends too obliquely, altering its Ver-
il much more than its Altitude; which is in-
nmodious in this operation, where the Ver-
il is to be determined by the Altitude. There-
e when the Sun is in the Summer Solſtice, a
cle is to be drawn upon the Horizontal plane,
as that the extremity of the ſhadow of the
le or Gnomon may fall upon it, when the
i alters its Altitude pretty much, and his
imuth but little; and therefore ſeveral con-
itric Circles are to be drawn, that the moſt
nmodious may be choſen.

To determine the Meridian line drawn for
fixing your more accurate and perfect In-
ments, this operation ſhould be repeated, and
ers made uſe of, to be ſeen amongſt ſeveral
ers, or eaſily thought upon by any one:
d this is to be underſtood of any other Obſer-
ions, that are fundamental, and upon which
ng train of others are built. The longeſt
les or Gnomons are the fitteſt for the making
Obſervation: And inſtead of an opake ex-
nity or top, in a large Building, a hole let-
g the Rays of the Sun paſs thro' it into the
ifice, would be more proper; as *Caſſini* did
the Church of St. *Petronius* at *Bononia*.

PROPOSITION XVII.

*TO determine by Obſervation the Height of the
Pole and Situation of the Equinoctial, in regard
he Horizon.*

Where one of the Poles has a conſiderable
vation, [*Fig.* 5.] let the greateſt and leaſt
itudes OS and $O\sigma$ above the Horizon HO
any Fix'd Star that never ſets, (both of which
pens in the Meridian,) be obſerved by an

Inſtrument fix'd in the plane of the Meridian, (determined by the preced. *Prop.*) And PS or $P\sigma$ half the difference $S\sigma$ of theſe two Altitudes, is the diſtance of the Star from the Pole: This diſtance $P\sigma$ therefore, or PS added to the leſſer Altitude $O\sigma$, or ſubtracted from the greateſt OS, gives OP the Height of the Pole ſought. If the Star, when it is at its greateſt Height is ſituated upon the other ſide of the Zenith Z, between Z and H; then inſtead of the greateſt Altitude of the Star, you are to make uſe of its complement to a Semicircle, in the foregoing Calculation.

Great care and diligence is to be uſed in making this Obſervation; For upon this are all the other obſervations of the Sun and Fixed Stars, and conſequently all Aſtronomy is founded. Such a Fix'd Star therefore is to be choſen as is leaſt liable to Refraction in its leaſt Altitude; that is, the neareſt to the Pole, which does not approach the Horizon too much: For the Refraction of Stars are liable to make their places obſerved near the Horizon uncertain; as ſhall be ſhewn hereafter.

If neither of the Poles be elevated enough above the Horizon, then the thing is to be done by the Meridian Altitude of a Star, whoſe Declination is known; as ſhall be ſhewn in the *Corollary* to the following *Propoſition*. And in moſt places, where but few Obſervations are deſign'd to be made, this method is uſually taken. But ſince the Obſervation of the Declination of the Stars, depends upon the knowledge of the Height of the Pole, 'tis more proper to lay down ſuch a manner of obſerving the latter as does not depend upon the former; which alſo muſt neceſſarily be uſed by all cautious Obſervers, that deſign to determine the places of the Stars,

s, and without truſting the Obſervations of
ers at all, ſince they may repeat them.

The Height of the Pole being found,the Alti-
e of the Equator,or *HE* the Arc of the Meridi-
intercepted between the Horizon *HO* and the
uator *E Q* is determined ; For it is the Com-
ment of the former to a quarter of a Circle:
becauſe *HZO* is a Semicircle, and *PE* a
adrant, *PO* and *HE* together will be equal
a Quadrant. The ſituation therefore of the
uator in regard of the given Horizon *HO*, is
ermined: For it interſects it in the points of
true Eaſt and Weſt, which are marked out
a Right line perpendicular to the Meridian
und by the preced. *Prop.*) and the Plane of it
clined to the Plane of the Horizon by an
gle, whoſe meaſure is the Arc *HE* lately
nd.

SCHOLIUM.

By the help of theſe two Propoſitions the
gnitude of the Earth may be known. If a
rney made along the Meridian (drawn by
p. 16.) be exactly meaſured, and by an Ob-
vation of the Altitude of the Pole made at
beginning and end of the journey, the ra-
of the Arc of the Terreſtrial Meridian inter-
pted, to the intire Meridian be determined;
Circumference of the intire Terreſtrial Meri-
n will not be unknown: And conſequently,the
rth's Diameter, and from thence (by Geo-
try,) its Surface and Solidity will be known.
Beſides, becauſe in practiſe, a journey taken
der any Meridian, or the length of the Way,
very where very nearly proportional to the
nge of the Elevation of the Pole by Obſer-
ion; 'tis evident that the common Section of
ſurface of the Earth, and of the plane of any
ridian is a Circle, and conſequently that the

figure of the Earth is Spherical. We abſtract now from the little elevation at the Equator, (concerning which in *Prop.* 31. *B.* 1) which in the before-mentioned operation will be inſenſible.

PROPOSITION XVIII.

TO *obſerve the Declination of any Celeſtial Body.*

Let the Meridian Altitude of the Star propoſed be obſerved. If the Star be in the quadrant ZH of the Meridian [*fig.* 6.] that the Equator EQ interſects the Meridian in, as at σ; the difference of the Altitude of the Equator HE and of the Altitude of the Star $H\sigma$ (namely $E\sigma$) is the Declination of the Star ſought. But if the Star be in the quadrant ZO of the Meridian, in which the Pole P is, as in S; then the difference of the Altitude of the Pole P, and of the Altitude of the Star OS, (namely, PS,) or the diſtance of the Star from the Pole, is the complement of the Declination ſought. *Q. E. I.*

We add nothing at all here about the actual making the Obſervations and the conſtruction of the Inſtruments, in which the chief difficulty of theſe things conſiſts. For we ſuppoſe the Maker very well verſed in *Practical Geometry, Mechanics* and *Optics*. And it is evident that it will be moſt commodious for any one that is about to determine the Places of the Fix'd Stars, to have a Quadrant or rather a Semicircle intirely immovable in the Plane of the Meridian, and divided into Degrees, and their leſſer parts according to art, and an Index moving upon its Center, furniſhed with Teleſcopic Sights, for the ready meaſuring the Meridian Altitude of any Star, when it comes to the Plane of the Meri-

eridian: For this Obſervation of the Decli-
ation of the Stars is of great uſe in correcting
eir Places.

COROLLARY.

As the Declination of an Heavenly Body
found by the Elevation of the Pole or E-
uator, and the Meridian Altitude of the Ce-
ſtial Body being given; ſo, on the contrary,
e Elevation of the Pole or of the Equator
ay be had, by the Meridian Altitude and De-
ination of the Star being given. For in the
uadrant *ZEH*, the ſum or difference of the
ines $H\sigma$, $E\sigma$ gives the Elevation of the Equa-
r *HE*. But in the Quadrant *ZPO*, the ſum
difference of the Lines *OS* and *PS* gives the
ltitude of the Pole.

PROPOSITION XIX.

TO *obſerve the Inclination of the Ecliptic to the Equator or the Obliquity of the Ecliptic, and determine its Situation and conſequently that of the*
lures, Tropics and Polars, in reſpect of the Equator.

Let the Sun's Declination be daily obſerved
out both Solſtices: (by the preced.) This,
here it is greateſt, is the Obliquity of the E-
iptic ſought. For ſince the Ecliptic is the
ay of the Sun, as big as the greateſt Declina-
on of the Sun is, ſo big is that of the Ecliptic
ſelf; conſequently the one being obſerved,
e other is alſo obſerved. *Q. E. F.*

But ſince from the preceding Praxis, the De-
ination of a Celeſtial Body can't be obſerved,
cepting when that Star is in the Meridian;
the Solſtice ſhould happen to be celebrated
hen the Sun is not in the Meridian of the
bſerver, (which will generally happen) there
ill be ſome error; but ſo ſmall as to be not
orth regarding. For we gave inſtructions

that the Declination ſhould be taken any Noon; and conſequently the greateſt error will be when the Solſtice is celebrated at Midnight; that is, at that point of time which is moſt remote from Noon. The error therefore, when it is greateſt, will be equal to what the Sun's Declination 12 hours before or after the Solſtice wants of its greateſt Declination: But this does not ariſe o above four Seconds; tho' the Obliquity it ſelf is not determined to a minute, or even two, by Artiſts themſelves. Of the Solſtices that is to be choſen, in which the Sun may be beſt obſerved without any danger of error, and in this regions near the North Pole, the Summer Solſtice; becauſe the Sun in the Winter Solſtice, even at Noon it ſelf, is not out of all danger of Refraction; which is a matter of very great moment in this kind of Obſervation. There are ſome places, where to find the Obliquity of the Ecliptic, it may be adviſable to ſubtract the Meridian Altitude of the Sun, when in one of the Solſtices, from its Meridian Altitude in the other, if the Sun in both caſes be on the ſame ſide of the Zenith; but if on different ſides, from the Complement to a Semicircle of the latter Meridian Altitude, to have the diſtance of the Tropics remaining, half of which is the Obliquity ſought.

To get the Obliquity of the Zodiac with great exactneſs, Obſervations continued for ſeveral Years, ſays the skilful and induſtrious *Hevelius*, and ſuch as are compleat on all accounts, taken by your larger Inſtruments, and ſuch as may be depended upon, are neceſſary. And upon this account it is that now-a-days, when Inſtruments are both great and accurately divided and furniſhed with Teleſcopic Sights, there is a ſenſible difference obſervable between the

he Obliquities of the Zodiac ſet down by dif-
erent Artiſts. For *Hevelius* makes it 23^{deg}. 30′.
0″. The ſame does *Ricciolus* in his *Reformed*
ſtronomy. *Mouton*, who has writ almoſt an en-
re Book upon this Argument, chooſing, after
veral repeated Obſervations, the middle be-
ween the extremes, fixes upon 23^{gr}. 30′, as
icciolus in his *Almageſt* does from ſome ſelect
bſervations: This is what Mr. *Street* alſo, in his
ſtronomia Carolina, ſets down, and moſt Aſtro-
omers now make uſe of. Notwithſtanding
Ir. *De la Hire*, in his *Aſtronom. Tables, Part firſt, of*
e Motions of the Sun and Moon, makes it an in-
re minute leſs; namely, only 23°. 29′, and
at from Obſervations near the Zenith, and
at of all danger from Refraction.

And were the judgment of the Ancients a-
ut this matter conſulted, ſo great a difference
ould be found between the Obliquity obſerved
them and that taken from the Heavens now,
at ſome Authors have thought it to be changa-
e, beſides the change made in it twice a Year;
hich we don't here diſpute about, becauſe al-
oſt inſenſible: Since from the time of *Ariſtar-*
us the *Samian*, who flouriſhed near 300 Years
fore Chriſt, it was reckon'd to be about 24°:
ut in *Ptolemy*'s time, about 140 Years after
hriſt, not above 23°. 51′. And again, in
lbategnius's time, near 900 Years after Chriſt,
ly 23^{gr}. 35′. And *Tycho* by his Obſervations
akes it to be but 23°. 31′. 30″. Yet the moſt
duſtrious obſervers of the preſent Age, will
ve it, notwithſtanding, to be conſtant; name-
, *Gaſſendus*, *Ricciolus*, *Horrox* and *Hevelius*, who
es not think, in his *Prodromus*, that we ought
depend ſo much upon their Obſervations,
ce they did not make uſe of the beſt Inſtru-
ents, nor proceeded in the ſhorteſt method.
Nay,

Nay, ſome of the Ancients, and *Ptolemy* himſelf, in his *Almageſt*, think ſo too; *Albategnius* for the ſame reaſon with *Hevelius*, prefers his own Obſervations to thoſe of the Ancients. And *Gaſſendus* in the *Life of Peireskius*, ſhews, by ſome Obſervations of his own making, that the Obliquity of the Zodiac is the ſame now-a-days, as it was in the time of *Alexander the Great*; becauſe he found by an Obſervation made at *Marſeilles* by himſelf, that the ratio of the Gnomon to its ſhadow at Noon in the Summer Solſtice, was the ſame with that obſerv'd in the ſame City at the ſame time of the Year by *Pytheas Maſſilienſis*.

Beſides, the points in the Heavens, where the Ecliptic interſects the Equator are they, in which the Sun appears when the days are equal to the nights to an Obſerver, to whom either of the Poles is elevated above the Horizon. This therefore is the ſituation of the Ecliptic, that it may cut the Equator in the ſaid points, and be inclined to the Equator in the Angle above determined, half of it tending to the North Pole, in which the Sun is, when the Days are longer than the Nights to the Inhabitants of the Northern Hemiſphere of the Earth. The ſituation of the Ecliptic being given, the ſituation of the Tropics is immediately given; becauſe they are parallel to the Equator, and touch the Ecliptic, and are as far from the Equator, as the Obliquity of the Ecliptic is great, and the Polar Circles are as far diſtant from the neareſt Poles. The ſituation of the Colures is deduced from hence, for they paſs thro' the Equinoctial and Solſtitial points found before, and interſect one another at right angles in the Poles already determined by Obſervation.

PROPOSITION XX.

HE Obliquity of the Ecliptic being given, to
find by calculation, the Right Aſcenſion and
ination of a given point in it, as alſo the Angle
the Ecliptic makes with the Meridian at the ſaid
; and converſely, to define the point of the Eclip-
to which one of theſe agrees, and conſequently to
rmine the place of the Sun in the Ecliptic, by an
rvation made in Prop. 18.

et *E* ♋ [*Fig.* 7.] repreſent the Equator,
ſe Pole is *P*; *E L* the Ecliptic; *E* the point
ne of the Equinoxes; *L* a point given in
Ecliptic, thro' which ſet a Circle of Decli-
on, as *P L* ♋, be ſuppoſed to be drawn.
he Spherical Triangle *E L* ♋, there is gi-
the Side *E L*, namely the diſtance of the
t given from *L*, and the Angle ♋ *E L*, the
iquity of the Ecliptic; the Angle ♋ alſo
ven, namely a right one, becauſe *P* is the
of the Circle *E* ♋: The ſide ♋ *L* there-
will be found (by Spherical Trigonometry,)
h is the Declination of the point *L* ſought,
is North or South, according as the adja-
Pole is: In like manner the Side *E* ♋ will
ound, which (according to the ſituation of
oint *L* in this or that Quadrant of the E-
ic, and the nature of the point *E*) will be
er the Right Aſcenſion ſought, or its com-
ent to a quarter, or half, or three quarters,
n intire Circle. For the Angle ♋ *L E* is gi-
that the Ecliptic makes in the ſaid point
the Circle of Declination or Meridian.
converſe of this is done after the like man-
Yet we are to take notice, that there are
points of the Ecliptic,whoſe Declination to-
ds the given Pole is the ſame; and therefore
ſort of a determination is neceſſary.

SECTION

SECTION V.

Of the Fix'd Stars, and the determining their Places by obſervation, and ſome other things that belong hereto.

PROPOSITION XXI.

TO explain the ſeveral Orders of the Fix'd Stars, into which they are divided on account of their different apparent Magnitude; and to give an account of this difference.

In ſome of the preceding Propoſitions we have laid down the method, whereby the poſition of the chief Circles of the Sphere are determined in reſpect to the ſenſible Horizon; namely, of the Meridian, Equator, Zodiac, both Colures, Tropics and Polar Circles: We ſhould now ſhew the way, how the Places of the Fix'd Stars in reſpect of theſe Circles are to be found. But before that, we muſt ſay ſomething about the Fix'd Stars themſelves, their differences and order.

And firſt, there appears a vaſt difference among the Fixed Stars as to their apparent Magnitude and efficacy of their Rays, to any one that ſhall but look up to the Heavens in a clear night when the Moon does not ſhine. On this account Aſtronomers have made ſix Claſſes, or Orders of them, the greateſt of which they call *Stars of the firſt Magnitude*; the leaſt, *Stars of the ſixth*, and the intermediate in the ſame order. Some make the difference to lie in the real magnitude of the Stars, aſſerting ſome of them to be really bigger than others, while they are all equally diſtant, becauſe placed according to them

em in the ſame Spherical ſurface: Others fol-
wing the Ancients, make their different diſtance
be the cauſe of their different Magnitude.

And this opinion is very much favoured by
number of the Fix'd Stars of the firſt and
ond Magnitude. For if every Fix'd Star
the office of a Sun, to a portion of the
ndane ſpace nearly equal to this that our
commands, there will be as many Fix'd Stars
he firſt Magnitude, as there can be Syſtems
this ſort touching and ſurrounding ours;
is, as many equal Spheres as can touch
equal one in the middle of them. Now,
certain from Geometry, that thirteen
eres can touch and ſurround one in the
lle equal to them, (for *Kepler* is wrong in
ting, in *B.* 1 of the *Epit.* that there may
welve ſuch, according to the number of the
les of an *Icoſaedrum*,) and juſt ſo many un-
roverted Stars of the firſt Magnitude are
notice of by Obſervation. For Aſtrono-
are not as yet agreed upon their number:
lius reckons the *bright Star in the Eagle's*
ler of the ſecond magnitude, whereas *Tycho*
it of the firſt; and on the contrary *Heve-*
nakes the *little Dog* and *the right Shoulder of*
of the firſt magnitude, and *Tycho* of the
d. And there are others that *Hevelius*
lf doubts of.

ain, if it be asked how many Spheres equal
former can touch the firſt Order of
es, ſurrounding the Sphere placed in the
ning, (or rather a Sphere comprehending
former thirteen together with a fourteenth
Center;) the number of theſe will be
to be 52, or 4×13. For the centers of
Spheres of the ſecond Order are in a Sphe-
rface, which is quadruple of that in which

the centers of the Spheres of the firſt Order touching the inmoſt are, ſince the Diameter of the one is double that of the other; and nearly as many Fix'd Stars of the ſecond magnitude have been taken notice of; *Hevelius* making them to be about 50; *Kepler* following *Tycho*, 58; and *Ptolemy* not above 45; now we aſſume the middle between them: For there is a greater difference among Aſtronomers about Stars of the ſecond magnitûde than about Stars of the firſt. Nor will there be any great difference from Obſervation in determining the number of the Fix'd Stars of the other magnitudes according to this method. For the farther conſideration of this matter, it only remains that we ſhew, that there is the ſame Order obſerved among the Fix'd Stars of the firſt rank, as there is between the central Bodies of the Spheres touching and ſurrounding the inmoſt Sphere (near whoſe Center the Eye is placed;) and the ſame between the Fix'd Stars of the ſecond magnitude, as there is between the central Bodies of the Spheres ſecond in order from theſe; and ſo on. And here indeed the matter does not go on ſo well, (which made *Kepler* of another opinion,) as is evident even from hence, that upon the firſt caſt of our Eyes upon the Heavens, ſome tracts of the Firmament appear fill'd with innumerable Fix'd Stars, whereas others are found to be almoſt empty and void of any.

But there is no great error committed in the order of the Stars of the firſt and ſecond magnitude, as will appear to any one that makes a compariſon: For there are ſix Stars of the firſt magnitude in the Zodiac, three to the North, and four to the South, nearly, as it ought to be according to this Theory.

PROPOSITION XXII.

O enumerate the Conſtellations, into which the Fix'd Stars are diſtributed by Aſtronomers.

he Ancients either following the Shape and
ıre of the diſpoſition the Stars appeared in,
d by Religion, have diſtributed all the Fix-
tars viſible in our Temperate Zone, into 48
ges: Twelve of which are ſituated along
th of the Zodiac, and give names to its
ecatemories: And there are in the Northern
Aries, Taurus, Gemini, Cancer, Leo and *Virgo*;
e Southern, *Libra, Scorpio, Sagittarius, Ca-
nus, Aquarius* and *Piſces*. The other Images
placed in the Hemiſpheres ſeparated from
nother by the Zodiac: in the Northern
amely *the little Bear, the great Bear, Draco,
us, Bootes, the Northern Crown, Hercules,
Cygnus, Caſſiopeia, Perſeus, Andromeda, the
gle, Auriga, Pegaſus, Equuleus, Dolphin, Sa-
Aquila, Serpentarius, Serpens*. To theſe were
vards added the Conſtellations of *Antinous*
out of the unformed ones near *Aquila*, be-
it and *Capricorn* and *Sagittary*, and of
Berenices out of the unformed ones near
on's Tail. *Ptolemy* makes *Antinous* belong
uila, and *Equuleus* to *Pegaſus*. On the
ern part of the Zodiac there are 15 Con-
ions known to the Ancients, namely *Cetus,
us, Lepus, Orion, the Great Dog, the Little
he Ship Argo, Hydra, the Cup, Corvus, the
r, Lupus, Ara, the Southern Crown, and Piſces
us.*

theſe are lately added 12 Conſtellations,
e inviſible to us, being near the South Pole,
y the *Phœnix, Grus, Pavo, Indus, Apus, South
le, Muſca, Chamæleon, Piſcis volans, Toucan,
ſer Americanus, Hydrus, Xiphias* or *Dorado*.

There

There are ſome other Stars, which are not within the compaſs of the Images deſcribed, or reducible to them, theſe therefore they call *Unformed*; and out of theſe, ſome of the more conſiderable Obſervers of the Fix'd Stars, make now and then ſome new Conſtellations, and give them proper names. Thus the Accurate Mr. *Edmund Halley*, who firſt of all obſerved with exactneſs the Stars near the South Pole, while he was at St. *Helena*, has taken this liberty, in forming a new Conſtellation into the Image of an Oak, out of the unformed Stars that lie between the Ship *Argo* and the *Centaur*, and conſecrated it under the name of *Robur Carolinum*, or *the Royal Oak*, tranſlated into the Heavens in perpetual memory of the Preſervation of King *Charles* II. inthe hollow thereof. Thus *Bartſchius* in his Globe of four foot diameter, has two other new Conſtellations, *the Camelopard*, and *the Monoceros*, that *Hevelius* retains.

But *Hevelius* himſelf has gone farther in this matter, and has made up more new Conſtellations out of the unformed ones, than any of the Moderns beſides, as is evident from his *Firmament*; where he has plac'd *Leo minor* between Leo and Urſa major, and *Lynx* between Urſa major and Auriga above Gemini; the *Canes Venatici* after Urſa major and under its Tail; ſo that what to *Tycho* was an unform'd Star between the Tails of Helice and Leo (afterwards call'd *Cor Caroli* by the *Engliſh*,) by *Hevelius* is ſaid to be *in Annulo Armillæ Charæ*, or *Collar of Chara*; the *Lacerta* or *Stillio* between Andromeda and Cygnus; the *Sextans Uraniæ* between Leo and Hydra; *Scutum Sobieſciannum* between Aquila and Serpentarius; *Vulpecula cum Anſere*, between Aquila and Lyra under Cygnus; *Triangulm minus* between Triangulum Boreale and the Head

of

ries. He has altered ſome of the ancient
ſtellations a little, and added others to them;
has armed Antinous with a *Bow* and *Arrow*,
was before unarmed, and placed *Cerberus*
e the bending of *Hercules*'s left Arm, and
Mænalus under the feet of Bootes. The
ſtellations are ſet in the places deſcribed, as
ntinuation of the ancient Fables of the ad-
nt Conſtellations, leſt he ſhould break too
n in upon or diſorder the Rules and Judg-
s of the Aſtrologers, or to perpetuate the
ory of ſome noble Action.

hers again are for giving the Stars new
es, ſuch as are *Chriſtian* ones; a ſpecimen
s we have in the Venerable *Bede* in the
of the Zodiac; and *Julius Schillerus* has
nu'd in his *Cœlum Stellatum*, or Starry Hea-
publiſhed in the Year 1627. This deſign
een very juſtly diſapproved of by *Coperni-*
ycho, and *Hevelius* in his *Firmament*, and o-
ſtronomers, who think the ancient names
to be retained, for fear of confuſion,
y degrees loſing the Ancient Aſtronomy.
y this method, which has been conveyed
to us from the original of Aſtronomy,
tain what was of the greateſt importance
s matter; namely (as *Copernicus* ſays) *that*
t a multitude of Stars might be diſtinguiſhed
rts, and that theſe may have ſome names to
them by.

e *Via Lactea*, or *Milky Way*, comes under this
of Conſtellations; for the preſent Age has
it to be a collection of innumerable little
Stars, as ſome of the Ancients believed, as
ius informs us: 'Tis a broad Circle white
Milk, compaſſing the whole Heavens; ſome-
in a double, but generally a ſingle path,
g thro' Caſſiopeia, Perſeus, Auriga, the

Feet of Gemini, Orion's Club; the fore parts of Monoceros, the Tail of Canis Major, Argo Navis, Robur Carolinum, Crux and feet of the Centaur; aſter which, it is divided into two parts, over againſt Ara; its Eaſtern part paſſes thro' Ara, the extremity of the Tail of Scorpio, the Eaſtern foot of Serpentarius, the Bow of Sagittarius, Scutum Sobieſcianum, the feet of Antinous and Cygnus, where its greateſt part joins the other: But the Weſtern part paſſes thro' the former part of the Tail of Scorpio, the Right of Serpentarius and Cygnus; and ends its courſe in Caſſiopeia, where *Manilius* begins the deſcription of it. There are other leſſer diviſions of the *Galaxy*, the which may be ſeen in *Hevelius*'s *Firmament*. There are alſo two *Nubecula* ſomewhat reſembling the Milky Way, ſituated near the South Pole, inviſible in *Europe*, and called by Sailors, the *Magellanic Clouds*, are exactly like the whiteneſs of the *Galaxy*, as Mr. *Halley* aſſures us; and being view'd by a Teleſcope, they preſent us here and there with little *Nebulæ*, and ſmall Stars. The greater of theſe two is ſituated between Hydrus and Dorado, the leſs between Anſer Americanus or Toucan and Hydrus.

PROPOSITION XXIII.

THE *Theory of the Sun and a Pendulum Clock being given, to determine the Right Aſcenſion of the Fix'd Stars by obſervation.*

Having explained the difference of the Fix'd Stars, as well on the account of their apparent magnitude, as of the figure of the Conſtellations, into which they are commonly divided; what remains is to ſhew the method of determining the Place of any of them, in regard of the Circles of the Sphere, whoſe ſituation has been

en determined before, and firſt in regard of
Equator; for which purpoſe, we have ſhewn
w to obſerve the Declination of any one of
m in *Prop.* 18; it remains therefore that we
d the Right Aſcenſion: But, becauſe the be-
nning of the Equator, from whence the Right
enſion of the Stars is reckoned, is its inter-
tion with the Ecliptic; in which, though the
n be found once a Year, *viz.* in the begin-
g of our Spring, yet it preſently departs
m thence, and leaves that point obſcure or
rked out by no appearance that may come
ler the cognizance of our Senſes; and laſtly,
h an one as that the Aſcenſions of the Stars
't be immediately deduced from thence by
ervation; ſome Phænomenon therefore is
e pitched upon, whoſe Right Aſcenſion is
en every moment, that the Right Aſcenſions
the others may be obſerved from this. And
ll others, the fitteſt for this purpoſe is the
, becauſe its motion is the moſt ſimple, and
Right Aſcenſion is quickly found, by *Prop.*20,
place in the Ecliptic being given. If its
ory therefore be given, that is, if its place
he Ecliptic at a given time be given, and a
ck alſo whoſe Index moves round equably,
Right Aſcenſions of the Fix'd Stars may be
obſerved.

irſt, let the Clock be order'd ſo, as that the
ex may go over the 24 Hours, during the
that any Fix'd Star, departing from the
idian, returns to it again, which is a little
than a natural Day. Let the Index of a
ck thus order'd, be ſet to twelve a Clock,
n the Sun is in the Meridian, and let the
the Index points at, be taken notice of, that
fix'd Star whoſe Right Aſcenſion is ſought,
es to the Meridian at. Let theſe hours and

parts be turn'd into degrees of the Equator and their parts, as was ſhewn before: And you will have the difference between the Right Aſcenſions of the Fix'd Star and the Sun; this added to the Right Aſcenſion of the Sun, gives the Right Aſcenſion of the Fixed Star ſought.

If in the Clock fitted to the diurnal Motion of a Fix'd Star, inſtead of the 24 Hours, there are placed 360 Degrees, and their Sexageſſimal parts, and if while the Sun is in the Meridian, the Index be placed to the number of Degrees and Minutes, the Right Aſcenſion of the Sun then conſiſts of; it will then point at the Right Aſcenſion of the Fix'd Star, without any farther reduction, whenever it arrives at the Meridian.

PROPOSITION XXIV.

THE Declination and Right Aſcenſion of one Star, and the Declination of another together with their Diſtance being given, to find the Right Aſcenſion of the latter; and vice verſa, *to find the Diſtance of two Stars, whoſe Right Aſcenſions and Declinations are given.*

Let *E* ♋ repreſent the Equator, [*fig.*8.] whoſe Pole is *P*; and the Stars *A* and *B*, whoſe Declinations *CA*, *DB* being given, their Complements alſo are given, or the diſtances from the Pole, namely *PA*, *PB*, and by ſuppoſition, the Diſtance *AB* of the Stars is given. Wherefore in the Triangle *APB*, the Sides being given, the Angle *APB* will be found, and conſequently its meaſure the Arc *CD*; namely the difference of the Right Aſcenſions of the Stars *A* and *B*, which therefore added to, or ſubtracted from the Right Aſcenſion given of one of them, according as the Circle of Declination paſſing thro' this, is to the Eaſt or the Weſt of the

Circle of Declination paſſing thro' that,)
es the Right Aſcenſion ſought of the other.
nd on the contrary, the Right Aſcenſions
he Stars *A* and *B* being given, the difference
them *CD* is alſo given; that is, the Angle
B, whoſe meaſure that Arc is: And in the
angle *APB*, the Sides *PA*, *PB*, the Com-
ments of the given Declinations, together
h the Angle *APB* contained are given; the
er Angles will conſequently be found, and
Side *AB* the diſtance of the Stars ſought.
fter the like manner you may find the Di-
ce computed in a great Circle of two pla-
in the Surface of the Earth, whoſe Longi-
s and Latitudes are given, and the Angles
ch that great Circle (which anſwers the
mon Celeſtial Azimuth) makes with both
idians.

PROPOSITION XXV.

O determine by Obſervation the Right Aſcenſion
of a given Star, by a Phænomenon viſible by
and night.

et the Declination of the Phænomenon vi-
by day and night, be obſerv'd (by *Prop.* 18.
the diſtance as well from the Sun as from a
; the latter in the night, the former in the
From the Declination of the Phænome-
, the Diſtance from the Sun, and the Sun's
e, the Right Aſcenſion may be deduced, by
preceding; from that and the Declination
he ſame being obſerved, together with the
lination of the Star and Diſtance from the
nomenon being obſerved, the Rignt Aſcen-
of the Star ſought may after the ſame
ner be found. *Q. E. F.*

SCHOLIUM.

The Ancients, who had not an accurate meaſure of Time,(ſuch as *Prop.*23.requires, their Hourglaſſes, the inſtruments they made uſe of, being too fallacious,) ſought after the places of the Fix'd Stars by the Moon, taking the diſtance of the Moon from the Sun, computed according to the Equator or Ecliptic a days, and the diſtance of a Star from the Moon a nights, from whence they concluded upon the diſtance between the Star and the Sun, and conſequently (by the Sun's Theory) the diſtance of the Star from the beginning of the Ecliptic.

The Moderns, inſtead of the Moon uſe Venus, becauſe its Diameter being leſs will make the error leſs in obſerving the diſtance; and becauſe the Moon's place comes often to be obſerved by the places of the Fix'd Stars firſt corrected, that its Theory may be corrected, which is not as yet accurate, eſpecially about the Quadratures, which was the times that the Ancients made theſe Obſervations in; and alſo becauſe the Motion of Venus is neither ſo ſwift, nor ſo uncertain. Therefore it will be beſt to obſerve Venus in its greateſt Elongation from the Sun, when its Motion is nearly equal to that of the Sun; and when by reaſon of their Elevation the Sun and Venus are leaſt obnoxious to Refraction. In making theſe Obſervations ſpecial care muſt be taken to repeat them, as well when Venus is to the Eaſt of the Sun as when it is to the Weſt, ſo that by theſe different Obſervations conſpiring to determine the ſame place of the Fix'd Star, the true place may be at laſt certainly had. *Ricciolus* is for uſing *Sirius* inſtead of Venus, which, as he ſays, ſhines ſo bright as about the Equinoxes is viſible above eight minutes before Sun-ſet in *March*, and after

fter Sun-riſe in *September*. But both *Regulus* and
rcturus have been obſerved by the naked Eye
the Meridian, while the Sun ſhone, by the
ſtronomers of the *French* Royal Academy of
ciences, even when the Sun was elevated above
e uncertain Limits of Refraction: Which may
done every day in any of the brighter Fix'd
tars by the help of the Teleſcope.

PROPOSITION XXVI.

*HE Height of the Pole, Hour from Noon, and
Place of the Sun being given, if beſides, the
ltitude and Azimuth of a Star be known by Ob-
rvation, to find the Right Aſcenſion and Declination
that Star.*

Let *PZHO* repreſent the Meridian; [*fig.* 9.]
AO the Horizon, whoſe Pole is the Zenith *Z*;
R ♎ the Equator, whoſe Pole is *P*; *S* the Star,
ro' which we are to imagine the Vertical
ircle *ZSA*, and Circle of Declination *PSR*
be drawn. In the Triangle *SZP*, there are
iven the Side *ZP*, the Complement of the
levation of the Pole *OP*, the Side *ZS*, the
omplement of the Altitude of the Star *SA*;
nd the Angle *SZP*, that the Arc *AO*, given
the Azimuth, meaſures; therefore the Side
S, the Complement of the Arc *RS*, of the
eclination of the Star ſought is found; as alſo
e Angle *ZPS*, and conſequently its Meaſure,
amely the Arc *ER*. But the Hour of the
ay being given, the Arc of the Equator in-
rcepted between the Meridian and Circle of
eclination, drawn thro' the Sun, is alſo given,
nſequently the ſum and difference of the
iven Arcs are given. But this ſum, if the Sun
nd Star be on different ſides of the Meridian;
difference, if on the ſame, is the difference
f the Right Aſcenſions of the Sun and Star.
And

And the Sun's Place being given, its Right Aſcenſion (by *Prop.* 20.) is given; conſequently the Right Aſcenſion of the Star ſought is alſo become known.

PROPOSITION XXVII.

TO *find the Longitude and Latitude of a given Star.*

In Figure 10, let *E* ♋ repreſent the Equator, whoſe Pole is *P*; *EC* the Ecliptic, whoſe Pole is *B*; the Point *E* the common interſection of theſe Circles, the beginning of ♈ or ♎; *S* the given Star, thro' which the Secondaries *BSL*, *PSA*, of the Ecliptic and Equator are to be imagined to paſs. And the great Circle *PB*♋*C* connecting the Poles is the Colure of the Solſtices. Let *AS* the Declination of the Star be found (by *Prop.* 18,) then you will have its Complement *PS*; and (by *Prop.* 23 or 25) *EA* the Right Aſcenſion of the Star (at leaſt from the given Aſcenſion, the Right Aſcenſion *EA* will be given,) from whence you will have its Compliment ♋*A*, and the Angle ♋*PA*, whoſe meaſure it is. Therefore in the Triangle *BPS*, there are given the Side *PS* and the Side *PB*, namely the diſtance of the Poles of the Equator and Ecliptic equal to the Obliquity of the latter, together with *BPS*, the Angle contained by them; the Side *BS* therefore is known, and conſequently its Complement *SL*, the Latitude of the Star ſought. The Angle *PBS* alſo will be found, or *CBL* the next to it, and its meaſure the Arc *CL*, and therefore its Complement *LE*. But this Arc either is the Longitude itſelf, which is ſought, or its Complement to one, two, three or four Quadrants.

In like manner, by the ſituation of a Star in reſpect of the Ecliptic being given, its ſituation,

on, in regard of the Equator will be deter-
ined.

PROPOSITION XXVIII.

THE *Longitude and Latitude of two Stars, and the Diſtance of the third from both being*
iven, to find the Place of that third.

Let *CED* (in *fig.* 11.) repreſent the Ecliptic,
hoſe Pole is *P*; *A* and *B* the Stars, whoſe Lon-
itude and Latitude are given, thro' which let
e Circles of Latitude *PAC*, *PBD* be ima-
ined to be drawn: The Arc of the Ecliptic
D is the meaſure of the Angle *CPD*, con-
ined under them, the difference of the given
ongitudes, and conſequently is given. In the
riangle *APB*, the Sides *PA*, *PB*, the Com-
ements of the Latitudes *CA*, *DB*, and the
ngle *APB*, being given, the Side *AB*, and
ngle *PBA* will be found. Again, in the Tri-
ngles *ASB*, all the Sides being given, namely
A, *SB*, the diſtances of the Star *S* from the
vo Stars *A* and *B* being given, and *AB* be-
g firſt found, the Angle *ABS* will be found,
nd conſequently *SBP*, the difference of this
nd *PBA* found before: And laſtly, in the Tri-
ngle *SBP*, two Sides *BP*, *BS*, with the
ngle *SBP* contained between them, being
iven, *PS* will be found, and conſequently
s Complement *ES*, namely the Latitude
f the Star *S*; and the Angle *BPS*, and its
re *DE*, namely the difference of the Lon-
itudes of the Stars *B* and *S*, to be added
the Longitude of the Star *B*, that the Lon-
itude of the Star *S* may be had, if *PSE* be
ore to the Eaſt than *PBD*; but to be ſubtra-
ed, if more to the Weſt. There are other
aſes of this Problem, according to the vari-
ous

ous ſituation of the points *A* and *S*, but they are ſolved after the ſame manner.

And after the ſame manner, if the ſituation of the Stars *A* and *B* in reſpect of any other Circle, for inſtance the Equator, be given; that is, if their Right Aſcenſions and Declinations, and Diſtance of the third from both be given; the ſituation of it, in regard of that Circle, will alſo be given. And theſe things are done on the Earth after the like manner.

PROPOSITION XXIX.

TO deſcribe the method of making a Catalogue of the Fix'd Stars; and to give an account of the chief Perſons who have done ſo, and the methods they uſed; and to make a Celeſtial Globe.

Let every ſingle Star of a Conſtellation be deſcribed in order, according to the place it has in the Conſtellation; and over againſt each ſet down the Longitude and Latitude thereof, found by *Prop.* 27, and then annex its Magnitude. If there be ſeveral Stars in the ſame part of the Conſtellation, (for inſtance, in the head or the hand) let them in the deſcription be diſtinguiſhed into Northern and Southern, and into ſuch as precede, are in the middle or follow; where ſuch as are ſituated toward the Weſt are ſaid, by ſuch as follow *Ptolemy*, to precede; or ſuch as poſſeſs the preceding parts of the Zodiac, *i. e.* nearer to the beginning of Aries; that is, ſuch as precede in the Diurnal motion: They are alſo diſtinguiſhed by ſome by the Letters of the Alphabet annex'd. If there be any remarkable Star in the Conſtellation, which has a peculiar name, this name alſo is put down; as *Arcturus* in Bootes, *Regulus* in Leo, *Syrius* in the Great Dog, *&c.* After the Stars proper to each Conſtellation, the unformed Stars about it

re annexed, deſcribed after the like man-
by the nearneſs of this or that part (about
ich they ſtand,) together with their Longi-
e, Latitude and Magnitude.

The firſt that attempted to reduce all the Fix-
Stars into a Catalogue, and determine their
ces, was *Hipparchus of Rhodes*, about 120 Years
ore Chriſt, who, in *Pliny*'s judgment, *dared*
o a thing that God himſelf did not approve of, to
the number of the Stars for Poſterity, and reduce
n to a Standard. Tho' 'tis certain from *Pto-*
y that *Tymocharis* and *Ariſtyllus* left ſeveral
ervations about this matter, made 180 Years
ore *Hipparchus*. *Menelaus* alſo the Geometri-
n, in the firſt Year of *Trajan*, obſerved ſome
he places of the Fix'd Stars at *Rome*. After
ſe, *Claudius Ptolemy*, about the 140th Year of
riſt, began to obſerve and commit to Wri-
g, ſome things about the progreſs of the
ed Stars *in conſequentia*; the Catalogue of *Hip-*
chus, which riſes to 1026 Fix'd being ſtill re-
ned in regard of the place as to Longitude
Latitude; nor have we any Obſervations
he Ancients beſides theſe, which are deliver-
to us by *Ptolemy*. *Albategnius* the *Syrian* has
e the ſame about the 880th Year after
riſt: For in the Book that he has given the
e of the *Science of the Stars*, Chap. 52. Pag.
. he ſays that he has added 11 degrees and
f and a third part to the places in which he
nd them to be deſcribed in *Ptolemy*'s Book.
neither the *Perſian* nor *Alphonſine* Aſtrono-
rs, nor *Copernicus*, who is later than any of
m, obſerved all the Fix'd Stars, but reſted
sfied with the Catalogue of *Hipparchus*, only
ing in the reſpective Catalogues the portion
he Motion, which was made in the mean
ile from them all; of which we ſhall ſpeak
here-

hereafter. *Copernicus* reckons the Longitudes of all the Stars fromthe preceding of the two in the Ram's Horn, which he calls the *Firſt of all*, and which, by conſequence, are the ſame in all Ages, tho the Longitude of this firſt Star from the Equinoctial point continually increaſes.

All theſe Aſtronomers in the determining the Places of the Fix'd Stars, for the moſt part, made uſe of an Armillary Sphere, or a part of it, (which they called an *Aſtrolabe.*) Its deſcription may be ſeen in Chap. 1. Book 5. of *the Great Conſtruction.* This Inſtrument being fitted to the Latitude of the Place and the Cardinal points of the World, they placed the Armilla or Hoop repreſenting the Ecliptic, in the day time, by the help of theSun's Rays, (whoſe place they knew by the Theory of it,) in ſuch a ſituation as the Eclipticହad then, in regard of that Horizon; then they obſerved the place of the Moon in the Ecliptic, by the help of a movable Circle of Latitude in the Aſtrolabe. The next Night, by the help of the Moon, (whoſe place found before, they corrected according to its motion performed in the mean while,) they placed this Zodiacal Armilla in ſuch a ſituation as was agreeable to the preſent moment of time, (as before they had done by the mediation of the Sun;) thus they obſerved the Longitude of each Star reckon'd according to the Ecliptic, as they had before obſerved in the day time the Longitude of the Moon (looking along the moveable Circle of Latitude,) and its Latitude by the help of a moveable Sight in the ſaid Circle. You may ſee this method of Obſerving, fully deſcribed in Chap. 4. Book 7. of the *Great Conſtruction of Ptolemy*, and in *Chap. 14. B. 2. of Copernic. of the Revolut.*

Tho'

the Ancients thought the Moon was
ly neceſſary to determine the places of
rs in reſpect of the Equinoctial and
l points, and together with theſe the
gacious *Copernicus*, Chap. 14. B. 2. ſays
or without her there would be no way to
nd the places of the Stars, becauſe ſhe a-
hem all partakes of Day and Night: But
in a Book, which he entitles *Supple-*
Almanack, (after he had told us that the
ars were to be compared with the Moon
a Lunar Eclipſe, becauſe the Moon's
reſpect of the Sun, is at ſuch a time beſt
becauſe oppoſite to the Sun's,) very
ſubſtitutes Venus inſtead of the Moon
ffair, becauſe ſhe is both a day and a
tar, and has ſome other advantages,
n'd in the *Schol.* of *Prop.* 25.

next after *Hipparchus*, that obſerved the
tars themſelves a-new, was *Ulugh Beigh*,
ndſon of *Tamerlane the Great*. He ſays
reface, that he obſerved all that could
rved, beſides 27 in the South. This Prince
out 1449, and ſet down in his Catalogue
ated into Latin by Mr. *Tho. Hyde*) the
aces for the Year of Chriſt 1437.

ngth in the 16th Century appear'd the *At-*
at Age, the immortal *Tycho Brahe*, who by
igable labour and pains obſerved a-freſh
Fix'd Stars that are eaſily ſeen above the
n of *Uraniburgh*. And at the ſame time
the moſt illuſtrious *Landgrave* of *Heſſe*,
ted the ſame great Work at *Caſſel*, and
is Mathematicians *Rothman* and *Byrgius*,
ed in it for above 30 Years. He ſought
Longitude and Latitude of the Fix'd
y Calculation, from the Right Aſcenſion
clination firſt obſerved by the method in

Prop. 27; as *Tycho* aſſures us in *Chap.* 2. *Part* 1. *Progymn, pag.* 147; tho' he does not intirely approve of it, becauſe of the difficulty of reckoning the time exactly from Noon, which tho' often times tried by himſelf, by making Clocks, and other ways deſcribed in the before quoted Ch. of the *Progymnaſmata*, yet never ſucceeded according to his wiſhes. But tho' the Obſervations of the Prince don't extend to above 400 Fix'd Stars, and were made by ſmaller Inſtruments, yet *Hevelius* in *Prod. Aſtro.* prefers them to the labours of *Tycho* himſelf, becauſe in his opinion, the Prince and his Obſervers uſed more diligence than *Tycho* and his fellow Obſervers. The Prince afterwards performed this buſineſs by the help of Venus; but *Tycho*, tho' in *pag.* 146. *B.* 1. *Progymnaſm.* he charges *Cardan* with an error in ſettling the place of the more Southern and Bright Star of the Scales of Libra by Venus, greater than it is in the *Alphonſine Tables*; nevertheleſs confeſſes, that it aroſe from hence, that the ſituation of Venus was not at that time well determined by Numbers: And he himſelf uſes Venus for the ſame purpoſe, after the manner deſcribed in *Prop.* 25. Inſtead of a Zodiacal Armilla uſed by the Ancients in obſerving the different Longitudes of the Fix'd Stars and their Latitudes, *Tycho* for very good reaſons, ſubſtituted the Æquatorial Armilla; by which he might obſerve the differences of the Right Aſcenſions and the declinations out of the Meridian, the Meridian Altitude being always made uſe of to confirm the others. All the other cautions which this moſt diligent Obſerver took in this Affair, may be ſeen in *Chap.* 2. *B.* 2. *Progymn.* The Places of ſome of the Fix'd Stars being once laid down, found by the methods already delivered, he determined the Pla-

ces

es of the otherStars which were near the Eclip-
ic orEquator,after the manner related in *Prop.*24.
He ſought the Right Aſcenſion of any unknown
tar by two, whoſe places were already verified,
nd if the Right Aſcenſions of the ſame Star
hus found, were a little different from one a-
other, he always took the mean for the true
ne without any heſitation: Afterwards from
he Right Aſcenſion and Declination of a Star
e computed its Longitude and Latitude. As
or the Stars about the Poles he deduced them
om the former by the the Praxis deſcribed in
rop. 28; tho' he verified the Place thus found
y obſerving its diſtance from a third Fix'd Star
ſt determined. And univerſally, he never
ſted ſatisfied with the Place of a Fix'd Star
termined only by one obſervation; but
ch of them by the methods deſcribed before,
y his Equatorial Armilla, and by the Praxis
id down in *Prop.* 23, making uſe of the beſt
locks for meaſuring the time, Quickſilver and
ead being Chymically reduced into powder,
d ſhewing the time by its running thro' a
tle hole; all which you may ſee accurately
ſcrib'd in the *Progymnaſmata.*

Tycho's Catalogue of 777 Fix'd Stars made with
ch care and diligence as this for the Year
00, and taken from the Heavens themſelves,
laſt came forth, in the Year 1610, in *Aſtron.*
ſtaur. Progymnaſm. And afterwards in the Year
27, copied into the *Rudolphine Tables,* and
creaſed by 223 Stars from ſome other obſer-
tions of *Tycho*; ſo that this Catalogue in the
dolphine Tables riſes to the number of a thou-
d Stars. *Kepler* has added a ſecond Claſs of
ed Stars, that *Tycho* left out of the ancient
talogue of *Hipparchus,* repeated and corrected
Ptolemy, and calls them *Semi-Tychonic*: He

ſubjoins a third containing 12 Celeſtial Images, which are inviſible in our Northern temperate Zone. Theſe he has put into his Catalogue, being named by the *Portugueſe*, and corrected by *Peter Theodori*. 'Tis generally reported that theſe Southern Conſtellations were afterwards obſerved by one *Frederick Houtman, a Dutchman*, and the *Celeſtial Globe of Bleau* corrected according to thoſe Obſervations. But the Celebrated Mr. *Edmund Halley* was the firſt that took that pains about them as became an Aſtronomer, while he was at St. *Helena*, and publiſh'd a Catalogue of 350 Southern Fix'd Stars to the Year 1677 compleat, which is a true *Supplement* to *Tycho's Catalogue*. He obſerv'd the Diſtances of the unknown Southern Stars from ſome of *Tycho's*, and calculated their places from the aſſumed places of ſome of the *Tychonic* ones (by *Prop.* 28,) adding alſo the diſtances themſelves, which he had obſerved.

The next after *Tycho*, that preſented the World with an intire Catalogue of the Fix'd Stars of his own, was *R. P. Ricciolus*, in *Aſtron. Reform. Lib.* 4. to the Year of Chriſt 1700 compleat. But this Catalogue containing as many Stars as *Tycho's*, is taken from *Tycho's*, excepting in 101 Stars, which are all that he had obſerved in the Heavens. In the others, he recedes a little from *Tycho*, and has copied *Tycho's* manifeſt errors into his own, retaining at the ſame time ſome Stars as were really obſerved by *Tycho*, but were not viſible in the time of *Ricciolus*, as *Hevelius* has obſerved in *Prodr. Aſtr. p.* 133. and the following.

Laſt of all came out a *Catalogue of the Fix'd Stars*, by the Illuſtrious *Jo. Hevelius* of *Dantzick*, containing 1888 in all; namely 950 known by the Ancients, 603 which he calls his own, becauſe

cauſe they were determined by none before
mſelf as they ſhould be by proper Inſtruments,
d the 335 of Mr. *Halley*, inviſible in the Ho-
zon of *Dantzick*. In the making of this he ſo-
mnly profeſſes that he had no regard to the
ſervations of any Author, but entered upon
e work, as if there were no Catalogue at all
being. Nay, he intirely laid aſide all other
undations, and compoſed a-new the Theory
the Sun and Solar Tables, founded intirely
on his own obſervations. And laſtly, he
cerely declares, that he has borrow'd nothing
any body, or put into his Catalogue the Place
the ſmalleſt Star, that he himſelf had not ob-
ned by his different Diſtances and Meridian
titudes.

Hevelius drew up two ſuch Catalogues, the
ſt or larger, containing as well the Longi-
des and Latitudes, as the Right Aſcenſions
d Declinations, to the Year of Chriſt 1660
mpleat; to which are annexed the Places as
Longitude and Latitude, reduced to the ſame
ar, of the Fix'd Stars obſerv'd formerly by
ho Brahe, the Prince of *Heſſe*, *Ricciolus*, *Ulugh*
gh and *Ptolemy*; the ſecond or leſs, exhibiting
Places of the ſame Fix'd Stars as to Longi-
de and Latitude, to the Year of Chriſt, 1700
mpleat; to which he has annexed Mr. *Halley's*
alogue of the Southern Stars, partly corrected
to their Longitude, from obſervations at
ntzick, and reduced to the ſame time: So
t the larger Catalogue preſents us at the ſame
e with the Catalogues of all his Predeceſ-
s; by which means it is evident at ſight, how
the obſervations of them all, agree or diſ-
ree. To theſe he ſubjoins a Table of the Right
cenſion and Declination of ſeveral Fix'd Stars

to the Years of Chriſt 1660 and 1760, with their differences for that hundred Years.

By the help of ſuch a Catalogue of the Fix'd Stars as this, a Celeſtial Globe may be eaſily made: For the Ecliptic and its Poles being marked out upon the ſurface of the Globe, let the places of the Fix'd Stars taken out of the Catalogue be in like manner marked out according to their Longitude and Latitude in regard of that Ecliptic, draw alſo the tract of the Milky Way, thro' the places of the Fix'd Stars, thro' which it paſſes; and add the Equinoctial Circle, cutting the Ecliptic in the point aſſumed for the beginning of the the reckoning, and the oppoſite point, and duly inclined to it; as alſo let the Polar Circles and Tropics, and the other neceſſaries be added; of which we have ſpoken before in *Prop.* 13. And to this very purpoſe did *Hevelius* deſign his leſſer Catalogue, and from hence did he deduce his *Uranography* or Tables of all the Stars drawn upon a Plane, which was publiſh'd in the Year 1690, under the Title of the *Firmament of Sobieski*. For they differ no more from a Celeſtial Globe than Maps do from the Terreſtrial. Theſe *Uranographic* Schemes of the Conſtellations are delineated in *Plano*, juſt as if the convex part of the Heavens were ſeen by us, as it is in the Artificial Globe, as was uſual among ſuch as went before him, and made Catalogues of the Fix'd Stars. He ſays that *Bayer*, who in his *Uranometry* attempted the contrary, *thoughtleſly inverted all the Stars*; and that, *by his Figures he did but diſorder the whole buſineſs of the Heavens*; and that, *all the Stars which are placed on the left hand in the Globes, appear on the right hand.*

The making of a Celeſtial Globe by *Hevelius's Firmament* is much eaſier, becauſe the Figures of the Conſtellations are very aptly delineated it, together with the Milky Way, and its fferent Streams as it were. But we are to pect the moſt ample Catalogue of the Fix'd ars from Mr. *Flamſteed*'s Obſervations, which ve been continued for ſeveral Years with this ew, and made with large Inſtruments, and at e ſame time very fit for that purpoſe.

PROPOSITION XXX.

TO give an account of ſome of the more remarkable Changes as have happen'd among the Fix'd ars.

So long ago as the Age of *Hipparchus*, a new ar appeared among the Fix'd ones. And this, *Pliny* informs us, *was the reaſon why* Hipparchus *upon numbering the Fix'd Stars for the uſe of Poſte-y, and reducing them to ſome Rule, having invented ne Inſtruments for obſerving the Places and Magni-des of each; ſo that it might eaſily be known by this, t only whether they periſhed or had a beginning, but hether any at all paſſed away or had any Motion; alſo whether they increaſed or decreaſed, the Hea-ns being left in common to them all.*

And tho' *Ricciolus* reckons no leſs than ſix ew Stars between this and that famous one, hich appeared in *Caſſiopeia* in the Year 1572; et ſince they were either ſcarce obſerved at all, by Perſons not over-ſkill'd, there hardly re-ains any thing to be ſaid of them.

This new Star in *Caſſiopeia*, a rival to the ggeſt in the Heavens, firſt appeared about e beginning of *November*, in the Year 1572, d continued until *March* in the Year 1574. his, juſt as that of *Hipparchus*, was the occaſion *Tycho*'s obſerving the Fix'd Stars again, with ew and large Inſtruments; as he informs us mſelf in the *Progymnaſm.* where he treats ex-eſly of this Fix'd Star.

In *Auguſt* 1596, *David Fabricius* diſcover'd a New one of the 3^{d} magnitude in Cetus, which diſappeared about two months after.

In the Year 1600 a New Star appeared in the breaſt of Cygnus, firſt ſeen by others, but afterwards by *Kepler* himſelf, who has written an Aſtronomical account about it. This continu'd of the ſame magnitude *viz.* the 3^{d} until the Year 1659, as *Hevelius* aſſures us; (who had obſerved it from the Year 1638, with large Inſtruments,) but from the Year 1659 it ſenſibly decreaſed; and about the latter end of the Year 1661, it intirely diſappeared. Yet five Years after, namely in *September* 1666, *Hevelius* obſerv'd it again with his naked Eyes, of the 7th or 6th magnitude, retaining the ſame place in the Heavens preciſely.

In the Year 1604, about the beginning of *October*, a New Star was ſeen in the right Foot of Serpentarius, which appeared bigger than Jupiter and almoſt equal to Venus: It continued a whole Year; becauſe it was not ſeen after *October* in 1605. Concerning this *Kepler* has written a Book full of Aſtrological and Phyſical diſputes.

It was the Milky Way that preſented us with theſe three New Stars, namely in Caſſiopeia, Cygnus, and Serpentarius; it has therefore on that account been called by ſome, the Fund of new Stars.

There have been ſome other New Stars obſerved, namely in the Year 1612. that in the belt of Andromeda, by *Simon Marius*, who is the Author of *the World of Jupiter*, and another in Antinous by *Juſtus Byrgius* and others.

In the Year 1638, *John Phocyllides Holuuarda* of *Franequer* obſerved a New Star in the Whale's neck of the 3^{d} magnitude, or bigger. How it

diſap-

appeared every Year, and then appear'd, but
t always exactly at the ſame time, (tho' *Bul-*
ldus thinks otherwiſe,) and afterwards was
iſible for four Years, is fully evident, from
Hiſtory of it written by *Hevelius* and an-
xed to his *Mercury*, and from his *Climacteric*
r. Mr. *Caſſini* has found by Obſervation, that
ſame Phaſes of this Star return at the end
330 days; but ſo as that this return is ſome-
es retarded or anticipated by 15 days. This
r ſeems to be the ſame with that, which *Fa-*
ius obſerved in the Year 1612; for their
ces are the ſame.

n the Year 1670 in *July*, *Hevelius* diſcovered
ew Star of the 3^{d} magnitude under the
an's head, and in his Catalogue reduces it to
Vulpecula; but it was leſſen'd ſo much in a
rt ſpace of time, that about the end of *Au-*
1671 it was intirely extinguiſhed: But
t *March* it appeared again, at firſt like a ve-
ſmall Star, but by degrees it increaſed to one
the third magnitude, and afterwards grew
again ſenſibly as it did before; and in *Sep-*
ber 1672, totally diſappeared, and was ſeen
more.

ot only New Fix'd Stars now-a-days make
ir firſt appearance, but others of the ſixth,
n and fourth magnitude, known by the An-
nts and obſerved by *Tycho*, have intirely va-
ned, inſtances of which may be frequently
t with in the *Catalogue of Hevelius*. He him-
mentions four in *Prod. Aſtr.* namely a Star in
left thigh of Aquarius, the contiguous one
ceding in the Tail of Capricorn, the ſecond
he Belly of the Whale, and the firſt of the
ormed ones after the Scales of Libra.
ere are alſo other Obſervations which have
n made about them by the *French* Aſtrono-

mers. And even ſome conſiderable Fix'd Stars of the firſt, ſecond and third magnitude are found to alter their brightneſs and magnitude ſenſibly; as is evident from the different ſentiments of Authors about Stars of the firſt andſecond, or of the ſecond and third magnitude, That Poſterity may be more capable of judging about the change of the more conſiderable Fix'd Stars, with greater eaſe and leſs miſtake; *Hevelius* in his *Prodromus* has drawn up ſeveral things tending to that purpoſe obſerved by himſelf.

I paſs over other changes in the Fix'd Stars obſerved by the help of a Teleſcope, ſuch as are thoſe ſaid to be diſcovered lately by Mr. *Caſſini*; namely that ſome, as for inſtance, the firſt of Aries and the preceding head of Gemini, ſometimes appear divided each into two equal to one another, and diſtant from one another the length of the diameter of either of them; others, as ſome of the Pleiades and the middle in Orion's Sword, appear triple, nay quadruple ſometimes. For theſe Phænomena, beſides that they are not to be ſeen by the naked Eyes (of which we only treat at preſent,) ſeem to have another original, and conſequently are to be conſidered in another place.

PROPOSITION XXXI.

TO *define the true preceſſion of the Equinoctial Points, or the motion of the Fix'd Stars* in conſequentia; *and to determine from a given Catalogue of the Fix'd Stars, the place of any Fix'd Star at a given time.*

Hipparchus of old, as *Ptolemy* informs us, Ch. 1. Book 7. *of the Great Conſtruction*, from a compariſon which he made of his own Obſervations with thoſe of *Ariſtyllus* and *Tymocharis*, ſuſpected that the Stars of the Zodiac had a motion

n *in conſequentia* : But *Ptolemy* from a compa-
n of the Obſervations of *Hipparchus* and of
ers, with his own, has expreſly aſſerted it of
m all, and that it is made in Circles parallel
he Ecliptic ; which he proves in ſeveral of
more conſiderable Fix'd Stars, Chap. 2, and
comparing their places in regard of the E-
ator obſerved by ſuch as lived before him and
himſelf. And he finds that this progreſſive
tion is one degree in an hundred Years. *Al-*
egnius from the places of *Regulus* obſerved
Menelaus and himſelf, at the diſtance of 785
ars, gathers in Chap. 52 of his *Book of the*
owledge of the Stars, that the motion of the
'd Stars is one degree in 66 Years. *Ulugh*
gh, comparing his Obſervations with ſuch as
re made a great number of Years before,
ermines in the *Preface to his Tables*, that the
'd Stars move one degree in every ſeventy
lar Years. Later Aſtronomers comparing the
ſervations of the Ancients and Moderns, and
intermediate ones of *Albategnius* about this
tter, find that this motion is equable, quick-
than *Ptolemy* believed, but ſlower than *Alba-*
nius, and nearly the ſame with what *Ulugh*
gh aſſigns. For *Tycho* in his *Progymn.* Book 1.
thers, that it is one degree and 25′ in a hun-
ed Years, or 51″ in one Year. *Copernicus*, tho'
looked upon this motion to be unequal, made
e mean to be, from the Obſervations of the
ncients and his own compar'd, to be about
. 23′. 40″. 12‴ in 100 Years. *Ricciolus* makes
ſomething leſs, namely 1°. 23′. 20″. in that
ne, which Mr. *Flamſteed* retains. *Street* in his
tronomia Carolina makes it ſtill leſs, as not a-
ve 1°. 20′. *Bullialdus* in his *Aſtronomia Philo-*
ica makes it 1°. 24′. 54″ in 100 Years ; whilſt
evelius lately in his *Solar. Tab.* in *Prodrom.* will
ve it to be only 1°. 24′. 46″. 50‴. For

For every Year therefore, forward from the the Year to which the Catalogue is fitted, add 50″ (the middle round number) to the Longitude of the Fix'd Star found by the Tables, and you will have the Longitnde ſought: The Latitude in the mean while continues invariable, ſince both the Fix'd Stars and the Ecliptic in the Heavens are immovable, and only the Equator (originally a Terreſtrial Circle) being looſen'd as it were leaves its ancient ſeat. But for every ſingle Year, backwards in reſpect of the given Catalogue, ſubtract 50″ from the Longitude in the Tables, if you would find the Longitude for ſuch a given time. But if the order of the given Catalogue be ſuch, as that the Longitudes of each of the Fix'd Stars are reckon'd from the firſt Star of Aries, (as it is in *Copernicus*'s Catalogue of the Fix'd Stars, as alſo *Clavius*'s, and of ſome annexed to the *Caroline Tables*,) to the number of the Longitude in the Tables, (becauſe continuing always the ſame) add the Longitude of the firſt Star of Aries (which at the beginning of the Year 1701, according to *Hevelius* is 29°. 00′. 58″, and it is to be increaſed by 50″ every Year afterwards, but leſſened by as many for every Year before, as was ſaid before) for the time given, and you will have the Longitude ſought from the Vernal Equinox, caſting away 360 degr. if the ſum exceed that. But if the Right Aſcenſion and Declination at a time given be required, what is propoſed may be done by the converſe of *Prop.* 27. from the Longitude and Latitude found for the given time, as before.

SECTION VI.

cerning the reſolution of the more con-
iderable Problems of the Firſt Motion by
alculation.

PROPOSITION XXXII.

O determine by obſervation the difference of
the Meridians or Longitudes of two Places gi-
upon the Surface of the Earth, and to make a
logue of the moſt noted Places of the Earth, with
Latitudes and the Differences of their Meridians,
to make a Terreſtrial Globe.

et any inſtantaneous Phænomenon (as the
inning or end of a Lunar Eclipſe, or of an
pſe of any of Jupiter's Satellites,) be obſer-
by Obſervers in the given Places, noting
hour in both Places: The difference of the
rs converted into an Arc of the Equator gives
Difference of the Longitudes or Meridians
ght. And that of the Places propoſed is
e toward the Eaſt, that reckons more hours
r Noon; But if they reckon the ſame, they
under the ſame Meridian. For ſince the
e moment of time has differing names as to
hour in theſe two Places, the Noons of
e Places or any other hours of the ſame
e, will be as diſtant from one another, as
e different hours in the ſame Place; and
ſequently the Arc of the Equator intercept-
etween the Meridians of the Places is right-
und by allowing 15° for every hour. This
rval of Time is called the *Difference of the*
idians, appoſitely enough, becauſe it is ſo
he Noons.

In like manner, if a Clock that ſhews exactly the hours at one of the Places be carried to the other, to be compared with another ſhewing the hours there; the Difference of the hours reckoned by the Clocks, converted into an Arc of the Equator, gives the Difference of the Longitudes ſought: For thus, as in the former caſe, you know the interval between the hours reckoned at the ſame moment in the two Places. For the Clock, that is tranſported, is ſuppoſed to ſhew the hours after the ſame manner, as it would have done, had it continued in its place.

To each of the more noted places that are put into a Catalogue, add the Latitude proper to it, expreſſed by the ſame number of degrees (by *Prop.* 7,) as is the Altitude of the Pole obſerved there by *Prop.* 17. Add alſo in another column the Difference of the Meridians expreſſed by hours and their parts (found by what has been ſaid before) of the Place propoſed and of the Place aſſumed, together with the mark of Addition or Subtraction, according as the Meridian of the one lies to the Eaſt or to the Weſt of the Meridian of the other.

The Latitudes of the Places and the Differences of the Meridians being given, 'tis eaſy to make by them a Terreſtrial Globe: For the Poles and a great Circle upon them being drawn upon a Sphere made for that purpoſe, which will repreſent the Poles of the Earth and the Equator, thro' the Poles draw a Circle repreſenting the Meridian of the Place (to which the Meridians of the other Places in the Catalogue are referred,) and in it mark out a point diſtant from this feigned Equator, as many degrees as there are in the Latitude of the Place ſet down in the Table; and that point will there-

refore repreſent the ſaid Place. Any other
ce is to be put in a Circle paſſing thro' the
les, between which and the Meridians alrea-
ſettled, there are as many degrees of the
gned Equator (and that according as the ſe-
id Place is, towards the Eaſt or Weſt,)as there
in the tabular difference of Longitudes found
ove; and as many degrees from the feigned
uator towards either Pole, as there are in
tabular Latitude of that Place. In like man-
, the other more noted Places are to be
rked upon the ſurface of the Globe. The
ermediate or leſs noted Places are ſupplied
the common Chorography, and Itineraries,
king uſe of the Latitude of ſome for corre-
on; for this is more eaſily obſerved than the
fference of Longitude. The Globe being thus
iſhed, either the Meridian of our own place,
any other paſſing thro' the Azores or any of
Fortunate Iſlands, may be reckoned for the
t; that is, the degrees of the Equator are to
reckoned from the interſection of it with the
d Meridian (or beginning) towards the Eaſt.
en the other Circles and neceſſary appurte-
nces deſcribed in *Prop.* 14. are to be added ac-
rding to art.

PROPOSITION XXXIII.

*HE Elevation of the Pole, the Right Aſcenſion
and Declination of a Star, or any other point
the Heavens being given, to find its Aſcenſional
fference, and by that means its Oblique Aſcenſion,
ntinuance above the Horizon, and Eaſtern Ampli-
le.*

Having laid down in what goes before, the
thod of determining the ſituation of the Cir-
s of the Sphere for any habitation or place,
d of finding by obſervation the places of the

Stars in regard of the ſaid Circles in the Celeſtial Sphere, and the ſituations of Places on the Earth in reſpect of the ſame, conſidered on the ſurface of the Earth; we ought now, by the rules of Method, to ſhew how to reſolve by calculation ſome of the chief remaining Problems, about what has been determined and found out already, and according to their Example may any others relating to the Firſt Motion, be eaſily reſolved after the ſame manner.

In the preſent caſe, let *E ♋* (*Fig.* 12.) repreſent the Equator, whoſe Pole is *P*; *HOR* the Horizon, whoſe common interſection's, O and the oppoſite point, are the points of true Eaſt and Weſt, namely the Poles of the Meridian *PE♋*. Let *S* be a Star riſing or ſetting, and thro' it imagine a Circle of Declination *PSA* drawn. In the Triangle *OAS* there are given, beſides the right at *A*, the Angle *SOA*, whoſe meaſure is *HE*, the Altitude of the Equator; the ſide *AS*, the Declination of the Star given; conſequently *OA*, the Aſcenſional Difference ſought, may be found: For O is the point of the Equator that arrives at the Horizon together with the Star; *A* is a point of the ſame that arrives with it at the Meridian. This Aſcenſional difference therefore ſubtracted from the Right Aſcenſion of the Star declining towards the elevated Pole, but added to the Right Aſcenſion of the Star declining towards the depreſſed Pole, gives its Oblique Aſcenſion; and ſo on the contrary in the Oblique Deſcenſion. But this ſame Aſcenſional Difference added to the Quadrant, or ſubtracted from the ſame, gives the ſemidiurnal Arc of the Star, according as it declines towards the elevated or depreſſed Pole: And that turned into Time, gives half the continuance of the Star above the Horizon. In the Triangle *OAS* the ſide *OS* is alſo found, the Eaſtern or Weſtern Amplitude.

By reſolving the ſame Triangle *OAS* after
e ſame manner may the converſe of this Pro-
em be ſolved; that is, the Right Aſcenſion,
eclination and Continuance above the Hori-
n of any Star or Point being given, to find
Elevation of the Pole.

PROPOSITION XXXIV.

T*HE Elevation of the Pole being given, if the
Altitude of a Star be known by obſervation,
oſe Right Aſcenſion and Declination are known, to
the moment of Time, and the Azimuth of the Star.*

The ſame Circles of the Sphere being ex-
ſſed by the ſame letters, let the Vertical *ZSA*
g. 9.) be ſuppoſed to be drawn thorough
Star *S*. In the Triangle *PZS*, all the ſides
given; namely *PZ*, the complement of the
vation of the Pole; *ZS*, the complement of
Altitude of the Star above the Horizon; and
the diſtance of the ſame Star from the Pole,
he complement of the Declination: The
gle *PZS* therefore is found, which is mea-
d by *OA* the Azimuth ſought; as alſo the
gle *ZPS*, whoſe meaſure is the Arc of the
ator *ER*. But the Sun's place being given,
ight Aſcenſion is alſo given: Conſequent-
e difference between this and the Right
enſion of the Star, known alſo by ſuppoſiti-
is alſo given: Therefore the ſum or diffe-
e of theſe Arcs will be given; namely the
of the Equator intercepted between the
idian *PEH* and the Circle of Declination
n thro' the Sun, which converted into
, gives the time before or after Noon, ac-
ing as the Sun is to Eaſt or Weſt of it.

ou will have the ſame, if inſtead of the Al-
le of a Star its Azimuth be known by ob-
tion.

PROPOSITION XXXV.

THE Longitude of the Place, the Hour of the day, and the Sun's Place being given, to determine the ſituation of the Ecliptic in reſpect of the Horizon; that is, to find the Angle of the Ecliptic and Horizon,(or the Altitude of the Nonageſim degree of the Ecliptic from the riſing point above the Horizon,) the riſing point of the Ecliptic (and from thence the Nonageſim-degree) and the point of the Horizon, in which the Ecliptic interſects it.

Things remaining as before, let *S* be the Sun in the Ecliptic *M S E A* [*Fig.* 13.] Let the right Aſcenſion for the given hour be found (by *Prop.* 20.) or (making *P S B* the Circle of Declination,) the point *B* in the Equator, or conſequently its diſtance from *E* the next Equinoctial point, namely *E B*. And ſince the Hour is given, the Arc of the Equator *B Q* intercepted between *B* and the Meridian, and conſequently *B O* its complement to the Quadrant of the Equator *Q O*: But *B E* is given, from whence the Arc *E O* is known. Therefore in the Triangle *A E O* comprehended between the Ecliptic, Equator and Horizon, there are given the Angle *A E O*, the obliquity of the Ecliptic, and *A O E* the inclination of the Equator to the Horizon, and the ſide *E O* juſt found: From whence the reſt will be known; of which *E A O* or its complement to two right *M A H* is the Angle that the Ecliptic makes with the Horizon, meaſured by the Altitude of the Nonageſim degree; *E A* the diſtance of the riſing point of the Ecliptic from the Equinoctial point *E*, and from thence the higheſt point, or the Nonageſim degree from the riſing point *A* becomes known; and *O A* the diſtance of the riſing point *A* from the true Eaſt *O*, computed in the Horizon. *Q. E. F.*

PRO-

PROPOSITION XXXVI.

HE ſame things being given, to find the ſitua-
tion of a Star, whoſe Longitude and Latitude
given, in regard of the Horizon, or the Altitude
Azimuth; as alſo to determine the point and
le, in which the Vertical thorough the Star inter-
the Ecliptic.

et the place of the Star *S* given in re-
d of Longitude and Latitude (by the con-
e of *Prop.* 27) be reduced to the Equator;
ind its Right Aſcenſion and Declination.
ſince the Sun's Right Aſcenſion is given, the
rence of theſe Aſcenſions will alſo be given.
from the Hour of the day given, the Arc
e Equator intercepted between the Meri-
and Circle of declination drawn thro' the
is known: The Arc therefore (*Fig.* 14.)
of the Equator, between the Meridian
P S the Circle of declination paſſing thro'
alſo known; and conſequently the Angle
B, which it meaſures. In the Triangle
S, there are given the ſides *P Z*, *P S*, (the
the complement of the Pole's Elevation, the
the diſtance of the given Star from the Pole,
vn by its Declination,) together with the
le *Z P S* juſt now found; by which means
will be known the ſide *Z S*, and the An-
P Z S: *Z S* is the complement of the Alti-
E S, and *RE* the Arc of the Horizon, which
led the Azimuth, meaſures the Angle *PZS*;
ſince *R O* is a Quadrant, the Arc *O E*
liſtance between *O* the point of the true
and the Vertical *Z E* drawn thro' *S*, is al-
und. Beſides, in the Triangle *E C A*, com-
ended between the Vertical *Z S E*, Hori-
HOR, and Ecliptic *ACM*, the Angles at
d *A* are given, (the one being found above,

and the other a right angle,) together with the intermediate ſide *E A*, the ſum or difference of the known Arcs *O E*, *O A*: Therefore the other ſides are alſo given, namely *A C*, the diſtance of the point *C* from the riſing point *A*, which is known, the ſide *E C*, and the Angle *E C A*, which the Vertical *Z S E* drawn thro' the Star *S*, contains with the Ecliptic. *Q. E. F.*

Tho' this Problem be no more than the converſe of *Prop.* 26, yet becauſe of its frequent uſe, I thought fit to inſert it.

COROLLARY.

The ſame things being given, by the ſame labour may the Place on the Earth, to which the Star *S* is Vertical, be determined. For in the Triangle *Z P S*, in which all the Sides and Angles are known, the ſide *P S* denotes the diſtance of the Place on the Earth from the Pole *P*, from whence its Latitude is known; and the Angle *Z P S* is the diſtance of the Meridians of the Place given and ſought. But if *S* be in the Ecliptic, the Problem becomes more ſimple; and is ſtill the more ſo, if it be the Sun, or the Point oppoſite to it.

PROPOSITION XXXVII.

THE Elevation of the Pole and the Oblique Aſcenſion of the Star being given, to find its Coſmical and Achronical Riſing.

The Circles of the Sphere [*Fig.* 15.] being marked by letters, as oftentimes before, in the Triangle *E O A*, there is given the Angle at *E*, namely the Obliquity of the Ecliptic; the ſide *E O* the Oblique Aſcenſion of the Star *S* (at leaſt the diſtance of the point of the Equator that riſes together with it from the next Equinox, from the Oblique Aſcenſion given) and the Angle *E O A* the inclination of the Equator to the Horizon, or its complement to two

right

ht: Therefore the ſide EA will alſo be
en; and conſequently that point in the E-
tic that riſes with the Sun. Therefore when
Sun is in this point of the Ecliptic, the Star
iſes Coſmically; and the Sun being in the
oſite point, the ſame Star riſes Achronically.
d after the like manner may the Coſmical
Achronical Riſing of a Star be found from
Oblique Deſcenſion of the Star (found by
. 33.)

ut if the Coſmical Riſing and Setting ſignify
ſame thing with an Aſcent above Horizon
Deſcent below it; the time of the Coſmical
ng and Setting is determined by the prece-
g: For the Altitude of the Star (being no-
g in its Riſing or Setting) is given, and
efore the moment of time will be known.
d then the Achronical Riſing will ſignify
t it commonly does; namely the Evening
. And the Achronical ſetting all one with
Coſmical ſetting, as it is commonly called,
orning one.

PROPOSITION XXXVIII.

*O determine the Heliacal Riſing and Setting of
a given Star.*

hings remaining as they were, let D
. 16.] be the place the Sun is in when the
S riſes Heliacally. Therefore in the Tri-
le AFD there is given, beſide the right
le at F, the Side FD, *viz.* the Arc of the
's depreſſion requiſite to permit the Star of
ven magnitude to appear; alſo the Angle
F, which the Ecliptic comprehends with
Horizon, when the Star S riſes, found by
p. 35; therefore the Side AD will alſo be
nd. But this Arc added to the Arc EA
nd by the preceding) gives ED the diſtance

the Sun from the Equinoctial point *E*, when *S* riſes Heliacally. Therefore the time of the Heliacal Riſing of a given Star *S* will be known; namely the ſame as that wherein the Sun poſſeſſes *D* the point of the Ecliptic found. And after the like manner the Heliacal ſetting of it may be found.

PROPOSITION XXXIX.

THE Latitude of the Place and the Sun's Place being given, to find the beginning of the Morning and end of the Evening Twilight.

Let *ZEPN* [*Fig.* 17.] expreſs the Meridian; *E* ♎ the Equator; *HO* the Horizon, whoſe Poles are *Z* the Zenith and *N* the Nadir; *S* the Sun depreſſed below the Horizon: Thro' which let the Circle of Declination *PSR* and Vertical *ZSN* be ſuppoſed to be drawn. In the Triangle *ZPS* all the Sides are given; namely *PZ* the complement of the Height of the Pole; *PS* the diſtance of the Sun from the Pole, known by its Place; and the ſide *ZS*, made up of the Quadrant and depreſſion of the Sun, neceſſary for the beginning or ending of the Twilight: Therefore the Angle *ZPS* will be known, whoſe meaſure is the Arc *RE*, which converted into Time, gives the Hours ſought, to be reckoned after Noon to the end of the Evening Twilight; and their complement to 12 Hours, or the Arc *R* ♎, the Hours to be reckoned from Midnight to the beginning of the Morning Twilight.

PROPOSITION XI. LEMMA.

IF two great Circles of the Sphere cut another great Circle at equal Angles, the Arcs of all Circles rallel to this intercepted by the two cutting the third, e ſimilar to one another; and the Arcs of the inter- cants contain'd between two Parallels are equal.

Let two great Circles *HEO*, *HQR* [*Fig.* 18.] ut the great Circle *EQV* in *E* and *Q*, ſo as that e Angles *HEQ*, *HQV* may be equal. I ſay, e Arc *AP* of any Circle *PAL* parallel to *QV*, intercepted between *HEO* and *HQR* ſimilar to *EQ*; and the Arcs *PE*, *AQ*, of *EO*,*HQR* intercepted between the two Paral- s, are equal. Thro' the Poles of *EQV*, aw the great Circles *PB*, *AC*. And the Sine the arc *AC*,is to the ſine of the arc *AQ*, ſo is ne Sine *AQC* to Radius,ſo is Sine *PEB* to Ra- us ſo is) Sine *PEB* to Sine *PE*. But the arcs *B*, *AC*, are equal (by *Prop.* 10. *B.* 2. *Spher. eodoſ.*) and therefore their Sines; conſequent- their Arcs *AQ*, *PE* are equal. Beſides, in e Triangles *PEB*, *AQC*, in which the gles *B* and *C* (becauſe right) and *E* and *Q* (by ppoſition) are equal, and the ſides *PE*, *AQ* d *PB*, *AC* are already ſhewn to be equal, e ſides *EB*, *QC*, will alſo be equal, and con- quently the Arcs *EQ*, *BC* are equal. But *P*, *BC* (by *Prop.* 10. *B.* 2. *of the Sphæric, of eod.*) are ſimilar Arcs; therefore *QE* and *AP* e alſo ſimilar. And becauſe the Circle *PAL* taken at pleaſure, the Arcs of all the Circles rallel to *EQV*, comprehended by the Cir- s *HEO*, *HQR*, are ſimilar to *EQ*, and nſequently to one another. *Q. E. D.*

PROPOSITION XLI.

TO *find the Parallel the Sun is in, at the time of the Leaſt Twilight.*

It has been long ago obſerved, that the proportion of the increaſe and decreaſe of the Twilight is very much different from the increaſe and decreaſe of the Artificial Days. For the Days are upon the increaſe all the while the Sun returns from the Winter Tropic to the Summer one; but the Twilights do not. For theſe are leſſen'd after the Winter Solſtice to a certain limit before the Vernal Equinox, where they are ſmalleſt; and are aftewards increaſing while the Sun paſſes the Equator and moves towards the Summer Solſtice: And from thence they are upon the decreaſe to a certain limit after the Autumnal Equinox; and then again they increaſe to the middle of Winter. Let that Parallel then be propoſed to be ſought, that is deſcribed by the Sun in its Diurnal Motion, when the Twilight is ſmalleſt at a given Place.

Let $ZSNM$ [*Fig.* 19.] be the Meridian, SEM the Horizon, to which let a leſſer *Circle* CAR called that of the *Twilights*, be parallel, namely that wherein the Sun is at the beginning of the Morning and end of the Evening Twilight. Let AP be the Parallel ſought. Thro' the point A, where it interſects CR, ſuppoſe the great Circle $A\mathcal{Q}HO$ to be drawn meeting the Horizon HO in O and H, which cuts the Equator $E\mathcal{Q}$ in $\mathcal{Q}$, ſo as that the angle $A\mathcal{Q}E$ is equal to the angle PED or TEK. And therefore (by the preced. Lemma) the arc $E\mathcal{Q}$ is ſimilar to the arc PA; and the arc ap of any other Circle parallel to the Equator, intercepted between OES and OAL, will alſo be ſimilar to the ſame AP or $\mathcal{Q}E$; and each of

em will be the meaſure of the duration of the
wilight, while the Sun deſcribes the Parallel
A. But becauſe (by hypotheſis) it is the
aſt Twilight, none of theſe arcs, beſides P *A*,
ill reach to *C R*. (For while the Sun deſcribes
its Diurnal motion any other Parallel, the
eaſure of the duration of the Twilight is its
c *p b*, intercepted between the Horizon *SEM*
d the Circle of Twilights *C A R*; which ſince
is (by ſuppoſition) greater than the meaſure
A of the Leaſt Twilight, 'tis evident that *p a*
es not reach to *C R*, becauſe *p b* which is
eater, only reaches ſo far.) And therefore
e Circle of Twilights *C A R* touches the
eat Circle *O A H*: and becauſe part of it un-
r the Horizon, (namely *O A H*) is a ſemi-
cle, and *C A R* is parallel to the Horizon, 'tis
ident that *H L A* or *O A*, is a quarter of a
rcle. And therefore the Circle *N A T Z*, de-
ibed upon the Poles *O* and *H* thro' *A*, inter-
ſting the Equator at *K*, is a great Circle, and
ſſes thro' the Zenith and Nadir, and meets
e Horizon at right angles in *T*. Beſides, in
e Spherical Triangles *E T K*, *Q A K*, the an-
es *E T K*, *Q A K*, (by *Prop.* 15. *B.* 1. *Theod.*)
e right and conſequently equal, as alſo the
gles *T E K*, *A Q K*, (by conſtruction) are
ual, and the reſt *T K E*, *A K Q* are likewiſe
ual, (becauſe vertical:) From whence (by
herical Trigonom.) theſe Triangles are mu-
ally equilateral; that is, the arc *T K* is equal
K A, and *T E* to *Q A*: But (by the preced.
emma) *Q H* is equal to *E P*, therefore *T E* is
ual to *E P*. Beſides, in the Spherical Trian-
e *E T K* right angled at *T*, (by a known
op. in Spheric. Trigonom.) the Radius is to
e Co-Tangent of the angle *T E K* of inclina-
on of the Equator to the Horizon, as the Tan-

gent of the arc *TK* to the Sine of the arc *ET*: And by permut. the Radius is to the Tangent of half the Arc of the depreſſion of the Circle of Twilights below the Horizon, as the Tangent of the Pole's Elevation to the Sine of the arc *ET* or *EP*. Again, in the Triangle *EDP*, (making *DP* the Circle of Declination) right angled at *D*, the Sine of the arc *EP* is to the Sine of the Arc *DP*, as the radius to the Sine of the Angle *DEP*; that is, to the Co-ſine of the Elevation of the Pole: But in all arcs, the Tangent is to the Sine as the Radius to the Co-ſine; wherefore the Sine of the Arc *EP* is to the Sine of the Arc *DP* as the Tangent of the Height of the Pole to the Sine of it. And by inverting and permutation, the Tangent of the Height of the Pole is to the Sine of the arc *EP*, as the Sine of the ſaid Height is to the Sine of the arc *DP*. But (by what has been ſhewn before) in the caſe of the Leaſt Twilight, as the Radius to the Tangent of half the arc of the depreſſion of the Circle of Twilights, ſo is the Tangent of the Height of the Pole to the Sine of the arc *EP*; and therefore the Radius is to the Tangent of half the arc of the depreſſion of the Circle of Twilights, as the Sine of the Height of the Pole to the Sine of the arc *PD*; namely of the declination of the Parallel that the Sun deſcribes when the Twilight is Leaſt. *Q. E. I.*

SCHOLIUM.

If the depreſſion of the Circle of Twilights below the Horizon be 18^d (as it is commonly determined,) the Radius will be to the Tangent of 9^d as the Sine of the Latitude to the Sine of Declination of the Sun towards the depreſſed Pole at the time of the Leaſt Twilight. But the

Declination of the Parallel and Obliquity of the Ecliptic being given, the two points of the Ecliptic will be found (by *Prop.* 20.) and (by the Theory of the Sun) the times of the Year, when he Sun is in the given Places; that is, when he Twilight is Leaſt. Tho' at *London* the Leaſt Twilight is when the Sun declines from the Equator towards the South 6 d 7′, and in ♎ or ♓ 17d 30′, which happens in the preſent Age, about 22d of *Febr.* and 27th of *September* f the *Julian* Year. This was noted by *Blan-anus* in his *Sphere*, who ſays that the ſhorteſt Twilights happen about the middle of *Libra* and he middle of *Piſces*. *Peter Nonius* has propoſed his Problem and ſolved it (yet by two Propor-ions) in a *Book* concerning the *Twilights*, pub-ſhed in the Year 1541; who is followed by *Ambroſius Rhodius* and *Clavius*.

PROPOSITION XLII.

IN a Plane given by poſition, on which a Sun-Dial is to be drawn, to determine the Meridian Line, Angle of the Style, and the Subſtyle line.

Every Plane upon which a Sun-Dial is drawn, ſuppoſed to paſs thro' the Center of the Sphere, becauſe in reſpect of the diſtance of the Sun, s diſtance from the Center is inſenſible,) and onſequently its interſection with the ſurface f the Sphere is a great Circle.

Let *IRSMCT* [*Fig.* 20.] be ſuch a Plane aſſing thro' *T*, the Center of the World which roduced to the ſurface, forms the Circle *IRS MC*. Let *HIO* be the Horizon of the Place, *HZO* the Meridian (in which is the Pole *P*, and he Zenith *Z*) interſecting the former Circle in *M*. The poſition of the Plane *IRSC* being iven, its *Declination* is given, meaſured by the Angle *ITH*, or Arc *IH*, namely that whereby the Horizontal Line *IC* of the Plane *IRSC* declines from *HO* the Meridian line drawn on the Horizontal Plane, by *Prop.* 16. *ZR* is alſo given, namely the arc of the Vertical Circle perpendicular to *IRSC*, intercepted between the Zenith and the ſaid Circle, the meaſure of the *Inclination* or *Reclination* of the ſaid Plane. In the Spherical Triangle *MZR* rightangled at *R*, there is given *ZR*, and the the angle *RZM*, equal to the complement of the angle *ITH* to a right one: (For the Vertical Circle, *ZR* perpendicular to the Circle *IRSC*, declines from the prime Vertical (drawn thro' the Equinoctial Eaſt and Weſt) as much as *IT* does from *HT*; and conſequently the complement of the one, namely *RZM*, is equal to the complement of theother, namely *ITH*.) Therefore the ſide *RM* will be found, or the Angle (which it meaſures) in the plane of the Dial *RTM*. But *TR* is given by poſition; (for it is ſuccedaneous to a perpendicular on the ſame Plane, or a right line perpendicular to the Horizontal line *ITC*;) wherefore the Meridian line *TM* ſought in the Plane of the Dial, or the common interſection of the plane of the Meridian with the plane of the Dial, will be given. In the ſame Triangle *MZR*, are alſo found the ſide *ZM* and Angle *RMZ*, which will be of uſe by and by. Again, thro' the Pole of the World *P*, draw an arc of a great Circle *PS*, perpendicular to *IRSC*: And in the Triangle *RSM* rightangled at *S*, there is given the Angle *PMS* found juſt now, and the ſide *PM*, the ſum or difference of the Arc *IM* found before, and of the diſtance of the Pole from the Zenith, namely *ZP*: Conſequently there will be known both the arc *PS*, or the Angle, which it meaſures, *PTS*, the inclination of the Axis

T to the Plane *IRSC*, that is, the Angle of he Style ſought; and the ſide *MS*, or the ectilineal Angle *STM*, which it meaſures n the plane of the Dial, contained between M the Meridian determined before, and the ubſtyle *TS*, namely that upon which the plane f the Gnomon *PTS* ſtands, whoſe other edge s the Axis of the World *PT* caſting the ſhaow, and is perpendicular to the plane of the ial. *Q. E. F.*

PROPOSITION XLIII.

TO find the Angle, that the line of a given Hour makes with the Meridian in the plane of the ial; and thence to deſcribe a Sun-Dial upon a gien plane.

The ſame things remaining as before, let *BPb* *Fig.* 21.] be the Hour Circle, the interſection b of whoſe Plane with the Plane *IMC* gien by poſition is ſought. The Hour of this ircle being given, the Angle *HPB* or *MPb* given, contained by it and the Meridian. herefore in the Spherical Triangle *PMb*, there re given (by preced. *Prop.*) the ſide *PM*, nd the angle *PMb*, as alſo the angle *MPb*: herefore the arc *Mb* is found; and the rectiineal angle, which it meaſures, *MTb*, which he Hour line *Tb* makes with the Meridian *TM*. herefore the right line *TM* being given by *Prop.* preced.) the Hour line ſought *Tb*, in he plane of the Dial given, will be determined. *Q. E. F.*

And after this manner a Sun-Dial may be ade, by drawing a Meridian and Subſtyle om a point aſſumed at pleaſure for a Center, by preced. *Prop.*) and erecting upon the Subtyle a Style in an Angle found alſo by the reced. and drawing the other Hour lines in

Angles with the Meridian *TM*, determined by this Propoſition, and marking them with their proper numbers. Thus *Tb* is the eleven a clock Hour line after Midnight, if *PB* be a Circle to the Eaſt of the Meridian *PZH*, making with it an Angle of 15^d; that is, if *Tb* be to the Weſt of the Meridian line *TM*, and the Angle *MPb* in the ſurface of the Sphere of 15^d. And after the like manner *TX*, ſituated on the ſame ſide, will be the ten a clock Hour line, if *MPX* be 30^d, and ſo on.

PROPOSITION XLIV.

TO draw a given Hour line upon a Sun-Dial, whoſe plane paſſes thro' the Poles; and ſo to deſcribe a Polar Dial.

Becauſe all the Hour lines interſect one another in the Axis of the World, and this Axis is always in a Polar plane; 'tis evident that all the Hour lines, in ſuch a plane, are to be found in that Right line: But that they may be diſtinguiſhed, and not confounded with the Subſtyle, the Dial is not to be conceived as deſcribed in the plane *IMC* itſelf, [*Fig.* 22.] paſſing thro' the Center (as it is in all other caſes,) but in another plane *imγ*, parallel and near to the former; ſo that the opake right line that caſts the ſhadow and coincides with the Axis of the World, is diſtant from the plane *imγ*, by the right line *PS*: And ſince the diſtance of the Sun is look'd upon as Infinite, any given Right line as *PS*, vaniſhes in reſpect of it, and the parallel Planes paſſing thro' *IMC*, *imγ*, are eſteem'd as very near one another; and the portions *Pm*, *Pb*, &c. of the Hour circles *HPm*, *BPb*, &c. croſſing one another in the Pole *P*, intercepted between the Pole and the Circle *imγ*, are to be looked upon as Right lines; and the

part of the Spherical ſurface which theſe
rtions take up (being infinitely ſmall in com-
riſon of the whole) is to be look'd upon as a
ne.

In the Polar plane then, having found (*by*
p. 42.) the Subſtyle, erect the Style of any
adth taken at pleaſure, as PS, whoſe other
ge plac'd in the Axis of the World is paral-
to the Subſtyle; for in this caſe there is no
gle of the Style. To find the Meridian line,
ſerve that in the rectilinear Triangle PSm,
ſides the right angle at S, there is given the
PS, namely the height of the Style; and
angle SmP equal to the angle MPZ,
nd at *Prop.* 42; hence you may find Sm
diſtance of the Meridian line from the Sub-
le in the right line Sm, which is perpendi-
ar to the Subſtyle, becauſe MP (parallel to
) is perpendicular to PT; and you may al-
find the ſide Pm: A right line therefore
wn thro' the point m, parallel to the Sub-
le (becauſe, as was ſaid before, the Hour
es and Subſtyle, which alſo is one of them,
this ſort of Planes are parallel) will be the
ridian line in the Sun-Dial propoſed.

If any other Hour line given is to be drawn,
oſe Circle for inſtance is BP; in the Tri-
gle Pbm there are given the angles Pmb,
d mPb, the one found before, and the other
ual to the vertical angle ZPB, given becauſe
Hour of the circle PB is given;) as alſo the
Pm determined before: Hence therefore
y mb be found, whoſe extremity m is given,
nſequently b alſo will be given. A right
e therefore drawn thro' it parallel to the Sub-
le, will be the Hour line of the circle PB.
d after the like manner may all the other
Hour

Hour lines be drawn, and a Sun-Dial made upon a Polar plane. *Q. E. F.*

There are other eaſy methods of making Sun-Dials; but we have taken the moſt obvious, and one that is the fitteſt for the illuſtration of the doctrine of the Firſt Motion: And after this manner 'tis eaſy to deſcribe upon a Sun-Dial any other ſuch furniture as may be ſhewn by the ſhaddow of a Gnomon.

SECTION VII.

Concerning the Parallax of the Stars.

PROPOSITION XLV.

TO deſcribe the nature of the Parallax, and to enumerate the various ſorts of it.

Thro' all this Book we have ſuppoſed the Phænomena, whoſe places we have been treating of, to be ſituated among the Fixed Stars; or (if they are placed between the center or the Earth and ſurface or Fix'd Stars, that they are referr'd to the Fix'd Stars by the help of right lines drawn from the center thro' the Phænomena; that is, that they are ſeen from the Center. And tho' the Earth in reſpect of the Sphere of the Fix'd Stars, is taken for a center, and puts on the nature of one, yet in reſpect of the diſtances of the other Celeſtial Bodies it does not happen ſo; but an intermediate Phænomenon, ſeen from the ſurface of the Earth, will have a place among the Fix'd Stars (or in an infinite diſtant ſurface of a concave Sphere concentric with the Earth,) different from that, which it ſeems to have when ſeen from the center; and this diverſity of ap-

pearance is very fitly call'd the *Parallax*: Thus
or inſtance, if LT [*Fig.* 23.] repreſent the
Earth, whoſe Center is C; L a Place on the
urface, whoſe Zenith is Z; ZSH the Sphere
f the Fix'd Stars; the Phænomenon P ſeen
om L will be placed among the Fix'd Stars at
, but from the Center C, at S its true place.
he difference between its true place S and
e apparent one σ, or the Arc $S\sigma$, is the Paral-
x of the Phænomenon P: And it may be ex-
reſſed by the Angle $SP\sigma$, or LPC; for ſince
T in reſpect of the circumference ZSH va-
ſhes, zPb alſo (to which LT has a ſenſible pro-
rtion) is as a Center in reſpect of ZSH; that
the Arc $S\sigma$ may be conſidered as deſcribed
on the Center P. From whence it is evident
at by reaſon of the Parallax, any Phænome-
n appears nearer to the Horizon than it is,
a Vertical Circle, and that ſo much the more
(other circumſtances being alike) it is leſs
vated; and that a Phænomenon ſeen from
ſurface in L, in the Vertex, as in z, is refer-
to the ſame place among the Fix'd Stars, as
had been ſeen from the Center C, or that
re is no Parallax in the Zenith.

By reaſon of the Parallax before deſcribed
hich is call'd either *the Parallax of Altitude*,
alſo ſimply the Parallax) the Place of a Phæ-
nenon is alter'd as to Longitude and Lati-
e: For let the true place of a Phænomenon
C, [*Fig.* 24.] the apparent γ, ſo that the Pa-
ax is $C\gamma$; if you imagine the Circles of La-
de CE, $\gamma\varepsilon$ drawn thro' C and γ, meeting
Ecliptic EL in E and ε, the apparent Place
he Ecliptic will be ε the true E; and there-
the change of Longitude made by reaſon
he Parallax, is $E\varepsilon$, which is alſo called the
llax of Longitude. And the true Latitude CE

is changed into the apparent one $\gamma \epsilon$, the difference of which γo (ſuppoſing $C o$ parallel to the Ecliptic) is the *Parallax of Latitude.* But if it happen that the Vertical, as *Z D N*, paſſing thro' the Phænomenon *D*, is alſo perpendicular to the Ecliptic (which comes to paſs when it paſſes thro' the higheſt point of the Ecliptic *A*, or Nonageſim degree from the Eaſt) then the Parallax affects only the Latitude. For by the Parallax the Place is only changed in the Vertical, and becauſe (in the preſent caſe) this coincides with the Circle of Latitude, the Place is only changed in this, that point of the Ecliptic continuing unchanged, to which it is referr'd by the ſaid circle of Latitude: and in like manner only the Longitude is ſometimes affected, the Latitude continuing unchang'd; namely when the Phænomenon is in the Ecliptic paſſing thro' the Zenith. From whence it is evident, that the Longitude of a Phænomenon ſituated to the Eaſt of the Nonageſim degree of the Ecliptic, is increaſed by the Parallax, becauſe it is removed more to the Eaſt upon this account; but that the Longitude of one ſituated to the Weſt from thence is leſſen'd: Wherefore beſides all other motion which the Phænomenon may have, it will ſeem to be moved by reaſon of the Parallax, becauſe change the Parallax, and its apparent place will be changed. And this motion, becauſe not real but apparent, is called by ſome *the Parallax of Motion.* Now, what is to be underſtood of a Phænomenon in reſpect of the Ecliptic and its Secondaries, is to be underſtood after the like manner concerning the ſame in regard of the Equator.

PROPOSITION XLVI.

THE diſtance of a Phænomenon from the Earth
is to the Semidiameter of the Earth, as the
ne of the apparent diſtance from the Vertex is to
Sine of the Parallax.

Let T be the center of the Earth; [*Fig.* 25.]
: Place on the Surface L, its Zenith Z, Ho-
on TH; and let the Phænomenon be in P.
w (from the known property of a rectilineal
iangle) TP is to TL as the Sine of the
gle TLP (or of its complement to two
ht ZLP) to the Sine of the angle TPL;
t is, the diſtance of the Phænomenon from
center of the Earth is to the ſemidiameter
he Earth, as the Sine of the apparent diſtance
n the Vertex to the Sine of the Parallax.
E. D.

onſequently the Parallax agreeing to a gi-
Altitude being given, the diſtance of that
nomenon from the Earth will alſo be gi-
; and ſo *vice verſa.*

PROPOSITION XLVII.

HE Sines of the apparent diſtances of a Phæno-
menon from the Vertex are as the Sines of the
llaxes.

or (by preced. *Prop.*) the Sine of the appar-
diſtance from the Vertex is to the Sine of
Parallax, as the diſtance of the Phænome-
from the center of the Earth is to the ſemi-
eter of the Earth; that is (in the ſame Phæ-
enon) in a conſtant Ratio. Wherefore, as
ine of the apparent diſtance from the Ver-
n the firſt Obſervation is to the Sine of the
rent diſtance from the Vertex in any other
rvation, ſo is the Sine of the Parallax in
rſt obſervation to the Sine of the Parallax
e ſecond Obſervation. *Q. E. D.*

PROPOSITION XLVIII.

THE *Parallaxes of two Phænomena at unequal diſtances from the Center of the Earth, but whoſe apparent diſtance from the Vertex is equal, are reciprocally as the diſtances from the Center of the Earth.*

Things being expreſſed as before, let the two Phænomena be *P* and *p* [*Fig.* 26.] in an equal apparent diſtance from the Vertex *Z*, meaſured by the Angle *ZLP*. 'Tis evident that the Sine of the Angle *LPT*, is to the Sine of the Angle *PpT*, or *LpT*, that is, that the Sine of the Parallax of the Phænomenon *P*, is to the Sine of the Parallax of the Phænomenon *p* as *Tp* to *TP*, that is, reciprocally as the diſtances of the Phænomena from the Center *T*.

PROPOSITION XLIX.

THE *ratio of the Sine of the Parallax of one Phænomenon to the Sine of the Parallax of another Phænomenon is compounded of the inverſe ratio of the diſtances from the Center of the Earth, and of the direct ratio of the Sines of the apparent diſtances from the Vertex.*

For when the diſtance from the Center of the Earth is given, the Sine of the Parallax is as the Sine of the apparent diſtance from the Vertex, (by *Prop.* 47;) and the apparant diſtance from the Vertex being given, the Sine of the Parallax is reciprocally as the diſtance from the Center (by preced.) And therefore when neither is given, the Sine of the Parallax is as the apparent diſtance from the Vertex directly, and the diſtance from the Center of the Earth reciprocally, conjunctly. *Q. E. D.*

PROPOSITION L.

O *find the Parallax of a Phænomenon that doe*
not change its Declination, paſſing between the
le and Vertex.

Let ZP [*Fig.* 27.] be the Meridian, and in it
the Zenith, P the Pole; the true Place V,
d L the apparent Place of the Phænomenon,
ioſe Parallel deſcribed by the firſt Motion is
A; the Vertical fartheſt off from the Meridi-
ZA, touching the Parallel VA, PA an Arc
fall perpendicularly upon it, from the Pole P.
the Spherical Triangle ZAP, right angled
A, by the Side ZP, the Complement of the
ight of the Pole, and the Angle PZA, or
imuth known by Obſervation being given,
u will find the Side PA, or PV equal to it,
true diſtance of the Parallel from the Pole
the ſum or difference of this and ZP is ZV
true diſtance (or that ſeen from the Center
the Earth) from the Vertex Z: And the dif-
ence between this and the apparent one ZL,
nely VL, is the Parallax of the Phænome-
agreeing to the Altitude in V. *Q. E. I.*

PROPOSITION LI. LEMMA.

HE *ſum or difference of any two Arcs, together*
with the Ratio of their Sines, to find the Arcs
nſelves.

irſt, if the ſum of the Arcs be given, the
blem is all one with that common Trigo-
netrical one, where two Sides AB, AC,
. 28.] of a rectilineal Triangle being given,
ether with the Angle included BAC, the
er two Angles B and C, are required: For
en the Angle BAC is given, the Angle BAD
he ſum of the Angles B and C themſelves;
the ratio of the Sides AC, AB, is the ſame

with that of the Sines of the Angles *B* and *C*. This Problem therefore will be ſolv'd after the ſame manner as that; for 'tis known that the ſum of the Sides *AB*, *AC* is to their difference, that is, the ſum of the terms of the given ratio is to their difference, as the Tangent of half the ſum of the Angles *B* and *C*, or of the Arcs that meaſure them, to the Tangent of half the difference of them: But the ſum and difference of the terms of the given ratio are given, and the ſum of the Arcs being given, the Tangent of half the ſum is given; therefore the Tangent of half the difference will be known, and conſequently half the difference it ſelf; from hence, and half the ſum, will the Arcs themſelves be known. But if the difference of the Arcs be given, half the one and Tangent of the other will be given, and conſequently (from the former Proportion) the Tangent of half the Sum of them, and the half Sum it ſelf will be found; from whence the Arcs themſelves will become known, after the like manner as before. *Q. E. F.*

POPOSITION LII.

Both the Meridian Altitudes of a Phænomenon being given, which does neither change its declination nor ſet, to find its Parallax.

Let *ZH* be a quarter of the Meridian [*Fig.29.*] reaching from the Zenith to the Horizon, in which the Pole is *P*; the real place of the Phænomenon above the Pole *A*, the apparent α; its true place below the Pole *B*, its apparent β. If the Phænomenon paſs between the Pole and the Vertex, the difference between the obſerved diſtances from the Pole, namely $P\beta - P\alpha$, is the ſum of the Parallaxes in α and β. For a Circle deſcribed upon the Pole *P* thro' *A*, paſſes alſo thro' *B*, becauſe the true declination of the

Phænomenon is always the ſame. If farther
hro' α you deſcribe upon the ſame Pole the Cir-
le αC, then will $PC = P\alpha$; and therefore
$P\beta - P\alpha = (P\beta - PC = C\beta =)$ to the ſum of
he Parallaxes in α and β. But the ſum of the
Parallaxes in α and β being given, and the ratio
f their Sines, (being the ſame, by *Prop.* 47.
ith the ratio of the Sines of the apparent di-
ances from the Vertex,) the Parallaxes them-
elves ſought, will alſo be found by the preced.
Lemma.

If the Phænomenon paſs beyond the Vertex
n reſpect of the Pole, then the difference be-
veen $P\beta$, $P\alpha$ [*Fig.* 30.] the diſtances obſerved
om the Pole is equal to the difference of the
arallaxes. For ſince $B\beta$ is the Parallax in the
pparent place β, and $A\alpha$ or (the ſame prepa-
tion being made uſe of as before) BC in the
ace α, $C\beta$ (that is, $P\beta - P\alpha$) is the difference
the Parallaxes in the apparent Places β and α.
ıt the difference of the two Arcs being given,
gether with the ratio of the Sines (namely
e ſame with the ratio of the Sines of the ap-
rent diſtances $Z\alpha$, $Z\beta$ from the Vertex,) the
cs themſelves or the Parallaxes ſought will be
und.

But if the Phænomenon paſs thro' the Ver-
k it ſelf; that is, if the points Z, A and α
incide, alſo B and C, then $B\beta$ or $C\beta$ is the
rallax it ſelf of the apparent Place β.

PROPOSITION LIII.

*BOth the Altitudes of a Phænomenon, that does not change its declination, being given, and ob-
ved in the ſame Vertical Circle, to find its Parallax.*

The reſt being expreſſed as before, Let
AB [*Fig.* 31.] be the Vertical Circle, in which
e Phænomenon has been twice obſerved,

namely in α and β, but whoſe true Places are A and B, and AEB the Parallel of the Phænomenon deſcribed by the firſt Motion. Upon the ſame Pole P, thro' α, deſcribe another Circle αC, and the portions $A\alpha$, BC of the Verticals intercepted between them (by *Prop.* 13. *B.* 2. *Sphær. of Theodoſius*) are equal; and therefore $C\beta$ is either the ſum or difference of the Parallaxes $A\alpha$, $B\beta$. And to find it let the great Circles $P\alpha$, PC, be imagined to be drawn. In the Triangle $Z\alpha P$, ZP the complement of the Height of the Pole, $Z\alpha$ the obſerved diſtance of the Phænomenon in α from the Vertex Z, and the Angle contained αZP, from the Azimuth, known by obſervation, being given, you will find αP, to which the arc CP is equal. Again in the Triangle PZC, two Sides PZ, PC being given, together with the Angle PZC, ZC will alſo be known, whoſe defect or what it wants of the obſerved Arc $Z\beta$, is $C\beta$ the ſum of the Parallaxes in the places α and β, when PZ exceeds $P\alpha$; or their difference, when PZ is leſs than $P\alpha$. But in both caſes, when over and above, the ratio of the Sines of the ſaid Parallaxes is given, (namely the ſame, by *Prop.* 47. with the ratio of the Sines of the apparent diſtances from the Vertex,) both the Parallaxes agreeing to the apparent places α and β will be found.

PROPOSITION LIV.

TWO Altitudes of the ſame Phænomenon obſerved at the ſame moment of time in the common Azimuth of two given Places upon the Earth, to find the Parallax of the Phænomenon.

Let the two Places on the Earth be A and B, [*Fig.* 32.] whoſe Vertices are Z and X, the common Azimuth ZDX; and let the true Place of the

ie Phænomenon be D, δ the place obſerved
: A, d its place ſeen at B : And firſt let theſe
oints be between the Vertices Z and X. The
m of the apparent diſtances from the Vertex,
amely of the arcs $Z\delta$ and Xd,) known by
bſervation, exceeds ZX the diſtance of the
ertices (found by *Prop.* 24) by an exceſs equal
$d\delta$, the ſum of the Parallaxes. But if the
oints D, d, δ, be without the arc ZX, the dif-
rence of the apparent diſtances from the Ver-
x (namely $Z\delta$ leſſen'd by Xd,) exceeds the
ſtance of the Vertices XZ by exceſs equal to
e difference δd of the Parallaxes ſought.

Beſides the ſum or difference of the Paral-
xes found after the manner deſcribed before,
ere is given the ratio of the Sines of the ſaid
rallaxes; namely the ſame (by *Prop.* 47.)
ith the ratio of the Sines of the arcs $Z\delta$, Xd
the apparent diſtances from the Vertex. From
hence (by *Prop.* 51.) in both caſes the Paral-
xes themſelves will be found. *Q. E. F.*

PROPOSITION LV.

*Iven two altitudes of a Phænomenon obſerved
at given Places on the Earth, the ſame moment
time, even without the common Azimuth, to find
Parallax of the Phænomenon.*

The Longitudes of the Places being given, the
me moment of time neceſſary for the making
e Obſervation in the two Places, is expreſſible
the number of Hours reckon'd from Noon
Midnight reſpectively, and their difference
equal to the difference of Longitudes turn'd
to time, the more Eaſtern Place reckoning
ore Hours; which is determined during the
bſervation in the preced. when the Phæno-
enon is found in the common Azimuth, un-
ſs both Places be in the Equator. Let XZ
[*Fig.*

[*Fig.* 33.] be the common Azimuth of the two Places upon the Earth, whoſe Vertices are X and Z; from which at the ſame moment of time (tho' expreſſed differently,) let the Phænomenon be obſerved, whoſe true place let be D; and its place ſeen from that Place whoſe Vertex is X be d, from the Place whoſe Vertex is Z be δ. In the Triangle XDZ, the angles X and Z being given (contained between the common Azimuth of the Places and the obſerved Azimuths,) and the Side XZ the diſtance of the Vertices (found by *Prop.* 24.) the Arcs XD, ZD may be found, and they ſubtracted from Xd, $Z\delta$ obſerved, leave Dd, $D\delta$ the Parallaxes ſought. *Q. E. F.*

PROPOSITION LVI.

TO find very exactly the Parallax of a Phænomenon, by a method not much different from the two foregoing, by the help of ſome neighbouring Fix'd Star.

Since the Phænomenon whoſe Parallax is ſought, is in the common Azimuth XDZ [*Fig.* 34.] of the two Places on the Earth pitched upon for this purpoſe, whoſe Vertices are X and Z; let it be obſerved with ſome neighbouring Fix'd Star; and let S be the Place of the Fix'd Star, d the Place of the Phænomenon obſerved at the place below X; conſequently in this Obſervation there are given the arc Xd the apparent diſtance of the Phænomenon from the Vertex, the arc Sd the diſtance between the Phænomenon and the Fix'd Star, and the angle XdS contained between it and the common Azimuth. At the ſame moment of time from the place below Z, where the apparent place of the Phænomenon is δ, let $Z\delta$, $S\delta$ and the angle $S\delta Z$ be obſerved: And tho' the arcs Xd, $Z\delta$ can't

't be obſerved exactly,(becauſe there is need of
uadrant to meaſure them;) yet they may do
our purpoſe : But if they could with perfect
urateneſs, the Parallax of the Phænomenon
uld (by *Prop.* 54.) immediately appear. Not-
hſtanding the arcs dS, δS and the angles
d and δ, may be obſerved very accurately,
projecting the image of the Phænomenon
neighbouring Fix'd Star S, by the help of
culums or Lens upon a Plane, upon which
ght line is drawn repreſenting the common
imuth XZ; and being in the place of its
ge,(if it ſhould project any;) as is evident to
one acquainted withOptics andAſtronomi-
Obſervations. Therefore in the Triangle
δ, the ſides $S\delta$, Sd with the angles ſub-
ded by them d and δ being given, the Baſe
will be found, which is the ſum of the Pa-
laxes; from whence and the ratio of the
es of the Segments δD, dD, (D being ſup-
ed to be the true Place of the Phænomenon,)
nely the ſame with the ratio of the Sines $Z\delta$,
of the apparent diſtance from the Vertex
Prop. 47,)the Segments themſelves Dd, $D\delta$ the
rallaxes ſought, will (by *Prop.* 51.) become
own. Much after the ſame manner the diffe-
ces of the Parallaxes may be found, and from
nce the Parallaxes themſelves (by the ſaid
mma,) if D, d, δ, be without XZ, but yet in
continuation of it. If the true Place D
ig. 35.] of the Phænomenon be without the
mmon Azimuth XZ of the Places, the
ne things being expreſſed the ſame way
before, thro' the Vertices X and Z and the
pective apparent Places, let the Vertical Cir-
s Xd, $Z\delta$ be drawn, whoſe interſection is
true Place D of the Phænomenon. Draw
o thro' S, the place of the Fix'd Star, the
Ver-

Verticals XS, ZS, and join the points S, δ, d by the great Circles Sd, $S\delta$, $d\delta$. Let the Obſerver below X, obſerve accurately (beſides the arc Xd) the angles XdS, XSd and the Side SD, either by the method inſinuated above, or any other more exact; in like manner let the Obſerver below Z (beſide $Z\delta$) do to the Angles $Z\delta S$, $ZS\delta$ and the Arc $S\delta$. Beſides, the habitations being given (by *Prop.* 24.) ZX is given, and from the ſituation of this, by the Obſervation of the Fix'd Star S, the angles ZXS, XZS, are had; from whence the Angle XSZ will be known, which taken from the ſum of XSd, $ZS\delta$, will leave $dS\delta$, from which and the ſides Sd, $S\delta$, being known, the angles $Sd\delta$, $S\delta d$ and the ſide $d\delta$ are found. But SdX, $S\delta Z$, were known by Obſervation; conſequently the others $Dd\delta$, $D\delta d$ are known, by which and the ſide $d\delta$ known in the Triangle $Dd\delta$, the ſides Dd, $D\delta$, the Parallaxes ſought, are found.

PROPOSITION LVII.

TWO *Obſervations of a fix'd Phænomenon, that is, one that has only a diurnal motion, being given, to find its Parallax.*

Let $ECLI$ [*Fig.* 36.] repreſent the Ecliptic, B the true place of the Phænomenon, intirely fix'd in regard of the Ecliptic. And in the one Obſervation let ZBL be a Vertical Circle, in which Z is the Zenith, and β the apparent place of the Phænomenon; in the other, let XBL be a Vertical, X the Zenith, (for the Vertical point may be conſidered as moved in regard of the Ecliptic unmoved, as well as the Ecliptic in reſpect of the Vertex unmoved, and indeed more properly;) and b the apparent place of the Phænomenon. From the firſt Obſervation (by the Problems of the Firſt motion conſtructed before,) you may determine the point *L* of the Ecliptic, where the Vertical paſſing thro' *B* meets the Ecliptic; and the arc *Z L*, namely a portion of a vertical Circle, intercepted between the Zenith and the Ecliptic; and the angle *Z L E* contained between the ſaid Vertical and the Ecliptic. And in like manner in the ſecond Obſervation of the Phænomenon in *β*, the point of the Ecliptic *C*, the arc *X C*, and the angle *X C L* will be known. Therefore in the Triangle *B C L*, the angles *B C L*, *B L C* being given, and the ſide *C L* that lies between them (namely the diſtance of the known points in the Ecliptic,) the ſides *B L*, *B C* will be known, which taken from the arcs found before *Z L*, *X C*, will leave *Z B*, *X B*. And theſe again taken from *Z β*, *X b*, the known apparent diſtances from the Vertex, leave *B β*, *B b* the Parallaxes of the Phænomenon agreeing to the moments of the obſervations.

PROPOSITION LVIII. LEMMA.

LET *there be two rectilineal Triangles*, ABC, DEF [Fig. 37.] *having the Angles* ABC, DEF *given; and let the ratios of* AB *to* DE, AC *to* DF, *and* BC *to* EF, *be given: To find both Triangles from the ſide* AB *given.*

Make the angle *G B C* equal to *D E F*, and take *B G* ſuch, as that the ratio of *B A* to *BG* may be equal to the ratio compounded of the given ratios of *B A* to *E D*, and *E F* to *BC*, which is done by taking *B G* equal to a fourth proportional to *E F*, *B C* and *E D*. Then in *B C* given by poſition, chuſe ſuch a point *C*, as that the ratio of the lines connected *A C*, *G C* is equal to that compounded of the given ratios of *A C* to *D F*, and *E F* to *B C*, which may be done as before. Then let *B G* be to *B H* in the ratio of *B C* to *E F*, and let *H I* be parallel to *G C*: I ſay the Triangles *A B C*, *B H I*, are thoſe ſought.

'Tis manifeſt from the Conſtruction, that the ratio of *B C* to *B I*, is equal to the ratio of *B C* to *E F*; and that the Angles *A B C*, *H B I*, are equal to the given ones. Beſides, the ratio of *A B* to *B G* is compounded (by conſtruction) of the ratios of *A B* to *E D* and *E F* to *B C*: But the ratio of *A B* to *B G*, is compounded of the ratio of *A B* to *B H*, and of the ratio of *B H* to *BG*, or *E F* to *B C*; and therefore *A B* has the ſame ratio to *B H* as it has to *E D*. Again, the ratio of *A C* to *C G* (by conſtruction) is compounded of the ratio of *A C* to *D F*, and of the ratio of *E F* to *B C*, or *H I* to *G C*: But the ratio of *A C* to *C G* is compounded alſo of the ratio of *A C* to *H I* and *H I* to *G C*; and therefore *A C* has the ſame ratio to *H I* as it has to *D F*. Therefore the Triangles *A B C*, *B H I*, have the conditions required.

PROPOSITION LIX. LEMMA.

THE *ratio of the Sines of the Arcs* AB, CD, [Fig. 38.] *belonging to a given Circle, and the ratio of the Sines of the Arcs taken away* AE, CF, *and the arcs remaining* EB, FD *being given; to find the whole arcs and thoſe taken away*, AB, CD, AE, CF.

In Circles equal to the given ones, let there be ſuppoſed to be taken the arcs *G L*, *L H*, *I M*, *M K* reſpectively the doubles of *A E*, *E B*, *C F*, *F D*: Wherefore *G L H*, *I M K* will be reſpectively the doubles of *A B*, *C D*. Compleat the rectilineal Triangles *L G H*, *M I K*, in which are given the ratio of *G L* to *I M*, the ſame with the given ratio of the Sine of the arc *A E*,

to

the Sine of the arc *C F*; the ratio of *L H* to *K*, the ſame with the ratio of the Sines of the cs given *E B*, *F B*; the ratio of *G H* to *I K*, hich is the ſame with the given ratio of the nes of the arcs *A B*, *C D*. There are alſo gi- n the angles *L G H*, *M I K* inſiſting upon the own arcs, and therefore (by the prec. *Prop.*) e angles *L H G*, *M K I* will be known, and om thence the arcs *G L*, *I M*, and their lves *A E*, *C F*: But the arcs themſelves *E B*, *D* are given; therefore the wholes *A B*, *C D* e alſo known.

And after the like manner, the ratio of e Sine of the whole arc to the Sine of e arc taken away, and the remaining arc elf being given, the whole arc and the arc ken away may be found. For ſuppoſe *GLH* e double of the whole arc, and *G L* of the c taken away; from whence you have *L H* uble the remaining arc. Connect *G H*, *L*, *L H*, and in the Triangle *L G H*, there given the ratio of the ſides *G L*, *G H*; the me with the given ratio of the Sines of the hole arc and that taken away: The angle *G H* is alſo given, inſiſting upon the gi- n arc *L H*. And therefore (by 41. *of the ita of Euclid*,) the angles *G L H*, *G H L*, e given; and therefore the arcs on which ey ſtand, *viz.* the complement of the arc *L H* to the whole circumference, and the c *G L*; and conſequently *G L H* and *G L* emſelves, and their halves, namely the arcs ught.

PRO-

PROPOSITION LX.

TWO Altitudes of a Phænomenon not altering its Declination, and the correſponding Azimuths being given, to find its Parallax.

In two obſervations of a Fix'd Star five things may be given; namely the two Altitudes, two Azimuths, and the Time between the obſervations; from any four of which the Parallax may be found, from whence ariſe theſe three following Problems.

For the ſolution of the firſt, let P [*Fig.* 39.] repreſent the Pole; *L* the true place of the Phænomenon; and in one of the obſervations let *L M* be the Vertical Circle, and *M* the place of the Phænomenon obſerved in it, and therefore *L M* the Parallax; in the other let *L N* be the Vertical Circle paſſing thro' *L*, and *L N* the Parallax of this obſervation. Let the great Circles *P M*, *P N*, *N M* be ſuppos'd to be drawn. From the Altitudes and Azimuths obſerved, and the Height of the Pole, the Arcs *P M*, *P N* the apparent diſtances from the *P*, and the Angles *P M L*,*P N L* contained between the Circles of Declination and the Verticals are known: There is alſo given the ratio of the Sine of the Parallax *L M* to the Sine of the Parallax *L N*; namely, the ſame (by *Prop.* 47.) with the ratio of the Sines of the diſtances obſerved from the Vertex. There are therefore given the ratio of the Sines of the whole Angles *P M N*, *P N M*, namely the ſame with the ratio of the Sines of the known Arcs *P N*, *P M*; alſo the ratio of the Sines of the Angles *L M N*, *L N M* taken away, the ſame with the ratio found before of the arcs *L N*, *L M*; as alſo the remaining Angles *P M L* *P M L* found before by Calculation. Therefore the Angles themſelves *P M N*, *P N M*, *L M N*, *L N M*.

NM will be known, (by the preced. Lemma.)
gain, in the Triangle *MPN*, the Sides *PM*,
N being given, with the Angles ſubtended by
em, the Baſe *MN* is found. And laſtly, in
Triangle *LMN*, the Baſe *MN*, and the
gles adjacent *LMN*, *LNM* being given,
Sides *LM*, *LN* the Parallaxes ſought will
found.

f one of the Obſervations was made when
Phænomenon was in the Meridian, then
of the Triangles *PML*, or *PNL* is changed
an Arc, and the Calculation becomes much
pler.

But if the Phænomenon *L* change its Decli-
ion, and that equably, (which will be done
ne revolution of the Earth, or thereabouts,)
may ſtill find its Parallax, by correcting the
es of the Phænomenon, by a proportional
of the Declination thus: When the Phæno-
on firſt, next after the firſt Obſervation, ar-
s at the Azimuth *LM*, obſerve its apparent
tude, which will be different from that ob-
ed the Day before, otherwiſe it would re-
the ſame declination. Let the difference
A. Call the diſtance of time between the
firſt Obſervations made in *M* and *N*, *B*;
the time between the two Obſervations of
Phænomenon in the Azimuth *LM*, *C*. And
e difference of the aforeſaid Altitudes *A*,
n exceſs, to the apparent Altitude in *M*
$\frac{BA}{C}$; but if this difference be a defect, ſub-
it from the ſame. This ſum or difference
be the corrected apparent Altitude, which
n the preceding Calculation (inſtead of the
er Altitude in *LM*) gives the Parallaxes
ht of a Phænomenon that does not change
eclination.

PROPOSITION LXI.

Given two Altitudes of a Phænomenon, with the Azimuth of one of them, and the Time between the two obſervations, to find the Parallax of the Phænomenon.

Let P be the Pole, (*Fig.* 40.) *E* the true place of the Phænomenon, *F* the apparent place when *X* is the Zenith, and *H* when *Z* is the Zenith. There are given the two apparent diſtances from the Vertex, namely *XF*, *ZH*, and the Angle *PXF*, ſince one Azimuth is given, and the Azimuth *XPZ* is known by the time. In the Iſoſceles Triangle *PXZ*, the equal Sides *PX*, *PZ*, and the Angle *XPZ* being given, *PXZ*, *XZ* are found: And ſince *PXF* is given, *ZXF* is alſo found. Therefore in the Triangle *XZF*, *XZ*, *XF*, *ZXF* are given; and therefore *ZF*, *ZFX* become known. Therefore the ratio of the Sine of the angle *ZFH*, to the Sine of the angle *ZHF* is given; namely the ſame with that of the Sine of the arc *ZH*, to the Sine of the arc *ZF*; and the ratio of the Sine of the angle *ZHF* to the Sine of the angle *EFH*, or the Sine of the arc *EF* to the Sine of the arc *EH*, or the Sine of the arc *XF* to the Sine of the arc *ZH*. Therefore the ratio of the Sine of the angle *ZFH* to the Sine of the Angle *EFH* is given; and the difference of thoſe Angles *ZFE* is alſo given; and therefore (by Prop. 59.) the angles themſelves *ZFH*, *EFH* become known; and conſequently *EHF* alſo. And laſtly in the Triangle *EFH*, *HF*, *FEH*, *EHF* being given, the Parallaxes namely *EF*, *EH* become known. *Q. E. F.*

Pro-

PROPOSITION LXII.

GIven two Azimuths of a Phænomenon, together with the Altitude of one of them, and the ime between the two obſervations of the Azimuth, find the Parallax of the Phænomenon.

The ſame things being ſuppoſed in the ſame
iagram, after the ſame manner may PXZ
ual to PZX, and XZ be found. The two
zimuths being given, the Angles PXE, PZE
e given, and therefore the angles XZE, ZXE
come known; by which, together with XZ,
y XE be found. But becauſe the Altitude is
en, its complement XF is alſo given, and
refore EF the Parallax ſought will become
own. *Q. E. F.*

If, in the caſe of either of theſe Problems,
Phænomenon has any motion after an in-
revolution of the Earth, when the Zenith
eturns to the ſame point of the Heavens, the
enomenon will be obſerved, not in F (as be-
e) but in ſome other place, for inſtance, G;
FG, XFG become known by obſervation.
ke therefore, as a whole Circle is to FG, ſo
e Angle XPZ to FL, and draw a great Cir-
XL cutting ZH in M. In the Triangle
L, XF, FL, XFL being given, XL, FXL
alſo given; and therefore ZXL is alſo given.
ly, taking the points L, M inſtead of F
E, HM the Parallax of the Phænomenon
be found as before. *Q. E. F.*

here are other ſorts of Obſervations, where-
he Parallax of a Phænomenon may be Geo-
ically determined; as its ingreſs into the
ow of a Planet; and a Conjunction of Bo-
ſeen between the Phænomena. But ſince
things do not belong to the Parallax con-
ed in general, they more properly belong
nother place in the following Books.

PROPOSITION LXIII.

THE place of a Phænomenon in reſpect of the Ecliptic, and the Parallax of a given Altitude, as alſo the place of the Sun, and the hour of the Day being given, to find the Parallax of Longitude and Latitude of the Phænomenon.

Let *VH* be the Horizon, (*Fig.* 41.) *ECB* the Ecliptic, and *ZCV* a Vertical Circle paſſing thro' the Phænomenon. Let *A* be the true place of the Phænomenon in it, *a* the apparent; and therefore *Aa* the Parallax of Altitude. Thro' *A* and *a* draw the Circles of Latitude *AB ab*, meeting the Ecliptic in *B* and *b*. Now (by *Prop.* 36.) there are given the Angle *VCB*, and arcs *VA*, *VC*, and therefore their difference *AC*. But becauſe the Parallax of a certain given Altitude is given, the Parallax of the given Altitude *VA* will alſo be given (by *Prop.* 47.) namely the arc *Aa*. Conſequently the ſum or difference *Ca*, of theſe arcs *CA* and *Aa* are given. Therefore in the Triangle *aCb*, beſides the right Angle at *b*, there are given the Side *aC*, and Angle *aCb*; from whence the Sides *Cb*, *ab* will become known. But becauſe the place of the Phænomenon *A* in reſpect of the Ecliptic is known, and the point of the Ecliptic (by *Prop.* 36.) alſo known, the arcs *AB* and *CB* are known. Therefore the difference between *ab* and *AB*, namely the Parallax of Latitude becomes known; and the difference between *Cb* and *CB*, namely the Parallax of Longitude alſo. *Q.E.F.*

The converſe of this Problem is ſolved after the ſame manner; for if the Parallax of Longitude *Bb*, or the Parallax of Latitude, the difference between *ba* and *BA*, be given, *Cb* or *ab*, (from the Place of the Phænomenon in reſpect of the Ecliptic already given, and from thence

hence the arcs CB and BA,) will be given. And ſince beſides the Angle at C in the right ngled Triangle Cba being given, Ca will be iven, and conſequently Va, ſince VC (by *Prop.* 36.) already given: But VA is known; therefore he Parallax of Altitude Aa will not be unnown. If the Phænomenon, as the Sun, be n the Ecliptic, the caſe is much more ſimple.

But becauſe the Parallax of the Celeſtial Boies, even the neareſt, is a pretty ſmall Arc, herefore ſome conſtruct this Problem of findng the Parallax of Longitude or Latitude from he given Parallax of Altitude, more expeditiuſly thus: Thro' A imagine the line $A\beta$ to e drawn parallel to Bb. Therefore in the ſmall riangle $A\beta a$, which is looked upon as rectilieal, there being given, beſides the right angle β, the angle βAa equal to ACB, and one f the Sides Aa the Parallax of Altitude, the oer Sides $A\beta$ the Parallax of Longitude and a the Parallax of Latitude are found, by the re reſolution of a plane Triangle.

But if not the abſolute Parallax of the Phæomenon be given, but the relative in reſpect another more remote Phænomenon, that is, e exceſs of the Parallax of the nearer Phæomenon above the Parallax of the more reote; ſuch an exceſs in a vertical Circle being iven, the exceſs in regard of any other Circle ill alſo be given.

For the abſolute Parallax of a Phænomenon, the exceſs of the Parallax of the Phænomeon itſelf above the Parallax of the Circle of Sphere, which is nothing, becauſe it is ſupoſed infinitely diſtant.

SECTION VIII.

Concerning the Refraction of the Stars.

PROPOSITION LXIV.

ON the account of an Atmoſphere, which is denſer than the Æther, circumfuſed about the Earth, each Star appears more elevated above the Horizon in the ſame Vertical Circle, than it would appear were there no Atmoſphere at all.

Let *T* repreſent the Earth (in *Fig.* 42,) round which the Atmoſphere *AED* is diffuſed; *S* any Star; O the Spectator, placed on the Surface of the Earth. If there were no Atmoſphere at all, or if it were equal in denſity to the circumfuſed Æther, the Rays of Light would reach directly from *S* to *O*, nor would they ever be bent into a Curve line; becauſe in an homogeneous Medium. But if the Rays, after their paſſage thro' the Æther *SQ*, enter into a denſer Atmoſphere in *A*, they will be refracted towards the Right line *QAP*, perpendicular to the Surface of the Atmoſphere at *A*; as is evident from *Dioptrics*. But it is not probable that an Atmoſphere very conſiderably denſer than the Æther is immediately under it, and then continued all along to the Earth of the ſame denſity; or that all the Refraction of the Rays, that proceed from any Star, is perform'd in that ſingle Surface which ſeparates the Æther from the Atmoſphere, (as it is commonly believed,) the portions of the Ray from *S* to the outmoſt Surface of the Atmoſphere, and from thence to the Eye, continuing all the while right lines; as with us a Ray paſſing out of Air into Water is only

nly bent at the Surface of the latter, being rait both before and afterwards. The Atmo-here rather ſeems to extend itſelf to a very reat height, and to be rarer in its higher and ore remote parts, and denſer in the lower; n which account a Ray proceeding from a tar S, ſuffers ſome Refraction towards the per-endicular, in a diſtance from the Earth much reater than is commonly believed; and in its rogreſs (as at B, C &c.) it is refracted the me way, becauſe the medium of the Atmo-here, that it enters into, is more and more nſe, the nearer it comes to the Earth: And erefore a Ray after its entrance into the At-oſphere (by reaſon of the perpetual refracti-n at the Spherical ſuperficies concentric to the arth of a continually denſer medium) is curved all its parts, and concave towards T. But an ye plac'd at O ſees a Star in the laſt part of e Ray $ABCO$ from O produc'd, namely in σ; r this part only affects the Sight. But the rt OC of the Ray produced, or the Right ne touching the Curve $ABCO$ in O, namely, σ, is leſs inclin'd to the Horizon of the Spe-ator HO, than the Right line connecting S nd O; becauſe the line $ABCO$ is concave to-ards T: And therefore, to an Eye at O a Star ppears in σ, higher above the Horizon, by aſon of the circumfuſed Atmoſphere, than if ere were no Atmoſphere at all.

Beſides, the Curve that the Ray propagated blique thro' the Atmoſphere, puts on, is in e ſame Plane produced thro' the Star and the enter of the Earth. For in *Dioptrics* 'tis de-onſtrated that the former part AB of the Curve onſiſting, as it were, of Right lines) is in the lane that the Right lines AS, $A\mathcal{Q}$, (becauſe the Plane of Inflection;) that is, in the Plane produced and paſſing thro' the Star S and Center of the Earth T; for (by *Prop.* 5. *B.* 1. *Spher. Theodoſ.*) the Right line QAP produced, paſſes thro' the Center T; and its ſecond part BC is in like manner in the Plane paſſing thro' BA, and the perpendicular thro' B; that is, in the ſame Plane with the former: and ſo on in the other parts of the Ray $ABCO$. Therefore that intire Curve is in the ſame Plane produced, paſſing thro' the Star and the Center of the Earth; and therefore the Right line $O\sigma$ touching the Curve, (becauſe a ſmall particle of it produced in a Right line) is in the ſame Plane: But this Plane is vertical to the Obſerver O, (for it paſſes thro' the point O, and the Center of the Earth) therefore the Star, by reaſon of the Refraction made in the Atmoſphere, appears in the ſame Vertical, but higher above the Horizon. *Q. E. D.*

SCHOLIUM.

It appears, from the demonſtration of the preced. *Prop.* that the Refraction is the ſame, whether the Star be nearer the Earth, or farther off (for inſtance, in ſ or in S,) provided their apparent Altitude above the Horizon be the ſame: For in this caſe the Right line σO revolving above the Axe TO, reaches to the apparent place of ſ. Beſides, 'tis probable that the Atmoſphere ſpread about the Earth does not extend it ſelf to the Moon, much leſs to the more diſtant Stars; and therefore the ſtrait part ſA of the line Sſ$ABCO$, (before it begins to be bent into a Curve by the Atmoſphere) will graze upon the next Star ſ. And conſequently the Ray proceeding from it (namely ſA) is bent in the Atmoſphere into $ABCO$, after the like, nay, after the very ſame manner, as SA propagated from the more diſtant Star; ſince it is all one (as to the Refraction made in the Atmoſphere) whether the Ray begin from S or $ſ$. Therefore 'tis contrary to the nature of Light, hat the Rays of a nearer Star ſhould be refracted otherwiſe than (all other circumſtances emaining the ſame) that of a more diſtant one. Tis true that the Angle σOS is greater than he Angle $\sigma Oſ$, but the difference of them $ſOS$ s owing to the exceſs of the Parallax of the earer Star above the Parallax of the more diſtant Star, and not to the Refraction.

POPOSITION LXV.

THE lower Stars are more elevated by reaſon of the Refraction at the Atmoſphere, than the igher, other circumſtances remaining the ſame.

In *Dioptrics* 'tis demonſtrated, that in the ame Mediums, the more oblique the incidence f the Ray is, the greater the Angle between he refracted Ray and the incident Ray produed; that is, in the preſent caſe [*Fig.* 43.] that he part AB of the Ray $ABCO$, proceeding rom the lower Star S, contains a greater Angle vith SA produced, than the part FG of the Ray $FGKO$ proceeding from the higher R does vith RF produced; becauſe SAQ is a greater Angle than RFL, ſuppoſing that LF produced alls upon T. After the like manner BC will ontain a greater Angle with AB produced, han GK with FG produced, and ſo on; that s, the line $ABCO$ is more curved in all its parts han $FGKO$ in its correſponding parts, or ſuch s are equally diſtant from the Earth. And herefore a Right line touching the Curve $ABCO$ in O contains a greater Angle with the Right line OS, than a Right line at the ſame lace, touching $FGKO$ with the line OR. The lower Star therefore is more elevated by the Refraction than an higher. *Q. E. D.*

COROLLARY 1.

The Place of a Phænomenon when it is in the Zenith is not changed by the Refraction: For the Rays in this caſe falling perpendicularly upon all the Surfaces of the Mediums, of which the Atmoſphere conſiſts, are not refracted; but going directly to the Eye in O, make the Phænomenon appear in the ſame Right line as it would appear were the Atmoſphere away. And for the ſame reaſon the Eye being placed in the Center *T*, the Refraction made at the Atmoſphere would have no power to change the apparent place of the Star; for every point in the Heavens would be a Zenith to it.

COROLLARY 2.

An account may from hence be given, why the Sun or Moon appears of an Oval Figure near the Horizon. For ſince the upper Limb appear but a little more elevated than it ſhould be, and the lower a great deal, the latter by the Refraction, will appear to approach the former. And therefore the erect or vertical Diameter of the Luminary ſeems contracted or ſhortened, while the tranſverſe or horizontal one is not ſo, becauſe its extremities are equally elevated by the Refraction. For the ſame reaſon the diſtance of two Fix'd Stars by obſervation is ſenſibly leſs, (if it be meaſured by an Inſtrument,) when, both being in the ſame Vertical, one of them is nearer the Horizon, than when both of them, by reaſon of their greater Altitude, are free from all Refraction.

COROLLARY 3.

On this account it is alſo, that the Sun and the other Stars riſe ſooner above the Horizon, and ſet later, than they ought, according to their

heir places determined by the aforeſaid Mehods. For the Ray flowing from ſuch points of the Heaven as are but a little below the Hoizon, are ſo incurvated in the Atmoſphere, hat the Right lines touching them at their xtremities, where they enter the Eye, being roduced, are above the Horizon. For which eaſon the Moon has been ſeen to be eclipſed, ho' the Sun was above the Horizon; whereas ne of the Luminaries at that time being oppoite (by *Prop.* 18. *B.* 1.) to the other, and above he Horizon, the other muſt neceſſarily be beow it. 'Tis to the ſame Atmoſphere diffuſed ound the Earth, that its Inhabitants owe the un's ſhining to them longer than it would do ere there no Atmoſphere; and that when e Sun is kept from inlightening any given habitant, by reaſon of the Spherical Surface f the Earth, thick darkneſs does not immeditely cover him, but he enjoys a Twilight for conſiderable time, the Light being gradually tinguiſhed.

PROPOSITION LXVI.

TO define how much a Star appearing in a given Altitude is elevated by the Refraction; and to ake a Table of the Refraction of the Stars.

Chuſe a Star that has no ſenſible Parallax; amely ſome one of the Fix'd Stars that is evated much above the Horizon of the Obrver. Let its Place be determined (by *Prop.* 26.) obſervations made when it was elevated etty much, and therefore when it was out the Effects of a ſenſible Refraction. Let the ne be marked of its coming afterwards to the ven Altitude, either aſcending or deſcending, d find then by Calculation, the Altitude, for e time marked, of the Star (whoſe place is known,) and that will want of the obſerved Altitude, as much as the Star is elevated by the Refraction in that given apparent Altitude.

After this manner may a Table of the Refraction of all the Stars be beſt conſtructed, if the true Altitudes of the Fix'd Stars, agreeing to the times exactly reckoned by a Pendulum Clock, be compared with the Altitudes of the ſame Star, in the ſame apparent points of time: For the difference between the true and apparent Altitude is the Refraction agreeing to that Altitude. And ſince the Refraction of a Fix'd Star at the greateſt diſtance, and of any other Star at a leſs diſtance, is the ſame, in the ſame apparent Altitude (by *Schol. Prop.* 64.) 'tis evident that a Table of the Refraction of all the Stars is made. *Q. E. F.*

SCHOLIUM.

If the *Dioptric* denſity of the Atmoſphere were known in any given diſtance from the Earth, or the ratio of the Sine of the Angle of incidence to the Sine of the refracted Angle, in the paſſage of a Ray thro' its Surface concentric with the Earth, whoſe diſtance from the Center is known; then by Geometry you may find out the nature of the Line, into which the Ray is bent by reaſon of the continued Refraction of the Atmoſphere. Conſequently, the Angle being given by obſervation, by which a Star in a given Altitude is elevated more than it ſhould be on the account of the Refraction, the Angle may be defined, by which it is elevated in any other Altitude propoſed more than it ſhould be, the Atmoſphere continuing the ſame; that is, a Table of Refraction may be made for every degree of a Star's Elevation. But becauſe our ignorance of the Law of Refraction obſerved in the Atmoſphere hinders all

this

s, 'tis better to make the Table by Obſerva-
n, as in the preceding. But beſides, the di-
ſity of the Atmoſphere in different places,
l the change of it in the ſame place, on the
ount of which it happens that the ſame Ta-
is not fitted for all places, is an impediment
both methods of making a Table of Refra-
on. For in places near either Poles the Re-
ction is greater in the ſame diſtance from the
rizon, (as is well known from the obſerva-
n of ſome *Dutch Men* who Wintered in *Nova*
mbla;) and in the ſame place at different
es of the Year, 'tis certain, not only from
obſervation of the *Dutch*, but of other con-
erable Obſervers, who make the Winter Re-
ction greater than the Æquinoctial, and that
ain greater than the Summer one, and form
ire Tables of Refraction built upon this
ange. But the hourly change of the Re-
ction is demonſtrated by the Tops of Moun-
ns being ſometimes viſible at a given place,
d ſometimes inviſible, tho' there be no dark-
ſs or miſt between: To this purpoſe alſo do
e Experiments mention'd by *Hugens*, in his
eatiſe of Light, tend and ſerve; namely that
e lower part of a Tower or any other thing
at is immovable is ſeen in the Morning and
ening, but the upper at Noon thro' a Tele-
ope continuing fix'd all the while; and that
the ſame place, ſometimes the top of a more
mote Tower has been viſible, riſing above the
p of one nearer, and ſometimes it is inviſible.
ow all theſe things, beſides the hourly change
the Refraction of the Rays in the Atmo-
here, demonſtrate the Curvity of them, (from
eir nature, as was ſhewn above,) ſince they are
opagated obliquely from an higher to a lower
ace, and *vice verſa.*

It muſt be confeſſed that all this change of the Refraction is made within pretty narrow limits: For the beforeſaid obſervations of the change of the apparent Altitude of the Sun, Mountains and Towers are made only in a ſmall Elevation above the Horizon, nor do they reach much beyond three degrees above it. The viciſſitude and changeableneſs near the Earth, of Vapours ariſing from it, as alſo of Heat and Cold, by which they are rarified and condenſed, is ſo great, that nothing certain can be laid down about the denſity of the Medium. Beſides, the inclination of the Rays of Light is ſo very oblique at their incidence on the Atmoſphere, when the Phænomenon is not elevated above three or four degrees, that its place is changed very much by the Refraction. Nay, ſo denſe yet pellucid a Cloud as it were of Vapours lies ſometimes upon the Surface of the Earth, that to an Eye placed above without it, the viſible Objects below it, namely the very low ones, continue unſeen, becauſe the Rays do not penetrate the Surface of the upper and rarer Medium, but are reflected by it downwards; as it is evident from *Dioptrics*, that it ought to be: But ſuch things as are immediately above and without it, namely ſuch as are a little higher, beſides that they are ſeen by the direct Ray, appear alſo inverted by a reflection made at the Surface of the abovementioned Cloud, juſt as if it were an Horizontal Speculum. But theſe things neither belong to this place, nor directly to Aſtronomy. They ſhow in the mean while, that no credit is to be given to any Table of Refraction of the Stars below the third degree of Altitude; nor conſequently to Obſervations made in that

Altitude for determining the place of the Phænomenon.

If a Perſon has a mind to make a Table of the Refraction of the Fix'd Stars, not from Obſervation, but upon the Principles of *Dioptrics*, he may proceed the following way very ſafely. Let the Atmoſphere, which is extended 40 or 50 Miles high, be ſuppoſed to be divided by 8 or 10 parallel Superficies, into as many Mediums of different denſities; ſo as that the Medium which lies between the two next Superficies, may be of the ſame denſity, that at the ſaid Superficies may be changed inſtantly into a rarer above, and a denſer below: Not that it is really thus in nature; for no doubt the denſity of the Atmoſphere in its deſcent, is increaſed by the ſmalleſt degrees. Let theſe 8 or 10 Mediums be ſo tempered in regard of one another by trials, that in one, two or more Altitudes (for the more they are, the more accurate will the Table be) the total Refractions made at all theſe Mediums, may be the ſame with the Refractions that are found by the accurateſt Obſervations, to agree to thoſe reſpective Altitudes. In which caſe the Refraction made in any aſſumed Altitude at the ſaid Mediums, and known by Calculation, according to the Laws of *Dioptrics*, is very nearly equal to the Refraction in the Heavens, agreeing to the ſame Altitude. After this manner alſo will the Line be determined very nearly, into which the Ray is bent in its paſſage thro' the Atmoſphere. But in finding, by this method, the Refraction correſponding to an Elevation three or four degrees leſs, the lower, or two or three lower, ought to be ſuppoſed a little rarer; for they really are ſuch, in the vicinity of the Earth, which is heated.

SEC-

SECTION IX.

Of the Tables of the Firſt Motion, and of the Fix'd Stars, depending upon the Second Book.

THO' all the Particulars that belong to the Firſt Motion may be found, by the Methods demonſtrated in this Book, as occaſion requires; yet becauſe there are ſome things of more frequent uſe, *Tables* are made for them, that they might be more expeditiouſly taken out of them, which would otherwiſe require a new Calculation every time they are wanted. And beſides Tables taken from other diſciplines, as of Right lines inſcribed and circumſcribed in a Circle, of Logarithms, *&c.* Aſtronomical ones are alſo made; and ſuch of them as depend upon this Book are of two ſorts; namely ſuch as are the ſame to the Inhabitants of the whole Earth; which regard either the Firſt Motion, or the Fix'd Stars; or ſuch as only hold in ſome certain place of the Earth.

PROPOSITION LXVII.

TO deſcribe Tables common to all the Inhabitants of the Earth, relating to the Firſt Motion.

The Equator and Ecliptic are the ſame to all the Inhabitants of the Earth, and they are of conſiderable and frequent uſe in Aſtronomy: The former, becauſe it is the mean, and as it were the Standard of the Firſt or apparent Diurnal Motion of the Heavens; the latter, becauſe it is the Way and Path of the Sun, (who is the governor of the Days and Seaſons.) The mutual habitude and reſpect of theſe Circles to one another in all their points is diſpoſed into Tables: That is, to every degree of the Ecliptic, marked with its proper name, (that is, the number it bears in the Sign) are annexed to the correſponding Declination from the Equator, Right Aſcenſion, and Angle that the Ecliptic comprehends with the Meridian; as is ſhewn in *Prop.* 20. And therefore theſe three Tables are put together n one very commodiouſly, under the univerſal Title of *The Declination, and Right Aſcenſion of evey degree of the Ecliptic, and of the Angles of the Eliptic with the Meridian.* And becauſe in ſuch ables, either the ſame number recur, or the ame increaſed by a Quadrant, a Semicircle or ree Quadrants of a Circle; in the making of em there will be no need of carrying the Calulation beyond one Quadrant, ſince the reaining part of the Table flows naturally from e former, either by the change of the firſt Comn or by Addition only.

Beſides, ſince the intire apparent revolution the Celeſtial Equator makes a natural Day, d proportionally in the parts of this revolun, by reaſon of the equability of the diurnal otion of the Earth; they make a Table, out which the time correſponding to each part of revolution of the Equator, and on the conry, the part of the revolution correſponding any time, may be taken. And becauſe among iter Nations, where Aſtronomy is cultivated, natural Day is divided into 24 Hours, and the ur in 60 Minutes, every one of which again livided into 60 Seconds, and ſo on; the aforel Tables are accomodated to this diviſion of Time.

PROPOSITION LXVIII.

TO *deſcribe the Tables which relate to the Fix'd Stars, that are common to all the Inhabitants of the Earth.*

The Places of the Fix'd Stars, in regard of Longitude and Latitude, are diſpoſed of by the methods explain'd in *Prop.* 29. Such a Table (or Catalogue) as this, is the foundation of the whole Science of the Stars: And becauſe the places of the Planets are found, by knowing the places of the Fix'd Stars, Aſtronomers are very careful in noting the places of the Fix'd Stars, as are in the Zodiac; not only thoſe as are only ſeen by the naked Eye, but ſuch alſo as are ſeen by a ſmall Teleſcope.

Becauſe the Longitude of the Fix'd Stars continually increaſes, by reaſon of the regreſs of the Equinoctial Points, the *Table of this Regreſs* is annexed to the Catalogue of the Fix'd Stars; that is, 'tis ſhewn how much it is in a Year, any parts of a Year, or collection of Years, ſo that the Tabular Longitude of a Fix'd Star may be very eaſily corrected by this means. *Copernicus* and others following him, reckon the diſtance of the Fix'd Stars in the Ecliptic toward the Eaſt, from the preceding of the two in the Horn of Aries, as was ſaid before: By adding to this Arc (in any conſtant Fix'd Star) the diſtance of the Vernal Equinoctial point, perpetually augmented from the ſaid firſt Star of Aries, to be taken out of that other Table, you make the Longitude of the Fix'd Star from the Vernal Equinox.

Becauſe the Right Aſcenſion and Declination of ſome of the more conſiderable Fix'd Stars are of more frequent uſe; and to find them out by the Longitude and Latitude given (by Converſ. *Prop.* 27.) as often as there is occaſion of them, would

ould be very tedious; ſuch as have a mind to are others ſo much trouble, form to a given ne, *a Table of the Right Aſcenſion and Declination the more conſiderable Fix'd Stars*, adding the dif- rence of each for a certain number of Years, which they ſuppoſe their change, as to ſenſe, be equable.

To the abovementioned Tables of the Motion the Fix'd Stars is alſo added *a Table of the Re- ction of the Stars*, made for the different Alti- le of them above the Horizon (by *Prop.* 66.) e Altitude of any Star obſerved is to be leſ- ed by the correſponding Refraction, that it y become true; or the true Altitude (found m the true place, at a given time, by Calcu- ion) is to be increaſed by the ſaid Refraction, t it may become the apparent Altitude. This erent Altitude of the Star from Refraction in- es a diverſity of its Place in regard of Longi- e or Latitude, or in any other Circle, after ſame manner with that ſhewn before in the allax. We have reckoned this Table of Re- ction here among the Tables of the Fix'd Stars, it might more properly be refer'd to the pre- ing, as what is common to every Celeſtial enomenon: Others in the mean while will ık it ought to be reckoned among the Tables are to be deſcrib'd in the following Propoſi- , ſince different Tables of Refraction are re- ite for the different denſity of the Atmoſphere ifferent places.

PROPOSITION LXIX.

O enumerate the more common Tables, wherein the ſituation of the more conſiderable Places upon Earth, in regard of one another, and in reſpect of Equator, is deſcribed, and thoſe wherein the Time oned in any of them is reduced to the Time reckoned rding to the cuſtom of any other.

In the Catalogue of the moſt conſiderable Places, you have the Elevation of the Pole in that place ſet down by it, (determined by *Prop.* 17;) to which the Latitude of the Place is equal. There is alſo ſet down the difference (found by *Prop.* 32.) of the Firſt Meridian (whether it be ſuppoſed to paſs thro' the place where the Aſtronomer lives, or through the *Azores* or *Fortunate Iſlands*) and of the Meridian of that place, and it is expreſſed in parts of the Equator, or in Hours and parts of an Hour. Thus we have the ſituation of theſe places to one another, and to the Equator ſettled, and may make uſe of it as occaſion requires in Geography or Hydrography. The Hour reckoned in one of the Places, with the difference of the Meridians, expreſſed in Hours and parts of an Hour, added to it, gives the Hour reckoned in the more Eaſtern place; but ſubtracted from it, ſhews the Hour reckoned in a more Weſtern place, provided both places make uſe of the ſame ſort of Hours; but if they uſe a different ſort, they are to be reduced to the ſame by a proper reduction.

You have alſo Tables for the more expeditious reduction of any Time in one of the places expreſſed by any Calendar, and reckoned from any known Epocha, into Time of any given form in the other place. For ſince (by *Prop.* 11. and 12.) both the Epochas may be compared together, and the various forms of the Years among different Nations, the grounds and method of conſtructing theſe Tables (which are as neceſſary for Men of Buſineſs and Hiſtorians, as Aſtronomers) is evident.

Beſides, the Circles diſtinguiſhing the Climates and Parallels are reduced into Tables by ſome. For if you begin at the Equator, the quantity of the longeſt Day is had, at the end of each Cli-

ate and Parallel, or the beginning of the next;
at is, the continuance of the Solſtitial Point
wards the viſible Pole above the Horizon. There
e alſo given the Declination of the Solſtitial
int (namely the Obliquity of the Ecliptic) and
e Right Aſcenſion: The Elevation of the Pole
that place therefore is alſo given; that is, the
ſtance of the Circle bounding the Climate or
rallel propoſed from the Equator.

POPOSITION LXX.

TO deſcribe the more common Tables, by which the Problems of the Firſt Motion, in different places
the Earth, are more readily ſolved.

For the finding out the places and times of the
ſing and Setting of the Fix'd Stars and other
leſtial Bodies, whoſe Places are known, and
ving other Problems about them, Calculation
commonly uſed, as occaſion requires. But be-
ſe the uſe of the Sun's Motion is more conſi-
able and frequent, and the Sun itſelf in its
nual courſe, is in different degrees of the E-
ptic; Tables are made for all Latitudes, to
w the reſpect of theſe points to the Horizon:
t is, for every degree of Latitude from the E-
tor, the Difference between the right and ob-
e Aſcenſion of every degree of the Ecliptic
und by *Prop.* 33.) is added to that degree of the
iptic expreſſed by its name, out of all which
able is made, or rather a heap of Tables; for
eed every degree of Latitude ſhould have a
ble, extending to the ſeveral degrees of the
iptic. Theſe ſort of Tables are called *Tables*
he Aſcenſional Differences.

The Aſcenſional differences being thus reduced
o Tables, the oblique Aſcenſions and diurnal
cs of every one of the degrees of the Ecliptic
in like manner eaſily reduced into Tables by

bare Addition and Subſtraction; and from thence the Hours of the Riſing and Setting of the Sun, when in thoſe points. But theſe Operations are ſo eaſy, as that they are uſually left to the pains of every one that makes the Calculation; excepting, that generally an Aſtronomer, that is pretty much taken up in ſupputations, has them already calculated for the place of his own Habitation. And ſometimes alſo he has by him Tables (made by *Prop.* 33.) of the *Ortive Amplitudes* of the degrees of the Ecliptic, either for ſeveral Latitudes, or at leaſt for his own.

There is another ſort of *Table* (to be made by *Prop.* 35.) of the *Angle of the Eaſt*, namely that which the Ecliptic makes with the Horizon, for the different points of the Ecliptic riſing in a given Latitude, or (which comes to the ſame) of the Altitude above the Horizon of the Nonageſim Degree from the Eaſtern point. And becauſe in Calculating Eclipſes of the Sun and other things, there is very great uſe of this Angle, Tables of it are made for every degree of Latitude from the Equator to the Pole.

And ſometimes Tables of the Beginning and Ending of the Morning and Evening Twilight are made (by *Prop.* 39.) for the degree of the Ecliptic that the Sun is in, and for different Latitudes, or at leaſt for the Latitude of their own Habitation.

In making all theſe ſorts of Tables, there are ſeveral compendiums of the calculation and pains that occur, ſuch as are ſhewn in *Prop.* 67; and a ſhorter and more elegant way of ordering theſe Tables may be had from thence.

Plate 1. Book 2.

Fig. 1. p. 222.

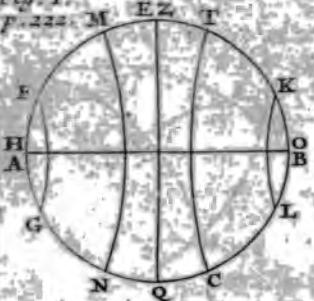

Fig. 2. p. 223.

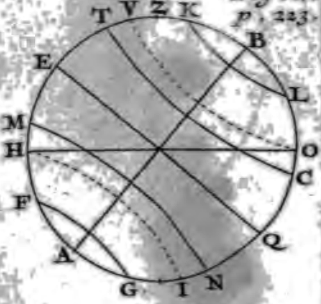

Fig. 3. p. 224.

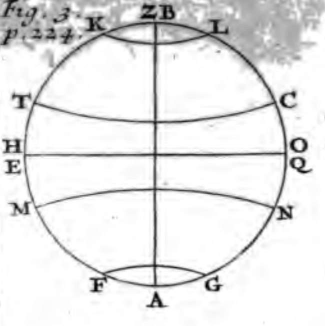

Fig. 4. p. 225.

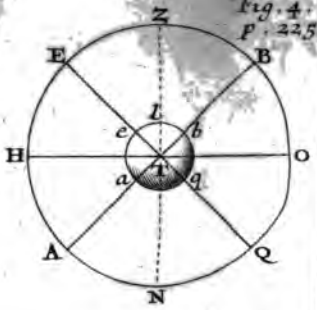

Fig. 5. p. 279.

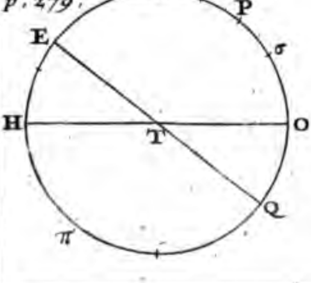

Fig. 6. p. 282.

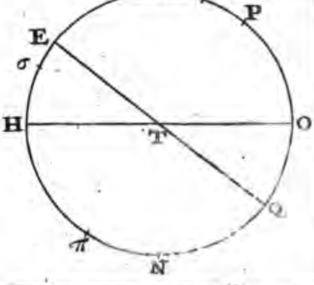

Plate 2. Book 2.

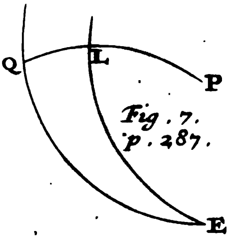

Fig. 7.
p. 287.

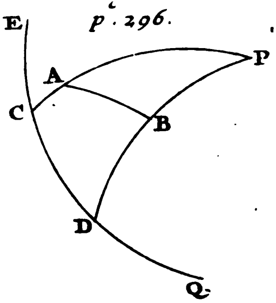

Fig. 8.
p. 296.

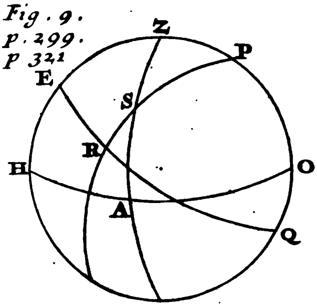

Fig. 9.
p. 299.
p 321

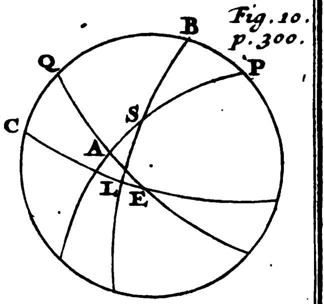

Fig. 10.
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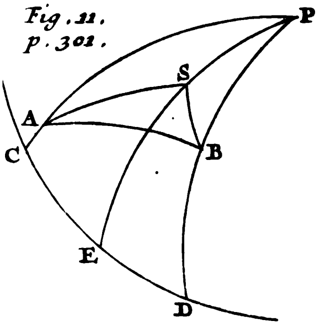

Fig. 11.
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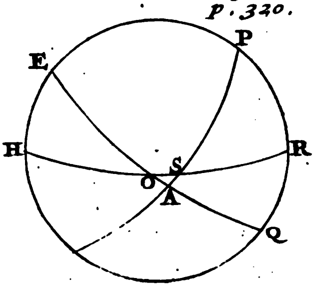

Fig. 12.
p. 320.

Plate 3. Book 2.

Fig. 13.
p. 322.

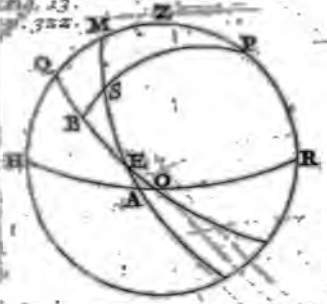

Fig. 16
p. 325.

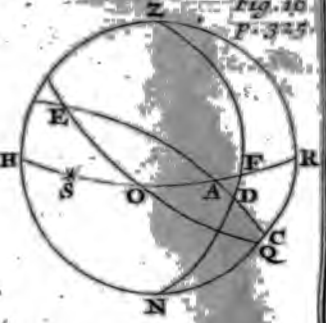

Fig. 14.
p. 323.

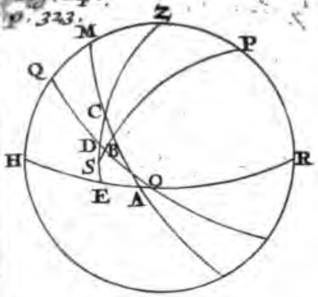

Fig. 17.
p. 326.

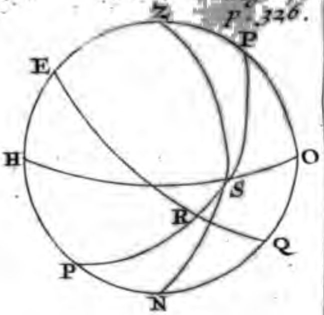

Fig. 15.
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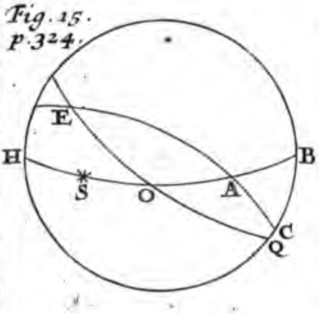


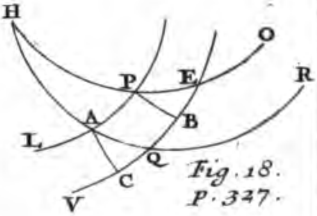


Fig. 18.
p. 327.

Plate 4. Book 2.

Fig. 19. p. 328.

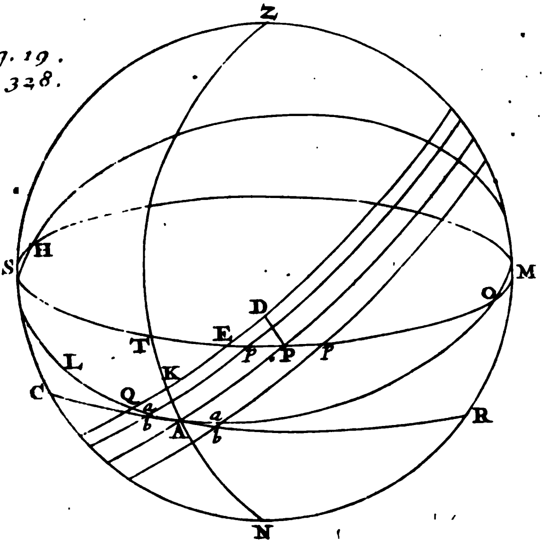


Fig. 20. p. 331.

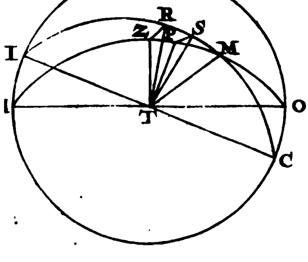


Fig. 21 p. 333.

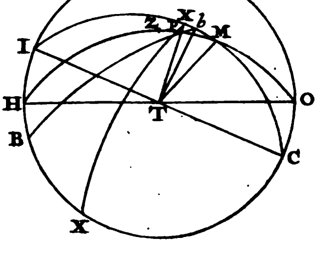


Plate 5. Book 2.

Fig. 22.
P. 334.

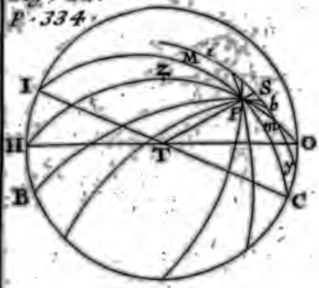


Fig. 25.
P. 339.

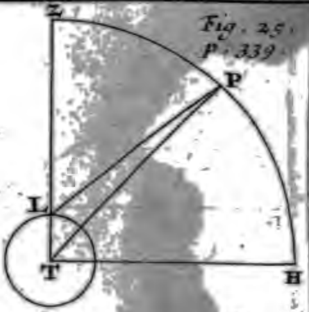


Fig. 26.
P. 340.

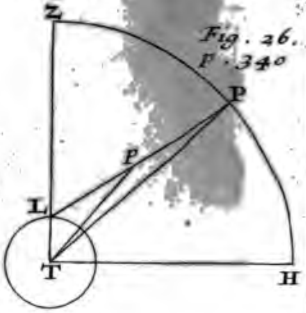


Fig. 23
P. 337.

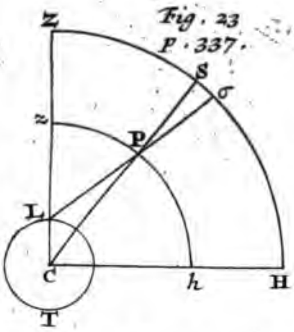


Fig. 24.
P. 337.

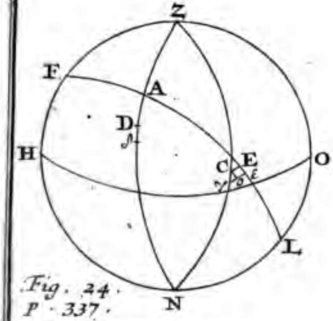


Fig. 27.
P. 341.

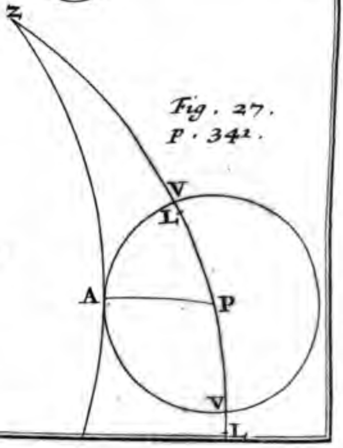


Plate 6, Book 2.

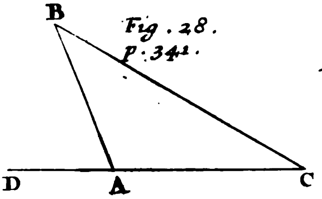


Fig. 28. p. 341.

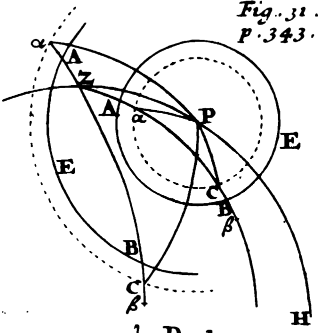


Fig. 31. p. 343.

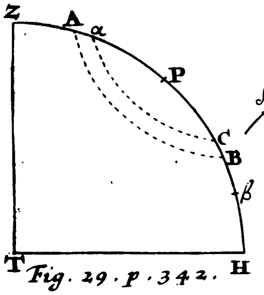


Fig. 29. p. 342.

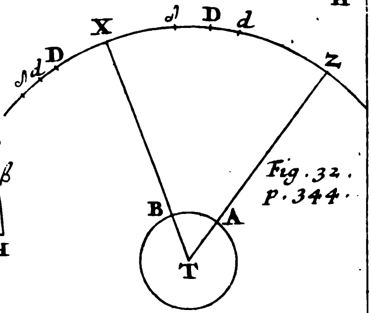


Fig. 32. p. 344.

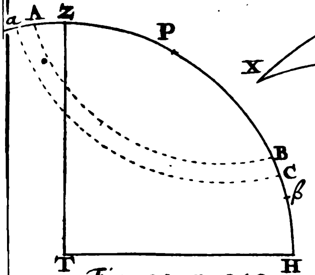


Fig. 30. p. 343.

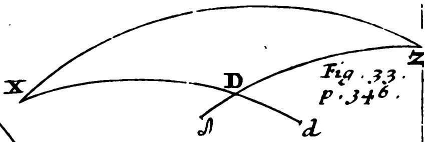


Fig. 33. p. 346.

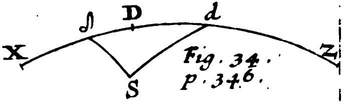
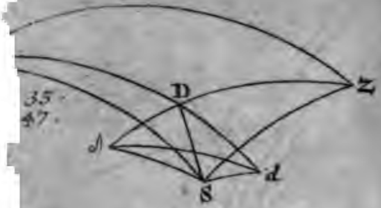


Fig. 34. p. 346.

Plate 7. Book 2.



35.
47.

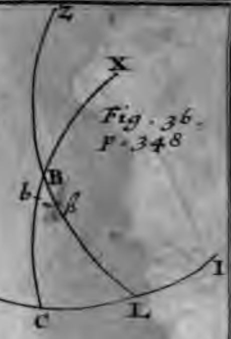


Fig. 36.
P. 348

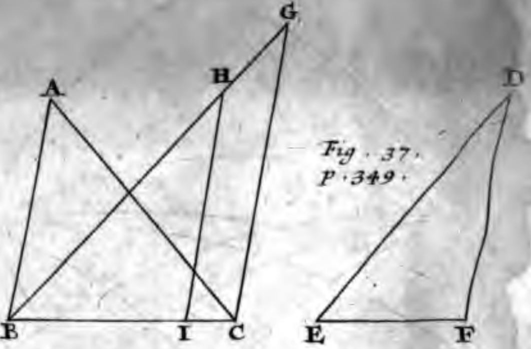


Fig. 37.
P. 349.

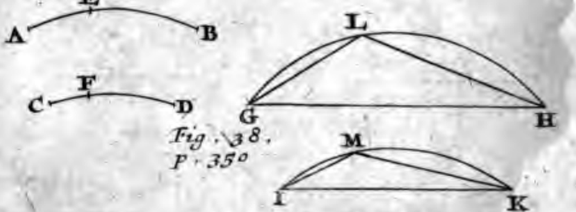


Fig. 38.
P. 350

Plate 8. Book 2.

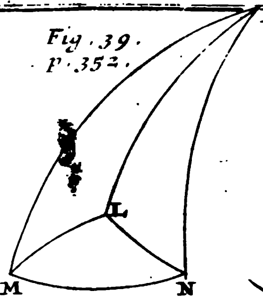


Fig. 39. p. 352.

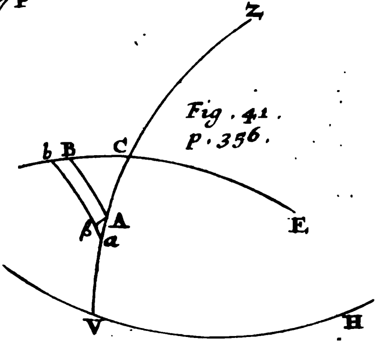


Fig. 41. p. 356.

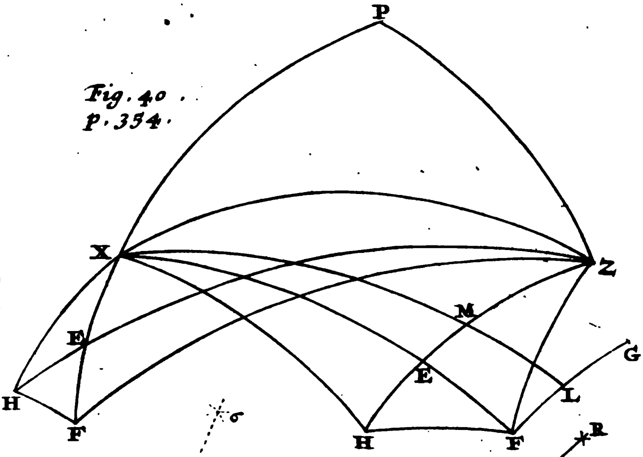


Fig. 40. p. 354.

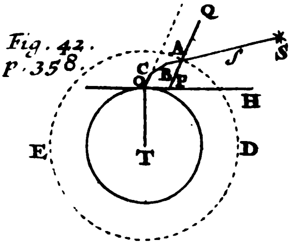


Fig. 42. p. 358.

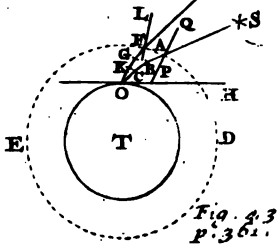


Fig. 43. p. 361.

THE
ELEMENTS
OF
Aſtronomy,
Phyſical and Geometrical.

The Third Book.

Of the Theory of the Primary Planets.

IN the Firſt Book, we have ſhewn what Laws a Planet muſt move by, to deſcribe an Ellipſe about the Sun placed in one of its Foci, which Figure only the Phænomena will admit of, as *Kepler* has ſhewn at large, in his *Commentaries* *f the Motions of Mars*, and all Aſtronomers now llow of. The ancient Aſtronomers themſelves, ogether with *Ptolemy*, (who were for admitting nothing but perfect Circles in the Heavens, which ſeem'd the moſt proper for Bodies that were not ſubject to generation or corruption, ſuch as they took the Heavenly Bodies to be,) almoſt come into this opinion, being compell'd by the Phænomena themſelves, ſince they have imagined a motion really unequal in the Excentric, but equal in the Equant Circle, whoſe Center was as far beyond the Center of the Excentric, as the Center of the true Motion was on this ſide of it; that is, ſince they biſſected the Excentricity, and deſcribed the Orbit of the Planet, from the point of biſſection as a Center. For ſuch an Excentric circular Orbit of a Planet does not differ from an Elliptic Orbit, whoſe Foci are the ſaid Centers, excepting that the former is ſomething thicker about the middle, but the latter ſomething ſmaller; which difference of the thickneſs of the Orbit is almoſt inſenſible in moſt of the Planets, by reaſon of the ſmallneſs of the Excentricity, and not diſtinguiſhable by the obſervations of the Ancients. Tho' *Kepler*, in *Chap.* 44. of the ſaid *Commentaries*, has made it out very clearly and unqueſtionably, from the obſervations of *Tycho*. 'Tis true *Ptolemy* uſes an Excentric in the ſimple Theory of the Sun, and does not think that the Excentricity of the Orbit is to be biſected; but in every one of the other Planets, beſides the Epicycle (by which he explains the Phænomena really ariſing from the motion of the Earth, which he did not acknowledge) he uſes (*Ch.* 5. *B.* 9.) an Excentric deſcribed upon the Center biſecting the Excentricity. Nay, *Copernicus* himſelf took the perfection of the motion and way of the Planets to be ſo firm, that upon this account he thinks (*Ch.* 2. *B.* 5.) the biſection of the Excentricity ought to be rejected, becauſe it is abſurd to grant *that an equality of circular motion can be made upon a ſtrange Center and not upon its own*, (which that it is was once *Ptolemy*'s opinion, is evident

from *Chap.* 3. *Book* 3. where he ſays *that all*
notions of the Stars are equal and circular by na-
; that is, (as he himſelf explains it) *that all*
ines, which are underſtood to carry about the Stars
eir Circles in all ſimply equal Times, intercept equal
les at the Centers of each Circulation, tho' he af-
ards changed, in *B.* 9. already quoted,) and
. 4. of the ſame Book, he would rather have
Epicycle and a ſimple Excentric introduced,
orming the ſame as an Excentric and Equant
or even a Concentric with two Epicycles.
thoſe exact and repeated obſervations of
o had not then been made, that *Kepler* wholly
ends upon when he demonſtrates that the Or-
of the Planets are Elliptic; tho' *Ricciolus* inſi-
tes by the by, that *Kepler* began to ſuſpect that
Way of a Planet thro' the Æther was Ovi-
n; his natural ſagacity taking the firſt hint
it from ſeeing the Oval Figure that *Reinholdus*
ed in the end of *Purbachius*'s *Theorics* belong-
to the Moon. Therefore in ſuch a true El-
ic Orbit of a Planet *ALP*, where *S* [*Fig.* 1.]
Focus is the place of the Sun, *ASP* the line
the Apſides, *A* the Aphelion, *P* the Periheli-
L the Place of a Planet taken at pleaſure,
Area *ALS* contained under the Right lines
, *SL*, and the Elliptic Curve *AL* (computed
m *SA in conſequentia*) is called the *Mean Anoma-*
that is, Equable; for this Area increaſes equa-
, namely in the ſame ratio with the time:
d the Angle *ASL*, contain'd by the line of the
ſides *SA*, and Radius *SL* that carries the Pla-
, is called the *Coequate* or *True Anomaly*. For it
roper that the Cultivators of a Phyſical Aſtro-
my, ſhould recede as little as may be from the
cient Terms of Art; being not only eſtabliſhed
uſe, but really the fitteſt for that purpoſe.

SECTION I.

Some General Things belonging to the Theory of all the Planets.

PROPOSITION I. LEMMA.

IF *upon the greater axe* PA, *of the Ellipſe* PBA [Fig. 2.] *whoſe Center is* C, *as a Diameter, you deſcribe the Circle* PDA, *and from a point* G *taken at pleaſure in the Circumference of the Circle, a Perpendicular* GE *be let fall on* PA, *interſecting the Ellipſe at* L; *and from ſome point in the Axe* AP (*for inſtance from one of the Foci as* S) *to* G *and* L, *the Right lines* SG, SL *be drawn; the trilinear Figure* AGS *contained under the Right lines* SA, SG, *and the Circular Curve* AG, *will be to the whole Circle, as the trilinear* ALS *contained under the Right lines* SA, SL *and the Elliptic Curve* AL *is to the whole Ellipſe.*

Draw eg parallel to EG, meeting the Diameter in e, the Ellipſe in l, and the Circle in g. Then (by *Prop.* 21. *Book* 1. *of the Conics of* Apollonius) el will be to CB in the ſubduplicate ratio of the rectangle under Pe, eA, to the Rectangle under PC, CA, or CD^q: And eg to CD is alſo in the ſubduplicate ratio of $Pe \times eA$ to CD^q; therefore (by 11 *Elem.* 5.) el is to CB, as eg to CD; and therefore $el : eg :: CB : CD$; that is, in a given ratio. And ſince this is always done, all the el inſiſting upon EA, will be to as many eg; that is, the Elliptic Semi-ſegment ALE to the Circular Semi-ſegment AGE, in the ſame given ratio of CB to CD. Again, (by *Prop.* 1. *Elem.* 6.) the Triangle LSE, is to the Triangle GSE as LE to GE; that is, (as has been already ſhewn) as CB to CD; from whence (by *Prop.* 12. *Elem.* 5.) ALE and LSE together, *i. e.* ALS is to AGE and

GSE together, or *AGS*, in the ratio of *CB* to *CD*: And after the like manner, in that ſame ratio of *CB* to *CD* are all the *el* inſiſting upon the whole *PA* to as many *eg*; that is, the Semi-ellipſe *PBA* to the Semicircle *PDA*. And therefore *ALS* is to *AGS*, as the Semi-ellipſe *PBA* to the Semicircle *PDA*, or as the Ellipſe to the Circle. And on the contrary the trilinear *ALS* is to the whole Ellipſe, as the trilinear *AGS* to the whole Circle. *Q. E. D.*

Conſequently, if the Semi-ellipſe *PBA* were propoſed to be divided into two Segments having a given ratio, by a right line *SL* proceeding from a given point *S* in the Axe, it is ſufficient to divide the Semicircle *PDA* (in ſome reſpect more known than an Ellipſe) by a right line *SG* in the ſaid ratio, and to let fall a perpendicular *GE* from the point *G*, on the Axis *PA* meeting the Ellipſe in *L*: For the right line *SL* divides the Semi-ellipſe *PBA* in the ſame ratio as the Semicircle *PDA* is divided by the right line *SG*: Or if *ABP* be an Elliptic Orbit of a Planet, in one of whoſe Focus *S*, the Sun is; as the Mean Anomaly of the Planet in *L* is repreſented by the trilinear figure *ALS*, ſo it may alſo be repreſented by the trilinear figure *AGS*.

PROPOSITION II.

THE Orbit of a Planet being given, to determine by calculation, as many Mean Anomalies and orreſponding True ones, as you pleaſe.

Let the Elliptic given be *ALBP*, [*Fig.* 3.] in one of whoſe Foci *S* the Sun is placed. Let *A* be the Aphelion, *P* the Perihelion, *C* the Center, *CA* or *CP* the Mean diſtance of the Planet from the Sun, *CBD* perpendicular to *AP*. In the Circle *ADP*, deſcribed about the greater axe as diameter, take at pleaſure from *A*, the arc *AG*, or angle *ACG*, expreſſed by the number of Degrees and Minutes (as is uſual.) And becauſe (by 23 *Elem.* 6.) the arc *AG* is to the whole circumference, as a ſector *AGC* to the whole circle, if the Circle as well as the Circumference be ſuppoſed to be divided into 360 equal parts, or circular ſectors correſponding to the degrees of the circumference; the ſame number that expreſſes the arc *AG*, expreſſes alſo the circular ſector *AGC*; the latter in ſmall ſectors, the former in ſmall arcs, either of theſe *Kepler* calls the *Anomaly of the Excentric.* But from the dimenſions of a Circle, (accurate enough for Calculation,) the ratio of the ſquare of the Semidiameter to the whole Circle is given; and from the ſpecies of the Orbit given, the ratio of the mean diſtance *BS* or *CA*, and alſo of the leſſer Semi-axe *BC* to the Excentricity *SC* is given; and from thence the ratio of the Triangle *BCS* to the ſquare of the Radius; and from the ratio of the arc aſſumed are *AG* to the quarter of the Circumference *AD*, is given the ratio of the Sines of them, *viz.* *GE* to *DC*, and conſequently of *BC* to *GE*, that is, (by 1. *El.* 6.) the ratio of the Triangle *BCS* to *GCS*. In the parts therefore, of which the Circle contains 360, there are given the ſector *AGC* and the Triangle *GCS*, (whoſe analogous one *LCS* is call'd the *Phyſical part of the Equation*, as the angle *SLC* the *Optical part*;) and therefore the trilinear aggregate of them *AGS*, which in the preceding *Prop.* is ſhewn to be proportional to the trilinear *ALS*, and conſequently may juſtly repreſent the Mean Anomaly. For if the whole area *ABP* of the Elliptic Orbit be imagined to be divided into 360 equal parts, the part of it *ALS*, namely the Mean Anomaly of the Planet in *L*, will contain as many parts of it, as *AGS* contains ſimilar parts of the circular Area *ADP*; and ſo will be expreſſed by the ſame number.

ber. Again, in the triangle *GCS*, the ratio of the Sides *GC*, *CS* being given,(the Mean diſtance of the Planet from the Sun to the Excentricity) and the contained Angle *GCS*, (the complement to two Right of the Angle *ACG* aſſumed,) the Angle *GSC* (by 41 of the *Data*) will be given; and therefore alſo *ASL* the Coequate Anomaly, namely theAngle whoſe Tangent *EL* is to *EG* the Tangent of the Angle *ASG*, found before, in the ratio of the leſſer ſemi-axe *CB* of the given Orbit, to the greater ſemi-axe *CD*. And ſince the mean and true Anomaly, found by calculation, agree to the ſame aſſumed Arc *AG*, they will alſo agree with one another; or one will correſpond to the other. And after the ſame manner any other arc may be aſſumed, and the agreeable Anomalies, (*viz.* mean and true) determined thence by calculation, which will conſequently be correſpondent to one another.

PROPOSITION III.

THE mean Anomaly of a Planet being given, whoſe Orbit is given, to find by calculation the true Anomaly and Diſtance of the Planet from the Sun, and the converſe.

Kepler, at the end of *Part* 4 of his *Celeſtial Phyſics*, ſays, that there is no direct method of Calculation for ſolving this Problem. How the calculation of the Problem may be perform'd by an indirect method, and fit enough for Aſtronomical uſes,is now to be ſhewn. In the Circle *ADP*, (*Fig.* 3.) touching the Orbit in the Aphelion and Perihelion, let the arcs *AG* or angles *ACG* of the Anomaly of the Excenter, be taken, exceeding one another by ſingle degrees. For every one of them (by the preceding) find the correſponding Anomalies, *viz.* the mean and the true. When a Canon of the Mean and correſponding Coequate Anomaly, for a given Orbit of a Planet, is made to each particular Arc of the Anomaly of the Excentric, look among them for the mean Anomaly given; the correſponding true Anomaly is that which is ſought. But if the mean Anomaly propoſed, is not found among thoſe in the Canon, the true Anomaly ſought will be found (after the manner commonly known) by a proportional part. If you are to work without Tables, *Kepler* adviſes the uſe of the Rule of Poſitions (call'd alſo the Rule of Falſe;) by ſuppoſing the Anomaly of the Excentric *AG* an arc, or *ACG* an angle, or *AGC* a Sector, (for theſe three are expreſſed by the ſame number) to be of as many parts as you pleaſe, and by adding or ſubtracting from it, ſo aſſum'd as there is need, an analogous Triangle congruous to the Phyſical part of the Equation, namely *GSC*, to make the mean Anomaly *AGS*, for if it comes out as great as that propoſed, the Anomaly of the Excenter was well put, and the true Anomaly that agrees to it is the Anomaly ſought; becauſe fitting the given mean Anomaly; but if it does not come out ſo great, the Poſition is to be corrected by that which does come out, and the operation is to be repeated. For finding the Diſtance of the Planet from the Sun, you have in the Triangle *LCS*, the Angle *LSC* given, namely the above found coequated Anomaly; alſo the Angle *LCS*, the Supplement of the Angle *LCA*, whoſe Tangent is to the Tangent of the Angle *ACG* of the Anomaly of the Excentric, as the leſſer axe of the Orbit to the greater; the Excentricity *SC* is alſo given, from whence the diſtance *SL* of the Planet from the Sun becomes known.

And after the like manner, may the Mean Anomaly be determined from the given True one, either by Tables already made, or by the Rule of Falſe Poſition.

PRO-

PROPOSITION IV.

TO *divide a Semicircle in a given ratio, by a right line drawn from a given point of the diameter.*

Let *AGB* [*Fig.* 4.] be a Semicircle, *C* its Center, *AB* its Diameter, and *S* a point given in the Diameter; from whence a right line as *SG* is to be drawn, cutting the circumference in *G*, ſo as that the Area *AGS* may be to the Area *BHGS* in a given ratio, namely as *p* to *q*. Suppoſe it done, and from *G* on *AC* let fall the Perpendicular *GE*. Let the Semidiameter *AC* or *CB* be called *r*; *AE*, *x*; let *m* be a fourth proportional to the Right lines *SC*, *CA*, and the Semicircumference *AHB*. Then Semcircumf. × Radius = twice the Semicircle $= SC \times m$, or $m = \frac{2AHB}{SC}$. Again, ince by ſuppoſition, area AGS : area $BHGS :: p : q$, th. Semicircle AHB : area $AGS :: p+q : p$; th. $p \times AHB = \overline{p+q} \times AGS$, and conſeq. $2AGS = \frac{p \times 2AHB}{p+q}$. Put a for $\frac{pm}{p+q}$; then is $a = \frac{p \times 2AHB}{\overline{p+q} \times SC} = \frac{2AGS}{SC}$; Or, $a \times SC = 2AGS$. Towards *B* take *CT*, ſo to *CB*, as *CB* is to *CS*; and call *AT*, *b*, then is $CT = b - r$, and th. $SC = \frac{rr}{b-r}$; and $SC \times a = 2AGS = \frac{arr}{b-r}$. And ſince $CE = r - x$, th. $SE = r - x + \frac{rr}{b-r}$; Alſo $2AGS = 2AGE + 2GSE = 2AGE + GE \times ES$. But, by the Doctrine of Series, in a Circle, whoſe radius is *r*, the Segment whoſe verſed Sine is *x*, that is,

$$2AGE = \tfrac{4}{3}\sqrt{2} \times r^{\frac{1}{2}}x^{\frac{3}{2}} - \frac{2r^{-\frac{1}{2}}x^{\frac{5}{2}}}{5\sqrt{2}} - \frac{2r^{-\frac{3}{2}}x^{\frac{7}{2}}}{28\sqrt{2}} - \frac{2r^{-\frac{5}{2}}x^{\frac{9}{2}}}{72\sqrt{2}} \text{ \&c.}$$

$$\text{And } GE = \sqrt{\overline{2xx-rx}} = \sqrt{2} \times r^{\frac{1}{2}}x^{\frac{1}{2}} - \frac{r^{-\frac{1}{2}}x^{\frac{3}{2}}}{2\sqrt{2}} - \frac{r^{-\frac{3}{2}}x^{\frac{5}{2}}}{16\sqrt{2}} - \frac{r^{-\frac{5}{2}}x^{\frac{7}{2}}}{64\sqrt{2}} \text{ \&c.}$$

Hence $2\,\triangle SEG = (SE \times EG = \overline{r + x + \frac{rr}{b-r}} \times EG)$

$= \overline{r+x} \times GE = \sqrt{2} \times r^{\frac{3}{2}} x^{\frac{1}{2}} - \frac{5 r^{\frac{1}{2}} x^{\frac{3}{2}}}{2\sqrt{2}} + \frac{7 r^{-\frac{1}{2}} x^{\frac{5}{2}}}{16\sqrt{2}}$ &c.

$+ \frac{rr}{b-r} \times GE = \frac{rr}{b-r} \times \sqrt{2} \times r^{\frac{1}{2}} x^{\frac{1}{2}} - \frac{r^{-\frac{1}{2}} x^{\frac{3}{2}}}{2\sqrt{2}} - \frac{r^{-\frac{3}{2}} x^{\frac{5}{2}}}{16\sqrt{2}}$ &c.

But $\frac{rra}{b-r} = 2\,AGS = 2\,AGE + 2\,SEG$, therefore, by reducing the Equation according to Art, you will find that

$a = \sqrt{2} \times b r^{-\frac{1}{2}} x^{\frac{1}{2}} - \frac{2 r^{-\frac{1}{2}} x^{\frac{3}{2}}}{3\sqrt{2}} + \frac{\sqrt{2} \times b r^{-\frac{3}{2}} x^{\frac{3}{2}}}{12} - \frac{\sqrt{2} \times r^{-\frac{3}{2}} x^{\frac{5}{2}}}{20}$ &c.

Th. $aa = \frac{2b^2}{r} x + \overline{\frac{b^2}{3r^2} - \frac{4b}{3r}} \times x^2 + \overline{\frac{2}{9r} + \frac{4b^2}{45r^3} - \frac{14b}{45}} \times x^3$ &c.

th. $x = AE = \frac{ra^2}{2b^2} + \overline{\frac{r^2}{6b^5} - \frac{r}{24b^4}} \times a^4 + \overline{\frac{r}{720b^6} - \frac{13r^2}{360b^7} + \frac{7r^3}{72b^8}} \times a^6$ &c.

Taking therefore *AE* equal to ſo many terms of this Series as you pleaſe, and from *E* erecting *EG* perpendicular to *AB*, it will meet the Circle in the point G ſought. *Q. E. F.*

And (by *Prop.* 1.) the ſame EG will meet any Ellipſe *AKB*, deſcrib'd upon the greater axe *AB*, in *L*, ſo as that *SL* being joyn'd will divide the Semi-Ellipſe *AKB* into two parts *ALS*, *BKLS*, having a given ratio to one another.

The leſs the diſtance of the points C and S (that is, the leſs the excentricity of the Ellipſe) and the ratio of *p* to *q*, the fewer terms of this Series will ſuffice: For the ratio of *p* to *q* ought to be ſmall, that the Series, expreſſing the value of the Segment *AGE*, may ſoon converge; that is, that few of its terms may be very near equal to the Segment.

I formerly publiſh'd this Solution of the Problem before us, in a *Geometric Exercitation concerning the Dimenſions of Figures*, Printed at *Edinburgh* in the Year 1684; but I thought it would not be a-miſs to inſert it again here, as its proper place.

'Twas firſt thus done by Mr. James Gregory, *and communicated to Mr.* John Collins, *in a Letter dated* April *the 9th* 1672, from St. *Andrews.*

PROPOSITION V.

TO *divide an Ellipſe that is not very excentric, by a right line thro' one of the Foci, ſo as that the portion of the Elliptic area contained between the greater Axe and the Line to be drawn, may be very nearly to the whole Ellipſe in a given ratio.*

The method delivered in *Prop.* 3. of finding the rue Anomaly by the mean, is reckoned tentative nd indirect: But any Geometric one, however legant, (as is that formerly given us by the pro-racted Cycloid by Dr. *Wallis,*) is not ſo fit for A-tronomical Calculations; and that which does t by Series (as *Prop.* 4.) is too laborious; on which ccount Aſtronomers have recourſe to Approxi-nations, eſpecially the following one.

Let the Ellipſe propoſed be *ALP*, [*Fig.* 5.] vhoſe Center is *C*, one Focus *S*, the other *F*. rom the Focus *S* a right line *SL* is to be drawn, o as that the Trilinear *ALS*, may have very early a given ratio to the whole Ellipſe *ALPA*.

At one of the Foci *F*, make an angle *AFL*, ha-ing the ſame ratio to four right, as the trilinear igure *ALS* ought to have to the whole Ellipſe, hoſe Leg *FL* meets the Ellipſe in *L*, and the ine *SL* will perform the thing propoſed very early. For if the Foci *S* and *F* coincide with ie Center *C*, it will be exactly, as is evident om *Prop.* 33. *Elem.* 6; the error therefore is as reat as that which proceeds from the Excentrici-y: If that therefore be ſmall, that is in an El-pſe not very excentric, the area *ALS* is to the vhole Ellipſe as the Angle *AFL* to four right ngles; that is in a given ratio.

SCHOLIUM.

Kepler himſelf, who firſt ſhew'd that a Planet deſcribed an Ellipſe, ſo as that in equal times equal areas were deſcribed, being cut off by right lines drawn to the Sun placed in the Focus, took notice alſo of this Approximation, and deſcribes it in *Chap.* 3. *Part* 2. *B.* 5. *Epit.* thus, *as if* L *were the Planet, the Angle* AFL *might be inſtead of the mean Anomaly almoſt.* But he did wholly forbear uſing it, both becauſe in the Planet Mars it plainly differs from the Heavens, and eſpecially becauſe his elegant demonſtrations of the motions from their Phyſical cauſes by this means would lie wholly neglected and uncultivated, becauſe intirely unknown; he was therefore willing to perſiſt in the true Phyſical Theory of the Planets, tho' forced to uſe an indirect method, as in finding the coequated Anomaly from the true, rather than to follow one foreign from the natural one, tho' the thing could be done by it directly and very eaſily.

Afterwards the Celebrated Aſtronomer *Seth Ward* made uſe of this Theory of the Planets, as one that Nature it ſelf took into her ſervice; and aſſumes that the Orbit of any Planet is Elliptical, in one of whoſe Foci is the Sun, which he acknowledges is the true and Phyſical inſtrument of all the Planetary motions; in the mean while the motion of each Planet is ſo tempered about the other Focus as to form at it, Angles proportional to the times; and upon this Hypotheſis he has built his *Geometrical Elliptical Aſtronomy* (a very excellent work, and what all ſound Aſtronomy is very much indebted to.) For he has applied Geometry (with which he was very well acquainted) ſo happily to Aſtronomy, as not only to find directly and Geometrically the true Anomaly from the mean one given, but to determine the Orbits of all the Planets, as well in regard of their figure and ſituation, as in regard of their magnitude, (taking the Orbit of the Earth as a meaſure.) But ſtill he *has not wiped off that* 'Αγεωμετρησία, *which ſtill cleaves to Aſtronomy*; For he ſolves this Problem of Aſtronomy, not according to the true and genuine Syſtem, whereby *Kepler* had challenged Geometers to ſolve them; but by a fictitious one, that comes very near the true one. Notwithſtanding, the method of his Aſtronomy is of very great uſe: Since the Orbits of the Planets are not very Excentric Ellipſes, but ſuch as come pretty near to Circles, nor does *Ward*'s Hypotheſis, wherein he makes the motion about the Focus, which the Sun is not in, equable, differ much from the accurate one; and will be of very great ſervice in determining the Orbits of the Planets very nearly, which may (by repeating the Calculation, as ſhall be ſhewn below,) again be made more accurate at pleaſure.

'Tis true that *Iſmael Bullialdus*, in his *Philolaic Aſtronomy*, while he makes every Planet move in an Ellipſe, and cuts this Ellipſe out of a Cone, ſuch as that the Axis of the Cone paſſes thro' one of the Foci of the Ellipſe; and laſtly, while (*Chap.* 15. *Book* 1.) he makes *the motion of the Planet about the Cone and in Circles parallel to the Baſis of the Cone, o be equal*; he has ſuppoſed the equable increment of the Angles at the Focus in which the un is not; (namely that through which the Axis of the Cone paſſes,) that is, the Hypotheſis f *Ward*, without knowing of it: Nor did he nderſtand it, till *Ward* inquired into the foundations of his Aſtronomy, and demonſtrates it in he firſt *Chap.* of his *Inquiry*. And *Bullialdus* himelf, who had commiſerated *Kepler* on the account f his miſtakes and defects in Geometry, there neets with what he deſerved at the hands of *Ward*, who ſhows *Bullialdus* (beſides the ſeveral errors

rors, which he had committed and ingenuouſly confeſſes in his *Found. of Phil. Aſtr. aſſerted againſt S. Ward*,) that he did not throughly underſtand his own Hypotheſis, while he ſuppoſes, without knowing it, that the motion about the Focus of the Ellipſe, in which the Sun is not, is equable; which notwithſtanding had he throughly known it, he would have finiſhed the greateſt part of his Aſtronomy much better and eaſier.

And *Count Pagan* has alſo ſeriouſly embraced this ſame Hypotheſis, namely of an equable angular motion of a Planet about the other Focus of an Ellipſe, in which the Sun is not placed, for true and genuine, and has publiſhed in *French* at *Paris An.* 1657, a *Theory of the Planets* built upon it; and at the end of his Theory, has boldly thrown upon the miſtakes of the Obſervers, the diſagreement of it with the Heavens, which is greateſt in the Octants from the Apſides. And tho' this *Theory of Pagan* comes a full Year after *Ward's Geometrical Aſtronomy*, and three at leaſt after the *Inquiry into the Foundations of Bullialdus's Aſtron. Philolaica*, (where he had laid down the foundations of all his Aſtronomy, and conſtructed moſt of the Problems;) yet *Pagan* goes no farther than the determining the place of a Planet in its Orbit being given at an aſſigned time, and its diſtance from the Sun; that is, the eſtabliſhing the principles themſelves of Aſtronomy; omitting all the comparative Aſtronomy, in the cultivating of which by Geometry, the Accurate *Ward* has well employed his pains.

PROPOSITION VI.

SUppoſing the approximation lately deſcribed to be accurate enough, (that is, in the Hypotheſis of Ward) *the Mean Anomaly being given, to find the coequated Anomaly, and the Planets Diſtance from the Sun in a given Orbit.*

Let *ALP* [Fig. 6.] be the Elliptic Orbit of the Planet about the Sun placed in *S* as a Focus; let its Aphelion be *A*, Perihelion *P*, greater Axe *AP*: From the Mean Anomaly the angle *AFL* given, the coequated Anomaly is to be found, namely the angle *ASL*, and the Diſtance of the Planet in *L* from the Sun, *viz.* *SL*; that is, the ratio of this right line to *FS* or *AP*.

Produce *FL* to *B*, ſo that *LB* may be equal to *LS*; and join *BS*. And becauſe (by *Prop.* 52. *B.* 3. *Conic of Apollonius*) *FL* and *LS* taken together are equal to the axe *AP*, *FB* will be equal to the ſame *AP*. And therefore in the Triangle *BFS*, the Sides *BF*, *FS*, and the Angle intercepted *BFS*, the complement of the Mean Anomaly given *AFL* to two right or four right angles, (according as *L* is in the firſt or ſecond Semicircle of Anomaly;) the Angles *FSB* and *FBS* or *LSB* will become known, whoſe difference *ASL* is the coequated Anomaly ſought. Beſides, in the Triangle *LFS* at firſt the angle *LFS* and ſide *FS* are given, and now the angle *FSL* is found, and the remaining one *FLS*; therefore the Side *LS* will be found in the parts of the given diſtance of the Foci *SF*; that is, the Diſtance of the Planet from the Sun. *Q. E. F.*

In finding the angles *FSB*, *FBS*, in the triangle *BFS*, when *BF*, *FS*, and the angle *BFS* are given; (by the common Rule of Trigonometry,) *BF* and *FS* together are to *BF* leſſen'd by *FS*, as the Tangent of half the Angles *B* and *FSB*, to the Tangent of half the difference of the ſame. But *BF* and *FS* taken together are equal to *AB* and *FS* taken together; and *BF* leſs *FS* is equal to *AP* leſs *FS*, or twice *SP*; and the ſum of the angles *B* and *FSB* is equal to the external *AFL*, namely to the Mean Anomaly; and their difference is equal to *FSL* the coequated Anomaly. And therefore the ſum of AP and FS is to $2PS$, or the half of the one SA is to the half of the other PS; that is, the diſtance of the Planet when in its Aphelion is to its diſtance when in the Perihelion, as the Tangent of half the mean Anomaly to the Tangent of half the coequated Anomaly: In which proportion the two firſt terms are conſtant in the given Planet.

After the like manner the mean Anomaly may be found, the coequated being given. But if you don't want the coequated Anomaly ASL that anſwers the given mean one AFL, but the angle FLS (called *the Elliptic Equation*;) it will be the double of the angle FBS found before. And this angle ſubtracted from the mean Anomaly AFL, the Planet deſcending from the Aphelion to the Perihelion, or in the firſt ſix Signs of the Anomaly, leaves the coequated ASL: But if it be added to the mean Anomaly in the laſt ſix Signs of the ſame it makes the coequated Anomaly; from whence it receives the name *Proſthaphæreſis*.

PROPOSITION VII.

TO *explain the correction of the preceding approximation made by* Bullialdus; *and to ſhew how (that being ſuppoſed) the coequated Anomaly and the diſtance of the Planet from the Sun may be found.*

Iſmael Bullialdus, in the defence of his *Philolaic Aſtronomy* againſt *Ward*, in a Book intituled, *The Foundation of the Philolaic Aſtronomy explained*, *Chap.* 1. and 2. ſhews from four Obſervations made by *Tycho*, that the Hypotheſis of *Ward* lately deſcribed does not agree to Mars, that in the firſt and third Quadrant of the mean Anomaly is more forward than it ought to be by this Hypotheſis, but in the ſecond and laſt Quadrant of the mean Anomaly is leſs than this Hypotheſis requires. But even now neither has he ſo much as looked back upon the ſagacious *Kepler*, according to whoſe Phyſical Syſtem, the matter ought to be thus in every Ellipſe, and in Mars ſenſibly, (becauſe its Orbit is ſenſibly excentric;) and as the very numbers themſelves which *Kepler* brings do plainly ſhow. But being reſolved upon correcting *Ward's* Hypotheſis, in requital of the likekindneſs from him, he brings the following correction of the mean Anomaly, found as above, in *Chap.* 4. of the ſaid Book. Let *ALBP* [*Fig.*7.] be the Elliptic Orbit of a Planet, whoſe Foci, Apſides, Center, *&c.* are expreſſed by the ſame letters as they have been frequently before: And let the angle *AFL*, the mean Anomaly in the Hypotheſis of *Ward*, be to four right angles, as the time in which the Planet deſcribes the Elliptic Arc *AL* to the intire periodic Time. And becauſe the point L in the firſt and third Quadrant of the mean Anomaly (that is, while the Angle *AFL* is leſs than a right, or greater than two, or leſs than three) s not ſo forward as Obſervations require, but in he firſt and fourth Quadrant too forward; that s, becauſe the angle *AFL* is in the firſt caſe too ittle, in the ſecond too great; therefore *Bulliallus* corrects this angle thus. Thro' *L* draw *LE* perpendicular to the Axe *AP*; meeting it in *E*, and he Circle deſcribed upon the diameter *AP* in *G*, oin *FG*; let the Angle *AFG* be taken for the mean Anomaly inſtead of *AFL*; the angle *AFG* *ullialdus* takes the liberty of calling the *True-Mean Anomaly*, leaving the name of the *Mean* to he angle *AFL*, which is proportional to the Time: nd therefore the Place of the Planet in its Orit corrected will be the point *V*, where *FG* meets he Orbit. The angle of the True-Mean Anomaly *AFV*, correſponding to the angle of the mean Anomaly *AFL* is found very readily, by aking an angle whoſe Tangent is to the Tangent of the mean Anomaly, as the greater axe of the Ellipſe is to the leſs; and this will be that which is ſought. For GE, the Tangent of the angle GFE, is to LE the Tangent of the Tangent of the Angle LFE, to the ſame radius FE, as DC or CA to BC. And in the Triangle VFS (juſt as in the preceding *Prop.* in the Triangle LFS) the Side FS, the external angle AFV, and ſum of the Sides FV and VS being given, the coequated Anomaly FSV is found, and the Side VS the Diſtance of the Planet from the Sun corrected. *Q.E.F.*

And this is the correction of *Ward's Hypotheſis* which *Bullialdus* brings; and 'tis well enough, if it be taken only for a correction of the approximation to the true Syſtem, as it ought to be; becauſe *that by which he had effected, that at length the coequated Anomaly might be found from the mean* à priori, *and at the ſame time the calculation ſatisfy obſervation; which no one had done before him* (according to *Mercator's* judgment) *in the Elliptic Hypotheſis.* But when *Bullialdus* puts it upon us as the true and genuine Syſtem, and derives the Phyſical cauſes of it (*Chap.* 3. and 4.) after his manner from the Cone, he not only leaves the Foundations of his *Philolaic Aſtronomy* unexplain'd, but diſputes as if there were no Phyſical foundations of Aſtronomy at all.

PROPOSITION VIII.

TO *deſcribe the Orbit, wherein that Celebrated Aſtronomer* Dominicus Caſſini, *thinks a Planet is carried about the Sun, and to explain its agreement and diſagreement with Natural cauſes and the Phænomena.*

Let the Diameter AB of the Circle $AKBH$ [*Fig.* 8.] be divided into two equal parts in the point C, and thro' it draw an indefinite line DE perpendicular to AB. Divide AB again in G,

ſo

hat *AG* may be to *GB* as the diſtance of the
let (whoſe Orbit is to be deſcribed) from the
when it is in its Aphelion, to the diſtance from
Sun when it is in its Perihelion; and take *CF*
al to *CG*. From *F* on *AB* let a Perpendicular
be erected, meeting the Circle in *H*. On
Center *F*, at the diſtance *FH*, deſcribe a
cle cutting the indefinite right line *DE*, in
points *D* and *E*, which are the extreams of
leſſer Axe of the Orbit to be deſcribed. To
its other points, draw thro' *F* any how, the
it line *IFK* meeting the Circle in *I* and *K*.
the Center *F* at the diſtance *IF* or *KF*, let
ircle be deſcribed; and on the Center *G*, at a
ance equal to one of the Segments of the
it line *IFK*, namely *KF* or *IF*, another Cir-
the interſections of theſe Circles, *L*, *L*, *L*, *L*,
l belong to the Orbit. And after this manner,
auſe two Circles are drawn upon both Cen-
, four points of the Orbit will be determined
he ſame time. After the like manner its other
nts may be determined, by drawing other
ht lines paſſing thro' *F*, and bounded by the
cle on both ſides. Becauſe the rectangle un-
the ſegments of the right line drawn thro'
point *F* and bounded by the Circle is given,
ng equal to the rectangle under *AF*, *FB* (by
p. 35. *Elem.* 3;) 'tis evident that the rectangle
der the right lines *FL*, *GL*, drawn from the
ints *F* and *G*, to any point *L* of the Curve
LB is alſo given, and equal to the rectangle
der *AF*, *FB* or under *AG*, *GB*; and therefore
it *AG* is to *GL* as *FL* to *GB* or *FA*. As in the
mmon Ellipſe the ſum of the right lines from
e Foci inclined to any point in the Circumfe-
nce is given, ſo the rectangle under the right
es drawn from the points *F* and *G* to any point
the Curve *ADBE* is given, on which account

Caſſini gave the name of *Foci* to theſe points *F* and *G*.

He would have this Curve to be the Orbit of a Planet, which it deſcribes about the Sun placed in the Focus *G*, ſo as that the angles at the other Focus *F* are proportional to the times; which he therefore calls the Focus or Center of the mean Motion, as he does the other *G*, of the true Motion, where he places the Center of the Sun and of the World. On which account any two diſtances of a Planet from the Focus of the true Motion are reciprocally as its diſtances from the Focus of mean Motion. In this Orbit, the true Anomaly, namely *AGL* is determined from the mean Anomaly or Angle *AFL*, if in the Triangle *LFG*, the Baſe *FG*, and one of the adjacent angles *LFG* (the complement of *AFL* the mean Anomaly,) and the rectangle under the two other ſides *FL*, *FG* being given, the other things may be found.

This *Caſſinian* Hypotheſis, briefly delivered in a *Treatiſe of the Origen and Progreſs of Aſtronomy*, has this Phyſical diſadvantage, that tho' a Planet, by the action of a centripetal Force of a certain kind toward the Sun, (whoſe Law may be known by *Cor.Prop.*37.*B.*1.) may deſcribe the Orbit *ALDBE* about the ſame placed in *G*, yet this force will be intirely different from that uſed in moſt of the operations of Nature, and from that whereby the common Ellipſe, having the ſame Foci and Vertices, is deſcribed about the Sun placed there; and this over and above, that a Planet deſcribing the Curve *ALD*, as *Caſſini* ſuppoſes, (namely ſo as that a line *FL* being drawn every where to the Planet, the angle *AFL* is made proportional to the time) does not all obſerve that univerſal Law demonſtrated in *Prop.* 11. *B.* 1. ſo as to make the area *ALG*, which the radius *GL* proceeding from the he Sun, deſcribes, proportional alſo to the Time. On which account this Hypotheſis, as well as the bove deſcrib'd Hypotheſis of *Ward* and *Paganus*, s Phyſically impoſſible. 'Tis utterly impoſſible hat a Planet, tho' it be acted upon by any cenripetal Force tending toward the Sun, ſhould decribe the Perimeter of the common Ellipſe, or of this *Caſſinian* Ellipſe, ſo as that the angle at he Focus where the Sun is not be alſo proporional to the Times. For this being ſuppoſed, the reas at the Sun would not be proportional to the Times, (as has been ſhewn above of the common Ellipſe, and ſhall be ſhewn hereafter of both) which notwithſtanding is done, as has been demonſtrated in *Prop.* 9. *B.* 1.

But we will demonſtrate that in the Figure above decribed ALDBE *(or in any other that is concave every where toward the Center, whoſe four parts made by the wo axes are equal and ſimilar, and the two Foci* F *nd* G *equi-diſtant from the Center,) every angle at the Focus* F *is not to four right angles as the area at the Focus* G *to the whole Figure* ADBE.

Let us ſuppoſe the angle at the Focus *F* to be o four right as the area at the Focus *G*, to the inire Figure [*Fig.*9.] Draw the right line *FL* perendicular to *AB*; by which means the angle *AFL* will be a fourth part of four right, or half of two ight; and therefore the area *ALG* will be by uppoſition a fourth part of the intire Figure *ADBE*, or a half of the half *ADB*, that is, equal to *LDBG* what it wants of half. From *G* raw *GM* parallel to *FL*, meeting the Orbit in *M*; and then *GM*, from the nature of the Orbit, will be equal to *FL*; and conſequently by *Prop.* 33. *El.* 1. *ML* (which intirely falls within he Curve *ADBE*, ſince by ſuppoſition it is every where concave towards the Center *C*) is equal and arallel to *GF*; and (by *Prop.* 34. *El.* 1.) the Triangle

angle *MGL* is equal to the Triangle G*FL*. ſince the Figure *AFL* is, from the nature o Orbit, equal to the Figure *BMG*; the Fi *ALG* is equal to the Figure *BMLG*, containe der the right lines *BG*, *GL*, *LM*, and the C *BM*. But it was juſt now ſhewn, that the Fi *BMDLG*, contained under the right lines *BG*, and the curve *BMDL*, is equal to the Figure *A* and therefore the Figure *BMLG* is equal t Figure *BMDLG*; that is, a part equal to the wh which is abſurd. Therefore the Figure *AL* not a quarter of the whole Figure *ADBE*, w the Angle *AFL* is a quarter of four right; o very angle at the Focus *F* is not to four ri angles as the area at the Focus G to the whole gure. *Q. E. D.*

Beſides the aforeſaid Hypotheſis labours un this difficulty, that the true Anomaly, and correſponding mean Anomaly, and the diſta of the Planet from the Sun, do not in the leaſ gree with the Phænomena in the Heavens. ſince the common Ellipſe does agree with t Phænomena, that other, the *Caſſinian* one, wh is of a different nature, cannot agree with th For the place of a Planet, at a given time, in common Ellipſe *AYBZ*, [*Fig.* 10.] will d from that in the *Caſſinian* Orbit *ADBE*, deſcri with the ſame Vertices *A*, *B*, and the ſame *F*, G, as well in regard of the true Anomal Angle made with *AB* at G, as of the diſtance f the Sun, in reſpect of the diſtance of the Aphel *GA*. And tho' in Orbits, where the Excentri is very ſmall, the difference is not very ſenſib where the Excentricity is great, things are oth wiſe. In the former caſe, the Circle *AQB* deſcribed upon the diameter *AB*, almoſt ſatis the Phænomena. Yet all Aſtronomers now days acknowledge that a circular Orbit is broad about the mean Longitudes, as *Q* and *X*. But this *Caſſinian* Orbit *ADBE*, on the contrary is much too narrow in thoſe places. Suppoſe for inſtance, *AYBZ* to be the common Ellipſe, whoſe greater axe is to the leſs as 5 to 4; let a *Caſſinian* Orbit *ADBE*, be deſcrib'd upon the ſame Vertices and Foci; its leſſer axe *DE* will be leſs than the leſſer axe *YZ* of the true Elliptic Orbit, above a third part, whereas in the mean while *QX* the breadth of the Circular Orbit exceeds *YZ* by a leſſer part; namely only by a fourth part. Therefore this Orbit, ſince Phyſical reaſons don't ſuit with it, (becauſe it is intirely impoſſible to be deſcribed after the manner that *Caſſini* its Author requires, namely ſo as that the Angles at the other Focus, than where the Sun is, may be proportional to the Times; for thus the area deſcribed by the radius would not be proportional to the Time: Nay, to deſcribe it at any rate, a Centripetal force towards the Sun is neceſſary that is altogether different from that which is uſed by nature, which, like every other natural Virtue, propagated from a Center or to a Center, in right lines thro' the circum-ambient ſpaces, is reciprocally as the ſquare of the diſtance,) and ſince the Orbit does not agree with the Phænomena and Celeſtial Obſervations, by reaſon of the ſhortneſs of the leſſer axe, and the narrowneſs of the middle Orb ariſing from thence; it ought, I ſay, to be thrown out of Aſtronomy.

Since the celebrated Mr. *Caſſini*, in his before-mention'd Treatiſe, propoſed to Aſtronomers this Curve, as the Orbit of a Planet, there have been various diſputes about the nature thereof, and the Law of Gravity requiſite to deſcribe it. In my ſecond thoughts upon it, its different Species, and ſome properties of them not ſufficiently known, preſented themſelves to me.

'Tis ſufficiently known that this is the nature of this Orbit. If from two given points *F* and *G* [*Fig.* 11.] to any point *H* of the Curve, the right lines *FH*, *GH* be drawn; the rectangle under *FH*, *GH* is equal to a given ſpace. The right line *FG* produced both ways till it meets the Curve, ſhows the Vertices *A* and *B*; and *AB* is the principal Axe; and the point *C* which is the middle between the Vertices, is the Center of the Figure, and *DE* thro' *C* perpendicular to *AB*, the leſſer axe; and the points *F* and *G* the Foci.

In this Figure if the leſſer axe exceed the diſtance of the Foci, the Curve terminating the Figure is every where concave towards the Center, as it is commonly taken to be. *If*, the principal Axe remaining the ſame, the diſtance of the Foci is leſſen'd, the leſſer Axe will be encreaſed, but it continues leſs than the Axe of the Ellipſe deſcribed upon the ſame principal axe and the ſame Foci; till at length the Foci meeting it becomes equal to the greater axe, and the Figure turns into a Circle. But if, on the contrary, the diſtance of the Foci be increaſed, the leſſer axe will be leſſened, and will become equal to the ſaid diſtance, ſince the diſtance is to the principal axe as unity to a mean proportional between unity and three.

If the diſtance of the Foci be farther encreaſed, the leſſer axe will ſtill be leſſened, and the Curve will be no longer concave towards the Center but convex, at the extremities of this axe, as in *Fig.* 12; till the diſtance of the Foci being ſo far increaſed, that it is to the greater axe as the Side of a Square to the Diagonal of the ſame, the leſſer axe will become nothing, and the Curve reach to the Center on each ſide.

If the diſtance of the Foci be greater than in the ſaid ratio, the leſſer axe is impoſſible, and the Figure turns into two conjugate ones, as in

Fig. 13. which, upon the increaſe of the diſtance of the Foci, will be leſſened, till at length the Figure run into two conjugate points.

The diſtance of the Foci increaſing, the two conjugate Figures do again emerge, and they increaſe after the ſame manner as they decreaſed before, differing from the former in the order of the Foci and of the Vertices, and are increaſed till they become infinite. And afterwards this Syſtem will again approach to a Circle by the ſame degrees as it receded from it.

From what has been ſaid, 'tis evident at firſt ſight, that this Figure is by no means fit to be the Orbit of a Planet. Not to mention the caſes wherein it paſſes into two conjugate Figures, and lays aſide the nature of an Orbit, namely whereſoever its Excentricity is ſo great, as Comets (if they revolve round the Sun like Planets, as is moſt probable) require to deſcribe their courſe: to paſs over theſe caſes, I ſay, there are alſo in thoſe caſes, where it returns into it ſelf and makes an Orbit, ſome with an Excentricity ſo large, as that he Curve near the points *D* and *E* [*Fig.* 12.] becomes convex towards the Sun; and therefore he Planet would need a Centrifugal force from he Sun to deſcribe this part of its Orbit, wheres in the mean while, in the nearer and more remote places *B* and *A*, a Centripetal force towards he Sun is requiſite. That is, that the circumſolar bodies may be moved by that Law, it muſt be ranted, that in equal diſtances from the Sun, ere the force is centripetal, but there centrifugal; which, how foreign it is to all the Laws of Na- ure, any body may eaſily ſee. And tho' the Ex- entricity of no Planet be ſo great, yet 'tis well nown to Geometers, that of a Figure, all whoſe pecies beyond a certain limit are unfit for the iſcharging any office in nature, the remaining

Species on this ſide of that limit, can't be admitted as fit for that office. 'Tis neceſſary *that this Caſſinian Curve be thrown out of Aſtronomy*, not only upon the account of the reaſons mention'd before in this Propoſition, namely, *becauſe it does not agree with Celeſtial Obſervations, by reaſon of the ſhortneſs of the leſſer axe, and no Phyſical reaſons anſwer it, ſince to deſcribe it there would be need of a centripetal force toward the Sun, intirely different from that made uſe of by Nature*; but becauſe of its abſolute impoſſibility. For it is impoſſible that any Species of this Figure can be deſcrib'd by a Planet, ſo as that the angles at the other Focus, where the Sun is not, may be proportional to the times; for thus the area deſcribed by the radius that carries the Planet along would not be proportional to the Time. For 'tis not true that increaſing the angle at one Focus by equal increments, the increments of the area made at the other at the ſame time will alſo be equal, as I thought not long ago.

In the *Figures* 12 and 13, the greateſt breadth of the Figure is found, by deſcribing a Circle upon the Center C thro' the Foci; for it will cut the Curve in the points ſought L, L. And the greateſt Ordinate KL is a third proportional to the right lines GF and FD, in the firſt of them, or a fourth proportional to the right lines GF, GA, and AF in both.

If DE remains in being, the Ordinate from the Focus FP is equal to the leſſer Semi-axe CD, when the leſſer axe is to the diſtance of the Foci, as the Side of a Square to its Diameter. If the diſtance of the Foci be greater than in that ratio, FP will exceed CD.

SECTI-

SECTION II.

Concerning the determining the Orbit of the Earth, and the Theory of the Earth ſeen from the Sun, or of the Sun ſeen from the Earth.

PROPOSITION IX.

TO *define by obſervation, the Species of the Orbit of the Earth and the Poſition of its Axe*; *as alſo the eriodic Time of the Earth.*

One deſign of this Book being to determine the pecies of the Orbits of the Primary Planets about the Sun; the Orbit we ought to begin with s that of the Earth about the Sun, as well becauſe t is the ſame with that which the Sun ſeems to eſcribe in the ſpace of a Year about the Earth, s eſpecially becauſe the Species and Poſition of his is neceſſary to determine the ſame things in he reſt of the Planets. For ſince the Eye of the bſerver is carried along together with the Earth, he Way of the Earth ought firſt of all to be nown, if you would be well acquainted with the ays of the other Planets.

Let the apparent motion of the Sun be obſer-ed in the Ecliptic, when he is ſwifteſt; that is, hen in a given ſpace of time it deſcribes the eateſt arc toward the Eaſt; for in that caſe (by r.2.*Prop.*41.*B.*1.) the Earth is in the Perihelion P ig. 14.] but when its motion is found to be ſlow- t the Earth is in the Aphelion *A*. But the place the Earth ſeen from the Sun is exactly oppoſite the place of the Sun obſerved from the Earth, d therefore the place in the Ecliptic of the A-helion and Perihelion of the Earth ſeen from the Sun is had; that is, the Poſition of the line of the Apſides of the Earth. But in Book 2d we ſhew'd how the places of the Fix'd Stars in reſpect of the Ecliptic, that is, the cardinal and other points of the Ecliptic might be determined in reſpect of the Fix'd Stars, from whence the Poſition of the line of the Apſides of the Earth among the Fix'd Stars will be found. The ſame may be done by biſecting the angle contained between two right lines drawn to two places, where the apparent motions of the Sun from the Earth performed in a given time are equal; for ſuch points are equidiſtant from the points of the Aphelion and Perihelion. To find the Species of the Orbit, you muſt obſerve (by *Cor.*3.*Pr.*41.*B.*1.) that the apparent motion of the Earth in the Aphelion *A* ſeen from the Sun in the Focus *S*, is to the apparent motion of the ſame in the Perihelion *P*, as SP^q, to SA^q. And ſince the apparent motion of the Earth ſeen from the Sun, is the ſame with the motion of the Sun ſeen from the Earth; from the obſervation of this, while the Earth is in the Aphelion and Perihelion, the ratio of SA^q to SP^q will become known, and conſequently its ſubduplicate SA to SP; by compounding and dividing them, you will have the ratio of $SA+SP$ to SP, and $SA-SP$ to SP; or of AP to SP, and (ſuppoſing F to be the other Focus) of SF to SP, and conſequently the ratio of AP to FS is alſo given,

The ſame things may be determined from the obſervation of the apparent Semidiameter of the Sun: For the Earth is then in the Aphelion, when the apparent Semidiameter of the Sun is the leaſt; and conſequently the place of the Sun oppoſite to the place then obſerved, is the place of the Aphelion ſeen from the Sun: and after the ſame manner of the Perihelion. The middle point alſo between the two points, where the Sun is of the ſame apparent diameter, ſhews the place of the Aphelion or Perihelion of the Earth; all which are too plain to inſiſt upon.

To determine the Species of the Ellipſe, let *DI* be the diameter of the Sun ſituated in the Focus *S*. And the angle *DAI* is to the angle *DPI*, as *SP* to *SA*; but the two firſt terms of this Proportion are given; therefore the ratio of *SP* to *SA* is known: From whence, as before, the ratio ſought of *AP* to *FS*, determining the Species of the Ellipſe, may be drawn. Inſtead of the apparent diameters or ſemidiameters of the Sun, meaſured by the Angles or Arcs, the Tangents of the angles *DAS*, *DPS*, of the apparent ſemi-diameters may be taken, as being evidently more exact.

Beſides, the ſpace of time between the moments immediately following, wherein the Earth was at the ſame Apſis, to be determined from what has been ſaid in this Propoſition, is the periodic Time of the Earth. But this will be better determined, if ſuch moments are uſed as are at the greateſt diſtance from one another as may be, and the intermediate time be divided by the number of the revolutions the Earth has made in the mean while about the Sun; for then the Time of one revolution of the Earth will come out.

PROPOSITION X.

To determine the Time of either Equinox by Obſervation.

Juſt about the Equinox obſerve (by *Prop.*18.*B.*2.) the Sun's declination, which if it be nothing at all, will then be the Time of the Equinox itſelf: But if any thing, then from thence (by *Prop.*20.*B.*2.) the place of the Sun in the Ecliptic will be found. Let the ſame be repeated again, or even a third time, at ſome diſtance of time between. And by the help of the places thus found and of the intermediate time, the Time of the Equinox itſelf will be found.

In the performing this and ſuch like Problems, the obſervations of the Sun are to be corrected, (as always) by the Parallax and the correſponding Refraction, and all other things are to be cautiouſly avoided, that may render the calculation uncertain; *v. g.* not two obſervations made on the ſame ſide of the Time which is to be determined; but rather two obſervations, whereof one precedes and the other follows the Equinox, are to be made choice of.

PROPOSITION XI.

T*O determine the Time of either Solſtice.*

Becauſe the Tropic touches the Ecliptic in the Solſticial point, the 15th Figure will repreſent the little portion of the Tropic and Ecliptic near the Solſtice; where *T R* is the Tropic; *E C* the Ecliptic touching it in the Solſticial point *S*; and the little right lines *d l*, *d l* perpendicular to *T R*, will repreſent the deviations of the points of the Ecliptic *l*, *l* from the Tropic, or the changes of their Declinations: In which caſe, the little right lines *d l*, *d l* are (by *Prop.* 24. *Book* 1.) as the ſquares of the Arcs *S l*, *S l*, or of the right lines *S d*, *S d*, being nearly equal to them reſpectively. On which account a ſmall portion of the Ecliptic near the Solſtice *S*, will not ſenſibly differ from the Parabola *E S C*; that is a Figure, whoſe property it is to have *d l* every where as the ſquare of the right line *S d* reſpectively. Again, the Arcs *S l*, *S l* are as the times wherein they are deſcribed by the Sun: For, ſince the Apſides of the Elliptic Orbit, which the Sun deſcribes as to ſenſe, continue at this time pretty near the Solſticial points, the apparent motion of the Sun will be equable; that is, the arcs *S l*, *S l* will be as the times wherein they are deſcribed. From whence it may be fairly inferred, that a portion of the Ecliptic near the Solſtice (ſuppoſe that portion which the Sun deſcribes in about ten days, five before and as many after the Solſtice,) does not at all differ from the Parabola *S ll*, [*Fig.* 16.] in which the Abſciſſa's *SP*, *SP* are what the Declination of the Sun every moment wants of the greateſt Declination; conſequently their differences *PP*, *PP* are as the differences of their Declinations themſelves; that is, as the intervals of the Altitudes of the Sun above the Horizon, (or of the diſtances of the ſame from the Vertex) in the Meridian or any other vertical: And the Ordinates *P l*, *P l* are reſpectively as the times wherein the Sun acquires the foreſaid Declinations, and the intervals of the one are as the intervals of the other.

Theſe things being demonſtrated, let a Gnomon *A B* [*Fig.* 17.] be erected, whoſe ſhadow made by the Sun when 'tis near either of the Solſtices and received upon the Plane *C D* (to which a right line *B H*, connecting the Sun and the Vertex of the Gnomon *B*, is almoſt perpendicular) marks out the point *H*: Some few days after (for inſtance, three or four) obſerve again the point where the ſhadow of the extremity *B* falls, when the Sun is upon the ſame Vertical; and let that be *F*: And let the like obſervation be repeated again, and the point obſerved be *G*: Now from theſe three points *H*, *F*, *G* being given, the time of the Solſtice itſelf is to be found. 'Tis evident that the right lines *HF*, *GF* are the intervals of the diſtances of the Sun in the ſame Vertical from the Horizon, or from the Vertex; (that is, by what has been ſhewn above, the changes of the Declination.) For *FGH* does not in the leaſt differ from an Arc of a Circle deſcribed upon the Center *B* with the diſtance *BG*, becauſe the Angles *FBG*, *HBG*, are very ſmall. And if inſtead of the Plane *CD*, to which *BG* is perpendicular, the ſhadow ſhould be taken upon any other Plane *CE* inclined to *GD* by the angle *DCE*, not a very great one; for inſtance, in the points *h*, *f*, *g*; the right lines *hf*, *gf* will have the ſame ratio to one another, as *HF*, *GF*, becauſe the right lines *fF*, *gG*, *hH*, are almoſt parallel; ſince they meet only in the point *B*, which is at a diſtance great enough in reſpect of the diſtances *fF*, *gG*, *hH*. The Problem therefore for finding the Solſtice from the Points *F*, *G*, *H* given, together with the moments of Time, wherein the Sun, placed in the ſame Vertical, caſts the ſhadow of the point *B* to them, is reduced to the following Problem, which is purely Geometrical.

The diſtances AT, TE, [Fig. 18.] *of three parallel right lines* AB, TY, ES, *lying in the ſame Plane being given; to deſcribe the Parabola* KLM, *whoſe Axis* CVD *is parallel to* AB, TY, ES, *that cuts them in the points* K, L *and* M, *ſo as that, drawing the Ordinates to the Axe,* KH, LF, MG, *the portions* FG *and* FH *of the axis intercepted by them, may be equal to given right lines.*

For by this means, (if *FG* and *FH* are equal to *FG*, *FH* marked out by the obſervation deſcribed above, and the ratio of *AT* to *TE* the ſame with the ratio of the Time between the obſervation of the points *H* and *F*, to the Time between the obſervation of the points *F* and *G*) *KH*, *LF*, *MG* will repreſent the Times between the obſervations of the points *H*, *F* and *G*, and the moment itſelf of the Solſtice; that is, as the points

A,

A, *T*, and *E* repreſenting the moments of the Time wherein the Shadow of the Gnomon was obſerved in the points *H*, *F*, and *G*, the point *C* will repreſent the moment of the Solſtice itſelf. Call the Line *AT*, a; *TE*, b; *FH*, c; *FG*, d; *TC*, x. Therefore AC or $KH = a + x$, and EC or $GM = b - x$. Call the principal *Latus rectum* of the Parabola, r. Then from the nature of the Parabola $x^2 = VF \times r$, and therefore $VF = \frac{xx}{r}$. After the like manner $VH = \frac{a^2 + 2ax + x^2}{r}$, and $VG = \frac{b^2 - 2bx + x^2}{r}$. Wherefore $c = (FH = VH - VF =) \frac{a^2 + 2ax}{r}$; and $d = (FG = VG - VF =) \frac{b^2 - 2bx}{r}$. And getting away r, by the help of theſe two Equations; you will find $\frac{a^2 + 2ax}{c} = \frac{b^2 - 2bx}{d}$. And ordering the Equation as is uſual, x will be $= \frac{b^2c - a^2d}{2ad + 2bc}$; that is, the ſpace of time between the known moment repreſented by *T*, (namely, when the ſhadow of the Gnomon was obſerved in *F*,) and the moment of the Solſtice ſought; from whence the time of the Solſtice itſelf will be given. *Q. E. F.*

But if the time between the obſervation of the Shadow in *H*, and the Solſtice itſelf be ſought, add *AT* to *TC* found, and the ſought $HC = (a + x =) \frac{b^2c + 2abc + a^2d}{2ad + 2bc}$. After the like manner will EC be $= (b - x =) \frac{b^2c + 2abd + a^2d}{2ad + 2bc}$. But if the order of the obſervations be ſuch, as that the obſervation of the Shadow of the Gnomon in *F* is exactly

actly in the middle between the obſervations of the ſame in H and G; that is, if $AT = TE$, or $a = b$, the Equation will be more ſimple; for the the diſtance of the Times of the Solſtice and of the Shadow obſerved in F will be equal to $\frac{ac - ad}{2d + 2c}$ the diſtance of the Solſtice from the time of the obſervation of the Shadow in H equal to $\frac{3ac + ad}{2c + 2d}$ and from the moment of the obſervation of the Shadow in G equal to $\frac{ac + 3ad}{2c + 2d}$; that is, in the Figure, as $2FH + 2FG$ to GH or $4HF - 2GF$ to GH, or $2HF - GH$ to $\frac{1}{2}GH$, ſo is the half of the time between the firſt obſervation of the Shadow in H and the laſt obſervation of the ſame in G, to the time between the middle obſervation of the Shadow in F and the moment of the Solſtice itſelf ſought.

The ſagacious Mr. *Edm. Halley* firſt found out and publiſhed this method of determining the times of the Solſtices, in the *Philoſph. Tranſact.* for the beginning of the Year 1695; and illuſtrated it by the example of two Solſtices determined by this, from the Obſervations of *Walther* and *Gaſſendus*; where he takes notice of ſeveral things well worth obſerving in the Practice, concerning the ſituation and advantageous form of the Gnomon, and concerning the accurate obſervation of the Shadow, and other things.

PROPOSITION XII.

TO define the quantity of the Tropical Year.

Since the Tropical Year is that ſpace of time wherein the ſame Seaſons of the Year return again; that is, wherein the Sun departing from any point of the Ecliptic (for inſtance that wherein

touches the Tropic) returns to it again; 'tis
ent that its quantity will be determined, if
un be twice obſerved in the ſame point of
Ecliptic, (for inſtance, twice in the ſame E-
ox, by *Prop.* 10, or in the ſame Solſtice, by the
,) and the intermediate time be divided by the
ber of revolutions of the Earth about the
made in the mean time. If there be any
r in determining the moment of the Equinox
olſtice, that it may not diſorder the meaſure
he Tropical Year, 'tis proper to make uſe of
Equinoxes or Solſtices as are at the greateſt
nces from one another, ſo that the error di-
d into ſo many parts (by the diviſion made by
eat number of intervening Revolutions)
become inſenſible.

ut becauſe the Times of the Solſtices ſeem'd
cult to the Ancients to be diſcerned, as *Ptole-*
nimſelf ſays, who therefore paſſed over the
tices of *Meton* and *Euctemon* and *Ariſtarchus* him-
; which alſo ſeem'd ſo true to the Moderns,
Ricciolus aſſerted that *Ptolemy* was too confident
oping that neither himſelf nor *Archimedes* had
d ſo much as a quarter of a day in their obſer-
ion or computation in the Solſtitials; and
velius, *Chap.* 4. *Prod. Aſtron.* ſays, *That the Solſti-*
tho' they be obſerved by the beſt and greateſt Inſtru-
ts, and even the moſt experienced Obſerver, can ne-
be determined in their ſmalleſt parts: Aſtronomers
e made uſe of the Obſervations of the Equi-
xes, whoſe moments they thought more accu-
ely determinable, on the account of the vaſt
d ſenſible change of the Declination of the Sun
out the times of the Solſtices. But, by the
p of the preceding Method, the times of the
lſtices themſelves may hereafter be determined,
leaſt as accurately as thoſe of the Equinoxes.

Whereas the Sun ſeems to move ſometi ſlower and ſometimes ſwifter, (according to different velocity of the Earth about the Sun, inſtead of that, it be imagined to move equab ſo as to deſcribe the Ecliptic in the time det mined by this Propoſition; ſuch a motion is v fitly called its *Mean Motion*, (being really ſuch one between the ſwifteſt and ſloweſt,) and place the Sun is thus in, is called its *Mean Place.*

PROPOSITION XIII.

TO *determine the quantity of the Sydereal Year.*

By comparing the place in the *Ecliptic* a Fix'd Star was formerly in, with its preſent place, find the Annual preceſſion of the Equinox; that is, how much each point of the Ecliptic recedes from a given Fix'd Star towards the Weſt in the ſpace of one Year, which was done in *Book* 2. *Prop.* 31: And by the preceding Propoſition determine the ſpace of Time requiſite for the Sun to deſcribe an arc of the Ecliptic equal to the Preceſſion thus found. This therefore added to the quantity of the Tropical Year, gives the quantity of the Sydereal Year, which conſiſts of the Tropical Year, wherein the Sun departing from a point of the Ecliptic returns to the ſame, that has in the mean while moved a little way forwards to meet it, and of that beſides wherein the Sun deſcribes that arc thro' which the ſaid point of the Ecliptic moved to meet the Sun; that is, wherein the Sun departing from an immovable point (for inſtance, a Fix'd Star) ſeems to return again to it.

PRO-

PROPOSITION XIV.

hree places of the Sun, determined by Prop. 20. B. 2. *and the Periodic Time of the Earth, which nearly equal to the Syderial Year, determined by* d. Prop. *being given, to find the Species of the f the Earth, the Situation of the line of the ps. and the Time when the Earth is in the Aphe-ion.*

The methods laid down in *Prop.* 9. for finding he Species and Poſition of the Orbit of the Earth, require ſuch accurate obſervations that they are hardly fit to be put in practice: We ſhall therefore add the following one better fitted for Aſtronomic uſes. Let there be given three Places of the Sun by obſervation, namely when the Earth is in the hree points *B*, *C*, *D* of its Orbit [*Fig.* 19.] Therefore he three lines *SB*, *SC*, *SD* are had by poſition, or the three angles *BSC*, *CSD*, *BSD*. And the Period of the Earth being given, together with he Times between the obſervations, the ratios between the whole area of the Ellipſe *ACPB* and its parts *BCS*, *CDS* and *BDS* are alſo given. The Problem therefore is reduced to this, to deſcribe ſuch an Ellipſe upon a given Focus *S*, as that its whole area ſhould be to its parts contained under the right lines given by poſition, *SB*, *SC*, *SD*, and the Curve of the Ellipſe, in given raios.

Let *AP* then be the Orbit of the Earth, *S* and *F* its Foci, *A* its Aphelion, *P* its Perihelion. On he Center *S* imagine a Circle deſcribed by a radius equal to the greater Axe, the Circumference whereof cutting the right lines *SB*, *SC*, *SD* produced in *R*, *Z* and *Y*. Join the right lines *FB*, *FC*, *FD*, *FR*, *FZ*, *FY*; and produce one of them *ZF*, untill it meet the Circumference again in *G*, and join *RG*, *GY*, *YZ*, *ZR*.

If $ACDPB$ be the true Orbit of the Earth, then (by *Prop.* 5.) the Angles BFC, CFD, BFD, will be reſpectively to four right angles nearly a the Times between the obſervations to the Periodic Time: And on the contrary, if the ſaid ratio hold in theſe angles, $ACDPB$ is the Orbit of the Earth ſought. What remains therefore to be done is, to find the Poſition of the right line SF or AP, and the Species of the Ellipſe, by having given, beſides the angles BSC, CSD and BSD, the ratios alſo of the angles BFC, CFD and BFD to four right angles.

Since there are given the angles RSZ, $ZS\Upsilon$, $RS\Upsilon$, namely the obſerved differences of the Longitudes, and the angles BFC, CFD, BFD, namely the Mean motions of the Earth almoſt for the ſpaces of the times between the Obſervations; therefore there are alſo given RFZ, $ZF\Upsilon$, $RF\Upsilon$ Arithmetical Proportionals between BFC and RSZ, CFD and $ZS\Upsilon$, BFD and $RS\Upsilon$ reſpectively. For ſince SR is equal to AP, by conſtruction, and the ſum of the right lines SB and BF equal to the ſame, (by *Prop.* 52. *B.* 3. *of the Conics of* Apollonius) BR and BF will be equal; and after the like manner may CZ, CF, as alſo the right lines $D\Upsilon$, DF, be ſhewn to be equal. Conſequently BRF and BFR, CZF and CFZ, $D\Upsilon F$ and $DF\Upsilon$ will be equal. But the angle $BFC = RFZ + BFR + CFZ$; and $RFZ = RSZ + BRF + CZF$. Since therefore BFC exceeds RFZ as much as that does the angle RSZ; 'tis plain that theſe are Arithmetically proportional. After the ſame manner are CFD, $ZF\Upsilon$, $ZS\Upsilon$; alſo the angles BFD, $RF\Upsilon$, $RS\Upsilon$ are Arithmetically proportional. Beſides, in the Triangle ΥFG all the Angles are given; namely ΥFG equal to the complement of the angle ΥFZ, to two right angles a mean Proportional between CFD and CSD, which are known, and *ϒGF* half the obſerved Angle *ZSϒ*: And therefore ſuppoſing *ϒF* to be of any number of parts whatever, *FG* will be given in thoſe parts. In the Triangle *RFG* all the angles being given, (namely *RFG* the complement of *ZFR* to two right, and *FGR* half the angle *ZSR*) and the ſide *FG*, *FR* will be given. And in the Triangle *ϒFR* there being given *Fϒ* and *FR*, and the angle *ϒFR*, (becauſe a mean Arithmetic proportional between *BFD* and *BSD*,) *ϒR* will be given and the angles *FRϒ*, *FϒR*. Then, in the Iſoſceles Triangle *ϒSR* all the angles are given (becauſe *ϒSR* at the vertex of the Triangle is given) and *ϒR*, therefore *ϒS* alſo will be given. Laſtly, in the Triangle *FϒS* the angle *FϒS* is given (the difference between the known ones *RϒS*, *RϒF*) and the ſides *Fϒ*, *ϒS*; therefore the angle *ϒSF* and ſide *SF* will be found. But the Poſition of the right line *ϒS* is given, namely the Place of the Sun in the third Obſervation, when the Earth was in *D*; therefore the Poſition of the line of the Apſides *AP* will be given, becauſe inclined in a known angle *ϒSF* to *ϒS* given by poſition. But in the ſame parts, in which *ϒS* is expreſſed, the right line *SF* is found; therefore the ratio between *ϒS* or what is equal to it *AP* the greater Axe of the Orbit, and *SF* the diſtance of the Foci is given; that is, the Species of the Orbit. Nor can any thing elſe be determined by the Obſervations: For every Elliptic Orbit ſimilar to *ACDP*, deſcribed upon the Focus *S*, whoſe greater Axe is the ſame in poſition with the right line *SF*, will ſatisfy the Obſervations propoſed.

In the Triangle *DFS*, the ſide *SF*, the angle *DSF*, and the angle *SDF*, the double of *SϒF* lately found, being given; *DFS* is found, and conſequently *DFA* contiguous to it. But the

Time which is to the whole Periodic Time as the angle *DFA* to four right, is that which happens between the moment of the obſervation in *D*, and the moment that the Earth is in its Aphelion: this therefore becomes known.

The method of determining the Species of the Orbit, and the Poſition of the Line of the Apſides here delivered, agrees as well to any other Planet revolving about the Sun as to the Earth, provided there are but three Obſervations, as much to the purpoſe in this other Planet, as thoſe three uſed about the Earth.

PROPOSITION XV.

TO correct at pleaſure the Poſition of the Line of the Apſides, and the Species of the Orbit determined above.

Let the Poſition of the line of the Apſides and the Species of the Orbit found by the foregoing, be taken for true and accurate, (for they do not differ much from thence by *Prop.* 5.) from them and the Periodic time and the times between the obſervations being given, find the angles corrected at *F*, agreeable to the times of the obſervations made in the points *B*, *C* and *D*; and if you will uſe only the firſt correction (namely *Bullialdus*'s,) this may be done by *Prop.* 7; and you may go to what exactneſs you pleaſe by *Prop.* 4. By the help of theſe angles thus corrected, inſtead of the former, that are ſimply proportional to the times; that is, in the language of Aſtronomers, by the help of the angles of the mean Anomaly thus equated, inſtead of the angles of the mean ſimple Anomaly made uſe of before, let the calculation of the former Propoſition be repeated, and let the Poſition of the Line of the Apſides and the Species of the Orbit be found, and they will be more accurate.

COROL-

COROLLARY.

From whence the Motion of the Apſides of the Orbit of the Earth may be determined: Namely by comparing of the Place of the Aphelion at preſent with its Place ſome Years ago, (both which are found by *Prop.* 14, and corrected by *Prop.* 15,) and by dividing the arc made in the mean while by the ſaid Aphelion by number of the intervening Years, the Annual Motion of the Aphelion will come forth, which alſo will be extended to parts of a Year.

PROPOSITION XVI.

TO *find the Place of the Earth ſeen from the Sun, and its Diſtance from the Sun at any given Time.*

The ſpecies of the Orbit of the Earth, and the place of the Aphelion for any known time being defined by the two laſt Propoſitions, find the place of the Aphelion for the time given: And if it be ſince the time to which the place of the Aphelion was determined, add the motion of the Apſides made in the mean while, and defined by the Corollary of the preced. *Prop.* to the place of the Aphelion determined before, or let it be ſubtracted, if the time given be before; and you will have the Place of the Aphelion for the given time. Then let the time when the Earth once was in the Aphelion be determined (by *Prop.* 14,) and ſince by the 13th the Periodic time of the Earth may be known; from thence alſo will be known the time when the Earth was in its Aphelion next before the Time propoſed, and conſequently the ratio between the Periodic time of the Earth and the time elapſed ſince the laſt Aphelion is known; that is, in the Figure *ALP* (*Fig.*1.) repreſenting the Orbit of the Earth, the ratio of the intire Ellipſe to the area *ALS*: And conſequently (by *Prop.* 3, 4, 6 and 7.) the angle *ASL* will become known.

And ſince the poſition of the right line AS is known, the poſition of the right line SL will alſo be known; that is, the Place of the Earth ſeen from the Sun. Beſides, by the Propoſitions above cited, the ratio of the right line LS to the right line AP becomes known; that is, the Diſtance of the Earth from the Sun in the parts of the greater Axe.

PROPOSITION XVII.

TO *explain the inequality of the natural Days, and to ſhew the Equation of time.*

Since it is the [illegible] of an Aſtronomer to ſhew the motion of th[illegible]s, and this is performed in Time, and can't be conceived without it; and ſince the moſt ſenſible part of Time is a natural Day, by the repetition of which a Year, and by the diviſion of which an Hour and the parts of an Hour are made; and ſince in computing their motions, all days ought to be ſuppoſed equal; as far as this ſuppoſition is from truth, ſo far even the Celeſtial motions taken out of the Tables will diſagree with the Phænomena, if no account of this inequality be taken. To underſtand this, let it be obſerved, that a Natural day is that ſpace of time, that intervenes between the Sun's leaving a given Meridian in the Heavens and returning to it again; that is, the ſpace wherein a revolution of the whole Celeſtial Equator is performed, and over and above, of that part of it, that anſwers to ſuch a portion of the Ecliptic, as the Sun deſcribes in the mean while by its Annual motion towards the Eaſt. But becauſe this portion of the Equator, added to the whole Equator, is not every where equal, as well on the account of the Obliquity of the Ecliptic, as that the apparent Annual motion of the Sun about the Earth is not equable; neither will theſe Natural days be equal: And would therefore then be equal, if the e Sun ſeen from the Earth in its Annual motion oved both equably and in the Equator; and is evident in this caſe, after that the whole E-uator has paſſed the Meridian, there remains hat portion ſtill to paſs, which the Sun in the ean while would deſcribe by its Annual motion owards the Eaſt, which portion would always e equal to itſelf, ſince we ſuppoſe this motion o be equable; and therefore even the Time mea-uring the paſſage by the Meridian of the whole quator and this additional part, would be al-vays equal to itſelf. Let us therefore ſuppoſe a-other Sun, as was ſaid before, moving in the quator, and conſequently meaſuring the equal r mean Time: 'Tis evident that the difference f the Time, between the appulſe of the true and maginary Sun to the Meridian, or any other Hour Circle, is that which makes the mean Noon f the apparent Noon, and the apparent of the ean; that is, the *Equation of Time* itſelf. And ecauſe the true Sun and that point of the Equa-or, where its Right aſcenſion ends, arrive at the eridian together; and the Equation of Time that ſpace of Time, that flows while the Arc of e Equator contained between the extreme point f the Right aſcenſion of the true Sun and the place f the imaginary Sun, paſſes the Meridian; if is Arc be converted into time, you will have e Equation of time ſought. Beſides, the place of e imaginary Sun is as far diſtant in the Equator om the point of the Vernal Equinox, as the mean ace of the Sun in the Ecliptic is from the me: And therefore the foreſaid arc of the Equa-r, the meaſure of the Equation of the time, is qual to the difference between the Right aſcen-on of the true Sun and the diſtance of the mean ace of the Sun from the Vernal Equinox. But is commonly known, that the difference of any two Quantities *A* and *B* is equal to the difference between *A* and any aſſumed quantity *X*, and to the difference between this *X* and *B* together. Aſſuming therefore a third arc, namely the diſtance of the true place of the Sun from the Vernal Equinox, the Arc meaſuring the Equation of Time will be the ſum of the two arcs, one of which is the difference between the Right aſcenſion of the Sun and its true place in the Ecliptic, the other the difference between the true and mean motion of the Sun. But ſometimes it happens, that the difference of the two Quantities is equal to the ſum of them taken ſimply; namely when the one is negative and the other poſitive: Therefore an Arc to be converted into Time, *to produce* the Equation of Time, is always the ſum or difference of two arcs, one of which is the difference of the true Motion of the Sun and its Right aſcenſion, and the other is the difference of the true and mean Motion of the Sun.

But becauſe this Arc conſiſts of two parts, we will conſider them ſeparately. And firſt let us abſtract from the unequal motion of the Sun under the Ecliptic; that is, let us conſider it a little while as equal; and therefore the motion of the imaginary Sun in the Equator will be equal to the motion of the true Sun in the Ecliptic, or the true and imaginary Sun will be equally diſtant from both Equinoxes. And becauſe in the firſt Quadrant of the Ecliptic, or from the beginning of ♈ to the beginning of ♋, the diſtance of the Sun from the beginning of ♈ is greater than its Right Aſcenſion; the imaginary Sun will be farther diſtant from the beginning of ♈ toward the Eaſt, than is the Right Aſcenſion of the true Sun, and therefore the true Sun arrives at the Meridian ſooner than the imaginary, becauſe more to the Weſt; that is, the apparent time precedes the mean : hereupon when the Sun is in the firſt Quadrant of the Ecliptic, that part of the Equation of Time, which is the difference of the true Motion of the Sun and its Right Aſcenſion,turn'd into Time is to be ſubtracted from the apparent, to make the mean or equable Time. But when the Sun is in the ſecond Quadrant of the Ecliptic, its Right Aſcenſion is greater than the diſtance of the Sun from the beginning of ♈, (becauſe the complement of the Right Aſcenſion to a Semicircle is leſs than the diſtance of the Sun from the beginning of ♎;) and therefore the imaginary Sun (which, by ſuppoſition, is as far diſtant in the Equator from the beginning of ♈, as the true Sun in the Ecliptic is diſtant from he ſame) is leſs diſtant from the beginning of ♈ than the true Sun; and conſequently arrives t the Meridian ſooner, and therefore the mean Noon precedes the apparent ; that is, the mean Time reckons 12 Hours, when the apparent does ot yet reckon ſo many, tho' it be the ſame moment, but differently called. And therefore in his caſe, when the Sun is in the ſecond Quadrant f the Ecliptic, this part of the Equation of Time which ariſes from the obliquity of the Way of he Sun to the Equator, and is the difference between the Sun's motion and its Right Aſcention urned into Time) is to be added to the apparent, make the mean time, or to be ſubtracted from he mean to make the apparent. But if the Sun s in the third Quadrant of the Ecliptic, the ſame to be done as if it were in the firſt ; for here e diſtance of the Sun from the beginning of ♈ greater than its Right Aſcenſion, as in the firſt : nd the Sun being in the fourth Quarter, the me holds as to this part of the Equation of ime as if it were in the ſecond, and for the ſame aſons.

Let us next conſider the Inequality of natural Days ariſing from the unequal motion of the Sun in the Ecliptic, or the Equation of time ariſing from thence, laying aſide a little while the Obliquity of the Ecliptic; that is, conſidering the Sun as moved after the like manner, and with the ſame celerity in the Equator as it is found really to move in the Ecliptic, and that theſe two Suns the true and imaginary, left the Apogæum together. And ſince the imaginary one moves with the mean Motion, it muſt go before the true one, while it goes forward from the Apogæum to the Perigæum; that is, the imaginary one *is more to* the Eaſt than the true one: The true one therefore arrives at the Meridian *ſooner*; *therefore* the apparent Noon precedes the mean. And the difference between the places of the true and imaginary Sun, or between the true and mean Motion of the Sun, converted into Time, is the Equation of the time ariſing from this inequality of the Sun's motion: This Equation therefore, while the Sun goes from the Apogæum towards the Perigæum, or (in the language of an Aſtronomer,) in the ſix firſt Signs of the Anomaly, is to be ſubtracted from the apparent Time to produce the mean, or added to the mean to produce the apparent out of it. In the Perigæum the true Sun overtakes the imaginary, and from thence to the Apogæum is more to the Eaſt: The imaginary one therefore in all this time, namely in the ſix laſt Signs of the Anomaly, arrives at the Meridian ſooner; that is, the mean Noon precedes the apparent Noon, by as much as the arc between the imaginary and true Sun ſpends in paſſing by the Meridian; that is, as much as that arc, namely the difference between the mean and true motion of the Sun converted into time makes. This part of the Equation of time therefore

fore in the preſent caſe is to be added to the apparent time to make the mean, or to be ſubtracted from the mean to make the apparent.

And when both theſe cauſes of the Equation of Time hold, (as they really do) both theſe things are to be done; that is, to equate the Time you muſt conſider the Sun two ways, *viz.* both in what Quadrant of the Ecliptic it is in, that one part of the Equation may be added to or ſubtracted from the apparent, as was ſaid before; and in what Semicircle of Anomaly, that the part depending upon that, may be added or ſubtracted according as the nature of it requires. And on this account it is, that the Equation of natural Days (which, from what has been ſaid above, abſolutely ſpeaking, is the Difference of the mean place and the right Aſcenſion of the Sun's true place turned into time) is made up of two parts, And becauſe the vernal Equinox and Apogæum of the Sun, the heads or beginnings of theſe parts, don't always keep the ſame poſition to one another, but the former being carried towards the Weſt leaves the latter, (as was ſhewn in *B.* 1.) comes to paſs, that the (abſolute and only) Equation of Time made up of theſe is not perpetual, but uſeleſs in a few Years, the Sun not poſſeſſing the ſame degree of the Anomaly, or the ſame ſituation in reſpect of the Apſides, when it returns to the ſame point of the Ecliptic.

Ptolemy expreſly demonſtrates the above mention'd Equation of Time, *Chap.* 10. *B.* 3. *of Great Conſtruct*, notwithſtanding *Vendeline* and others have ſaid that it is all uſeleſs, and uſe the apparent and mean Time promiſcuouſly. *Kepler* on the contrary ſuſpected a third cauſe of equating the time, ariſing from the diurnal Motion of the Earth's not being intirely equable, but ſometimes quicker ſometimes ſlower, according as it is more or leſs diſtant from the Sun, the original of its motion; and calls the part of the Equation depending upon it, the *Phyſical* Part, leaving the name of the *Demonſtrative* to the other deſcribed above, which is made up of two parts. *Tycho* alſo had a ſuſpicion of the acceleration or retardation of the Firſt Motion, and was for ballancing the other part of the Equation depending upon the Sun's Anomaly, by means of it; for he only made uſe of the other ariſing from the Obliquity of the Ecliptic, which he calls the *Empyric*, becauſe it would ſuffice for the finding by Calculation the Eclipſes he had obſerved. But *Street* thought this Inequality of the Firſt Motion, to be ſufficient for the making uſe of the part neglected by *Tycho*, under a contrary Title, as he has put it in his *Aſtronomia Carolina*.

PROPOSITION XVIII.

T*O correct the Place of the Sun, ſeen from the Earth, found by* Prop. 16.

If the given Time is the mean, the Place found needs no correction; for it is hitherto the true. But if otherwiſe, let the apparent given Time be converted into the mean (by the preced. *Prop.*) by taking the Place of the Sun found above, *Prop.* 16. as accurate: (For that Place of the Sun is really accurate enough for the determining the Equation of Time, ſince it continues very nearly the ſame, whether the Place of the Sun be that which is found above, or the true and moſt correct:) And to this mean, again (by the ſaid 16th *Prop.*) find the Sun's place, and the place thus found will be the correct and accurate one. For the mean Time only is fit for Aſtronomic uſes, on which account it is call'd the *Aſtronomic Time*; this therfore ſhould be made uſe of: But to determine the mean when the apparent is given, the Sun's place is neceſſary; that therefore was firſt to be found, by uſing the apparent Time, as if it were the mean, to find the Equation of the Time. But this once found, and the apparent Time reduced to the Aſtronomic, ſeek for that Time (which is the ſame with the moment propoſed, but expreſſed by a number fitted for an entrance into the Tables) the Sun's place (by the ſame *Prop.* 16.) and the Diſtance of the Earth from the Sun, which is evidently the true and correct one.

The converſe of this Problem may be conſtructed much after the ſame manner, and the Time may by found, when the Sun ſeen from the Earth will be in a given Place.

SECTION III.

Concerning the determining the Orbits of the reſt of the Primary Planets, and compoſing the Theory of them ſeen both from the Sun and the Earth.

PROPOSITION XIX.

TO *determine by obſervation the Poſition of the Line of the Nodes of a Planetary Orbit, and the Diſtance of the Planet itſelf, when in the Node, from the Sun.* [*Fig.*20.]

In the two Figures fitted to the two caſes of a ſuperior and an inferior Planet, let *S* repreſent the Sun; *Tt* the Orbit of the Earth, whoſe Focus is *S*; *Nn* the Line of the Nodes of the Planet propoſed. And firſt let the Earth be in *T*, and from thence let the Planet be obſerved when in the Ecliptic, and therefore in its Node, ſuppoſe *P*: For the Planet will never be ſeen in the Ecliptic from the Earth, being in the Plane of the Ecliptic, unleſs when it is alſo in the Plane of the Ecliptic. After an intire revolution, let the Planet be obſerv'd in the ſame Node, when the Earth arrives at t. Let the right lines ST, Pt, be imagined to be drawn, as alſo St, PT interſecting one another in the point X. In the Triangle STX there are given the angles TSt by the Theory of the Earth and Time between the obſervations, and STP the obſerved Elongation of the Planet from the Sun, and the Side ST the Diſtance of the Earth from the Sun in the former obſervation. Therefore SX is found; and ſo Xt ſince St the Diſtance of the Earth from the Sun in the ſecond obſervation is alſo given: Again, in the Triangle tXP the angle tXP equal to the angle TXS in the former is given, and the angle StP the elongation of the Planet from the Sun in the ſecond obſervation, and the ſide tX lately found; and conſequently the ſide Pt will be found. Laſtly, in the Triangle StP, the ſides tS, tP being given, together with the angle contained PtS, SP the Diſtance of a Planet in a Node from the Sun will be found, and the angle tSP that the Centric Place (or that ſeen from the Sun) contains with St given by poſition, namely, the Place of the Earth ſeen from the Sun at the time of the ſecond obſervation; and the oppoſition to this is the Centric Place of the other Node; that is, the Poſition of the Line of the Nodes Nn.

By this means the Motion of the Nodes of the Orbs of the Primary Planets will be determined, by comparing the Place of the Node of each Planet found out by the obſervations of the Ancients, with the Place of the ſame determined by the obſervations of the Moderns.

After the like manner, if a Planet be obſerved twice in any other point of its Orbit from the Earth, the Place of the Planet ſeen from the Sun and its Diſtance from the Sun are determined.

PRO-

PROPOSITION XX.

TO *define by obſervation the Inclination of any Planetary Orbit to the Plane of the Ecliptic.*

Let *S* repreſent the Sun; the Circle *NBn*
ig. 21.] the Ecliptic among the Fix'd Stars;
d *NAn* the Path of the propoſed Planet, ſeen
m the Sun among them: And the right line
Sn will be the Line of the Nodes known by po-
ion, (by the preced. *Prop.*) Whenever the Earth
ives at the ſaid Line of the Nodes, which it
es twice a Year, the Time of which (by *Prop.*18.)
known; let the Geocentric Place *A* of the Pla-
t *P* be obſerved (by *Prop.* 27. *B.* 2,) whoſe La-
ude is *AB* an Arc perpendicular to the Ecliptic,
d Longitude ♈*B*. Since the Longitude of the
n ♈*N* is known, the difference alſo of theſe
ngitudes *NB* will be known. In the Spherical
riangle *ANB*, right angled at *B*, the Sides *AB*,
B are given; and therefore the angle *ANB*,
e meaſure of the Inclination of the Plane *NAn*
the Orbit of the Planet to the Plane of the
cliptic *NBn*, will be known.

PROPOSITION XXI.

FRom *the obſervation of a Planet in Oppoſition or Conjunction with the Sun, to get away its Second
ıequality and find its Diſtance from the Sun.*

Beſides the true Inequality in the motions of
ıe Planets viſible to an Eye in the Sun, placed in
ıe common Focus of all the Orbits, there is an-
ther Optical one, ariſing from the annual motion
f the Earth, relating to us living on the Earth.
he former, which is proper to the Planets, is
alled by a name proper enough, the *Firſt Inequa-
ity*; but the other is ſecondary, as it were to the
lanet, and therefore juſtly called by Aſtrono-
ners the *Second Inequality*: And he is ſaid to get

it away, who ſhews that Place of a Planet, which an Obſerver would ſee from the Sun. Let *S* (in *Fig.* 22.) repreſent the Sun, *T* the Earth, *P* the Planet in its Orbit, *NB* the Ecliptic among the Fix'd Stars, *NA* the interſection of the Orbit of the Planet with the Sphere of the Fix'd Stars; *N* the Node: And ſeeing the Sun is in the Plane of the Orbit of each Planet, the right line *SN* will be the Line of the Nodes. Let the Earth *T* be ſeen from the Sun among the Fix'd Stars at *B*, and the Planet *P* at *A*. And becauſe the Planet is either in Oppoſition to the Sun, (as in *Fig.* 1ſt) or in Conjunction with it (as in the 2d;) the arc of the great Circle connecting the *points A* and *B*, is the arc of the Circle of Latitude, and conſequently perpendicular to the Ecliptic. And in the Spherical right angled Triangle *ABN* there are given the angle *ANB* the meaſure of the Inclination of the Plane of the Orbit of the Planet to the Ecliptic, (found by the preced *Prop.*) and the ſide *BN*, namely the inclination of the right line *TS* given (by the Theory of the Earth) by poſition to *SN* the Line of the Nodes found by poſition (by *Prop.* 19.) Therefore there will be known the Side *AB*, the Centric Latitude of the Planet, and *AN* the Diſtance of the Planet in its Orbit, ſeen from the Sun, from the Node *N*, which is to get away the Second Inequality of the Planet. Moreover in the rectilineal Triangle *PST* there are given *ST* from the Theory of the Earth, and the angle *PTS* the Latitude of the Planet by obſervation, or its complement to a Semicircle, and the angle *PST* the Centric Latitude of a Planet found above; wherefore *PS* and *PT* the Diſtances of the Planet from the Sun and Earth will be known.

PRO-

PROPOSITION XXII.

BY any ſingle obſervation of a Planet whatever, to clear it of its Second Inequality, and find its Diſtance from the Sun.

Let the Planet *P* (in *Fig.* 23.) be obſerved, the Line of whoſe Nodes is *NS*, from the Earth *T*, with the apparent Latitude *PTB*. Let the Plane of the apparent Latitude be produced, till it cut the Plane of the Orbit of the Planet in *PN*, and the Plane of the Ecliptic in the right line *BTN*. From *P* let fall *PB* perpendicular to the right line *NB*, and from *T* erect *TO* perpendicular to the ſame *NB*, which (by *Prop.* 38. *El.* 11.) will be perpendicular to the Plane of the Ecliptic, becauſe the Plane of the Latitude *PNB* is perpendicular to the Plane of the Ecliptic. From *T* let fall *TE* perpendicular to the right line *NS*, and join *OE*, which is perpendicular to the ſame *NS*; and the Angle *OET* will be equal to the Inclination of the Planes of the Orbit of the Planet and of the Eliptic. In the Triangle *NST*, are given the Side *ST*, and angle *TSN*, by the Theory of the Earth and the known place of the Node, and by obſervation the angle *NTS*, the Elongation of the Planet from the Sun computed in the Ecliptic, or its complement to two right; therefore the Sides *TN* and *NS*, and the angle *TNS* will become known. In the Triangle *TEN* right angled at *E*, *NT* and *TNE* are given; and conſequently *TE*. In the Triangle *OTE* right angled at *T*, there are given *TE* the Side found juſt now, and *TEO* the Inclination of the Planes of the Orbit of the Planet and of the Ecliptic, known by *Prop.* 20. of this Book; therefore *OT* will become known. In the Triangle *OTN* right angled at *T*, there are given *OT* and *TN* found before; and therefore the angle *ONT* alſo. In the firſt angle *PNT*,

PNT, there are given *NT*, and the angle *TNP*, alſo *PTN* the apparent Latitude of the Planet known by obſervation, or its complement to two right; therefore *NP* will be known. In the Triangle *NPB* rightangled at *B*, the ſide *NP* and angle *PNB* lately found being given, *PB* and *NB* will be found. In the Triangle *BNS*, *NB* and *NS* and the angle *BNS* being given, *NSB* the Centric Longitude of the Planet computed from its Node, and the ſide *SB* will be known. Then in the Triangle *PBS* rightangled at *B*, *PB*, *BS* being known, *PS* the Diſtance of the Planet from the Sun will be known, and the angle *PSB* its Centric Latitude. *Laſtly*, in the Triangle *PNS* all the ſides being given, the angle *NSP* is known, the Centric Diſtance of the Planet in its Orbit, computed from the Line of the Nodes given by poſition (by *Prop.* 19.) in its Orbit. The Second Inequality of the Planet therefore is got rid of, and its Place ſeen from the Sun is determined: And beſides the Diſtance of the Planet from the Sun is found in the parts of the mean Diſtance of the Earth from the Sun, which is taken for the meaſure.

PROPOSITION XXIII.

TO *define the Periodic time of any Planet about the Sun.*

Let the Planet propoſed be obſerved in the Node (as was done in *Prop.*19.) and again, when it returns next to the ſame Node: The Time between theſe obſervations is very nearly the Periodic Time of the Planet ſought. For ſince the Lines of the Nodes are (by *Prop.* 43. *B.* 1.) almoſt at reſt, when a Planet has deſcribed its whole Orbit, it returns to the ſame Node, *&* *vice verſa.*

The Period of a Planet is determined not only by the obſervation of it twice in the ſame Node, but its Periodic time will become evident, if it be obſerved twice in any other point of its Orbit, cleared of the Second Inequality. But if the Centric Place of the Planet be computed from the Equinoxes, regard muſt be had to the Preceſſion of the Equinoctial points made in the mean while. In defining the Periodic times of a Planet, it will be proper to make uſe of that advice given, in determinining exactly the Quantity of a Tropical Year; namely to take the moſt diſtant Obſervations that can be had, that the error may be inſenſible, the diviſion being made by a great number of revolutions. And that an error of an intire revolution may not be committed, the knowledge of a Period near the truth, and expreſſed by a round number will ſuffice, which may be had, if in one revolution of a Planet two or more Centric Places be ſought in Oppoſition or Conjunction with the Sun, (namely when it has no Second Inequality) and the Periodic time be taken in that ratio to the Time elapſed between any two Obſervations, as the diſtance between the centric Places is to an intire Circle. If the Planet ſhould not happen to be obſerved in the Node itſelf, or in the Oppoſition with the Sun itſelf, (which generally happens;) then that moment is determined by obſervations made before and afterwards. If the period of a Planet be taken for that ſpace of Time, wherein the Centric place of a Planet becomes the ſame again in the Mundane ſpace or between the Fix'd Stars as it was before; tho' accurately ſpeaking, there is no ſuch Period at all, ſuppoſing the Nodes move; that is, that the planes of the Orbits themſelves change; notwithſtanding, if this Period be different from the Period defined above, it may be defined (conſidered in regard of Longitude) by the help of the obſervations made by the preced. *Prop.* when the Centric Place of the Planet is near the ſame Fix'dStars.

PROPOSITION XXIV.

TO *find the length of the greater Axe of the Orbit of any Primary Planet.*

We have hitherto defined the ſituation of the planes of the Orbits of the Planets to one another: We now come to deſcribe the Orbits themſelves in the planes thus defined. Which we ſhall do ſeveral ways, that this or that may be uſed according as the obſervations are, and the one may correct the other; and that the Orbits determined by all or ſeveral of theſe ways tending to one and the ſame thing, may be eſteemed more accurate. And firſt we ſhall ſhew how their greater Axes may be determined. The greater Axe of the Orbit of any primary Planet, is to be taken to the greater Axe of the Orbit of the Earth in the ſubſeſquiplicate ratio of the Periodic time of the propoſed Planet, (found by the preced. *Prop.*) to the Periodic time of the Earth about the Sun, determined by *Prop.* 13. For then the Cubes of the greater Axes of the Orbit of thePlanet and of the Earth, will be as the Squares of the Periodic times of thoſe Bodies about the Sun, which has been demonſtrated (in *Prop.* 40. *B.* 1.) to hold in Bodies revolving about the ſame Center, and acted upon by a Centripetal Force, which is reciprocally proportional to the ſquare of the diſtance from that Center: And that this is the condition of the Primary Planets revolving about the Sun, has been ſhewn in *Prop.* 42. *B.* 1.

PRO-

PROPOSITION XXV.

THree Centric Places of a Planet and its Periodic Time being given, to deſcribe its Orbit, and find the moment of Time when the Planet was in the Aphelion.

After the ſame manner as in *Prop.* 14. three Places of the Earth ſeen from the Sun (that is, Centric places of the Earth) being given, together with the Periodic time, the Species of the Orbit of the Earth, and the Poſition of the Line of the Apſides is determined; the ſame may be done by the help of the ſame like *Data* in the Planet propoſed, and the Orbit thus deſcribed, may be corrected by the method pointed at in *Prop.* 15. And as the Orbit of the Earth is deſcribed in the plane of the Ecliptic, determined before (in *Prop.* 19. *B.* 2,) ſo the plane of the Orbit of the Planet propoſed, wherein its Orbit is to be deſcribed, is determined (by *Prop.* 19 and 20.) Moreover, the ratio of the greater Axes of the ſame Orbits is found from the Periodic times of the Earth and Planet (by preced. Prop.) The magnitude of the Orbit of the Planet will be had from the aſſumption of the greater Axe of the Earth's Orbit (or its half, the mean Diſtance of the Earth from the Sun:) The Orbit of the Planet therefore is wholly determined. And the Time when the Planet was in the Aphelion, is determin'd after the ſame manner, as the like was done *Prop.* 14. in the Earth.

Becauſe the Species of the Elliptic Orbit of the Planet is only determined from theſe *Data*, (as is evident from *Prop.* 14,) and the magnitude of the Orbit in reſpect of the Orbit of the Earth is concluded by the preced. from the compariſon of the Periodic times of theſe Planets; the ſame may be inferred other ways; namely, by making an

Orbit of a certain Species (determined from three Centric Places) deſcribed about the Sun in a Focus, to paſs thro' the very Place of the Planet in the line of the Nodes, found by the 19th *Prop.* of this: For this point given in the Orbit of the Planet, ſerves as well in the determining its Magnitude in reſpect of the Magnitude of the Orbit of the Earth, as its greater Axis defined above: For the diſtance of that point from the Sun is determined in the parts of the lines of the Earth's Orbit; and any one of theſe ways of deſcribing the Orbit, may be examined and corrected by the other.

And from hence may the motion of the Apſides of any Primary Planet be defined; namely the ſame way as the motion of the Nodes was before.

PROPOSITION XXVI. LEMMA.

THE Focus and length of the greater Axe being given, to deſcribe an Ellipſe paſſing thro' two given Points.

Let *S* [in *Fig.* 24.] be the Focus of the Figure to be deſcribed, *AP* the length of the greater Axe, *L* and *l* the two points thro' which the Ellipſe ought to paſs. On the center *L*, at the diſtance equal to the exceſs of the right line *AP* above the right line *LS*, let a Circle *CF* be deſcribed; and after the like manner upon the Center *l*, with a diſtance equal to the exceſs of the right line *AP* above *lS*, deſcribe a Circle *RF* cutting the former in *F*; and *F* will be one Focus of the Ellipſe which is to be deſcribed: Conſequently an Ellipſe deſcribed upon the Foci *S* and *F*, having its greater Axe equal to the right line *AP*, will be the Ellipſe ſought. For ſince *LF* is equal to $AP - LS$, $LS + LF$ will be equal to *AP*. After the ſame manner *AP* is equal to $lS + lF$: Conſequently (by *Prop.* 52. *B.* 3. of

Conics,) an Ellipſe deſcribed upon the Foci *S* and *F*, having a greater Axe equal to the right line *AP*, will paſs thro' the points *L* and *l*.

'Tis eaſy from ſuch a Geometrical conſtruction as this to invent a method of calculating ſuitable to Aſtronomy. For three points *S*, *L* and *l* being given, that is, the three ſides of the Triangle *S L l* being known, the angles become known. Again in the Triangle *L l F*, all the ſides are given; namely *L l* as before, *L F* the difference of the given right lines *AP* and *LS*, and *l F*, the difference of *AP*, *l S*; therefore the angle *F L l* becomes known: But *S L l* was known before, therefore the angle *S L F* will be given. Therefore in the Triangle *S L F*, *L S*, *L F*, with the angle *S L F* being given, the ſide *S F* the diſtance of the Foci, and the angle *L S F*, between *L S* given by poſition and the greater axe, will be known; this axe therefore is given by poſition; and it is already given by magnitude: From whence all that belongs to the Ellipſe may be drawn.

PROPOSITION XXVII.

Given two Centric Places of a Planet and the Diſtances from the Sun, together with its Periodic Time, to determine the Orbit of the Planet.

Let *S* be the Sun [*Fig.* 25.] let the two Centric places of the Planet given be in the right lines *L*, *S l* given by poſition, and let the Diſtances of the Planet from the Sun be the right lines *S L*, *l* given in magnitude. Wherefore *L* and *l* are the points thro' which the Ellipſe is to paſs. Moreover, the periodic Time being known (by *Prop.* 4.) *PA* the greater axe of the Elliptic Orbit is determined. Deſcribe an Ellipſe therefore (by the preced.) whoſe Focus is *S*, having its greater axe of a given length, that may paſs thro' the given points *L* and *l*. The Centric places of the

Planet and its Diſtances from the Sun, either in Oppoſition or Conjunction with the Sun are determined as in *Prop.* 21; or from an Obſervation made any where, as in *Prop.* 22. Nevertheleſs that may be aſſum'd, which is determined *Prop.* 19. when the ſituation of the Planet's Orbit in regard of the Ecliptic was defined. But if you have more than two Centric places with their correſponding Diſtances from the Sun, then their Orbit being defined by any two, you muſt ſee whether they agree to it thus defined, and that is to be retained (if they diſagree) which is confirm'd by moſt Obſervations, as is uſually done in moſt Aſtronomical Calculations.

PROPOSITION XXVIII. LEMMA.

To the greater Axe AP *of the Ellipſe* ALP [*Fig.* 26.] *from the Focus* S *let a perpendicular* SR *be erected meeting the Ellipſe in* R, *thro' which let a right line* DR *be drawn touching the Ellipſe, and meeting with the Axe* AP *produced in* G, *and from* G *let* GF *be drawn perpendicular to the Axe. Theſe things being ſuppos'd, if from any point* L *of the Ellipſe two right lines* LS, LE *be drawn, one as* LS *to the Focus aſſumed* S, *the other* LE *perpendicular to* FG; *I ſay, that* LE *is to* LS *in the given ratio of* GP *to* GS.

Thro' the point *L* to the Axe *AP* draw the perpendicular *LC* meeting the Axe in C and the Tangent *DR* in *D*, and from P draw another line *PN* parallel to it: And *CD* will be equal to the right line *SL* (by *Prop.* 139. *B.* 4. of *Gregory St. Vincent*) and *PN* to *PS*. But *GC* is to *CD* in a given ratio, namely of *GP* to *PN*; and therefore *EL* is to *LS* in the given ratio of *GP* to *PN* or *GP* to *PS*. From hence it follows that *GA* is to *AS* in the ſame ratio of *GP* to *PS*; which is a particular Caſe of this Propoſition, namely when the point *L* taken at pleaſure coincides with A. And this is evident from hence, that (by *Prop.* 34. *B.* 1. *of Conics.*) RG being a Tangent, AS is to SP as AG to GP; from whence, by permutation and inverſion, GA is to AS as GP o PS. This Propoſition holds alſo in the other Sections of the Cone.

PROPOSITION XXIX. LEMMA.

TO deſcribe an Ellipſe about a given Focus, that ſhall paſs thro' three given Points.

Let the Points given be L, l, λ, (in *Fig.* 27.) ind the Focus S. Join the right line Ll, and pro-luce it to F, ſo as that LF may be to lF, as SL o Sl. So let λl joined, be produced to f, ſo as hat λf may be to lf, as $S\lambda$ to Sl. Join the right ine Ff, and upon it, from L and S let fall the erpendiculars LE, SG, divide SG in P ſo as GP nay be to PS as LE to LS, and produce it to A, ſo as that GA may be to AS in the ſame ratio: n Ellipſe deſcribed having A and P for its Ver- ces and S for its Focus; that is, having its great- r Axe PA, and its *Latus rectum* quadruple a right ne which is to SP as SA to AP (by *Prop.* 54. . 1. *of Conics.*) will paſs thro' the Points L, l, and λ.

From l and λ, upon FG let fall the perpendi- ılars le, λe. Since by conſtruction LF is to F as SL to Sl; LE will alſo be to le (by reaſon f the Parallels LE, le) as SL to Sl, or LE to SL s le to Sl. But by conſtruction alſo GP is to S, and GA to AS, as LE to SL; and therefore GP is to PS ſo is le to Sl. After the ſame man- er λf is by Conſtruction to lf (that is, λe to le) s $S\lambda$ to Sl; and therefore λe to $S\lambda$ as (le to Sl; at is, as was juſt now demonſtrated, as) LE to L. Since therefore SL is to LE, alſo Sl to le, nd $S\lambda$ to λe, as GP to PS, or GA to AS, and is is a property of an Ellipſe demonſtrated in e preced Lem. 'tis evident that the Points L, l, λ,

are in the Ellipſe, whoſe Focus is S, and extrem
of the greater Axe or Vertices, A and P. Ar
from hence it is eaſy to be known, when the Pr
blem is impoſſible, and when inſtead of an E
lipſe, a Parabola or Hyperbola comes forth.
a Calculation familiar to Aſtronomers be r
quir'd, it may eaſily be drawn from the preced
ing Conſtruction; for inſtance, after the follow
ing manner. Since $S\lambda$ is to Sl, as λf is to l
then dividedly $S\lambda - Sl$ to Sl as λl to lf: But t
three firſt terms of the Proportion are give
from the Points S, l, λ, being given; therefor
the fourth lf will be known. And after the lik
manner, becauſe $SL : Sl :: LF : lF$. By dividin
$SL - Sl : Sl :: Ll : lF$, and the three fir
terms being given the fourth term will be know
namely the right line lF. Moreover, the thre
Sides of the Triangle $Sl\lambda$ being given, the angl
$Sl\lambda$ is given, and the three Sides of the Triang
SLl being known, SlL becomes known; there
fore there is given both the Angle $Ll\lambda$ the diffe
rence or ſum of them, and the Vertical to it Fl
Therefore in the Triangle Flf, the Sides lF, l
juſt found, being given, together with the angl
Flf contained; the angle Ffl will be found, an
conſequently its complement to a right fl
But the angle $Sl\lambda$ being known before, its com
plement to two right Slf will be known; there
fore the ſum or difference els of the known An
gles Slf, elf, will be given, and conſequentl
lSG its complement (becauſe of the Parallels el
GS) to two right; that is, the inclination of th
greater axe to the right line lS, given by poſitio
Beſides, in the Triangle fel rightangled at e,
the angles are given, together with the ſide fl
therefore el, or GM is known, ſuppoſing lM pe
pendicular to GS. In like manner in the righ
angled Triangle SMl, the Angles and Side S

bein

being given, SM is given; and GS is the ſum or difference of GM, MS. Again, becauſe by the preced. Lem. $el.\ lS :: GP.\ PS$, by compounding $el + lS.\ lS :: GS.\ SP$, and the three firſt terms of this Proportion being given, the fourth SP will be known: In like manner, becauſe $el : lS :: GA : AS$, by dividing $el - lS : lS :: GS : AS$; from whence AS becomes known: The ſum of PS, SA is the greater Axe. The other Focus of the Ellipſe is as far diſtant from A, as S is from P. The Foci and Vertices A and P being given, the reſt will not be unknown.

PROPOSITION XXX.

THree Centric Places of a Planet, and the Diſtances from the Sun being given, to deſcribe its Elliptic Orbit.

Let the Centric Places of the Planet be given, and its correſponding Diſtances from the Sun; namely the right lines SL, Sl, $S\lambda$, in poſition and magnitude [*Fig.* 28.] ſuppoſing S to be the Sun. By the prec. Prop. let an Ellipſe, whoſe Focus is S, be deſcribed or determined by Calculation, paſſing thro' the Points L, l, λ. This will be the Orbit ſought. The relation this has to the Orbit of the Earth is therefore known, becauſe the magnitude of the Lines SL, Sl, $S\lambda$ is given, in the parts of the mean Diſtance from the Sun, and the ſituation of hePlane of the Orbit in reſpect to the Plane of the Ecliptic, is ſuppoſed to be determined by *Prop.* 19. and 20.

Upon this Propoſition depends the Method that Dr. *Halley* lays down, in the *Philoſop. Tranſact. An.* 1676. N° 128. for determining the Orbits of the Planets. Namely, if any Planet be obſerved thrice from the Earth in the ſame Point of its Orbit (which, when it may happen, is well enough known from its periodic Time nearly known) there will be had three Places of the Earth, ſeen from the Sun, and three correſponding Diſtances of the Earth from the Sun, expreſſed in the parts of the diſtance of the Planet from the Sun; and therefore the Orbit of the Earth itſelf is determined by the preceding: Which being known, if any Planet be obſerv'd twice from the Earth in the ſame Point of its Orbit, and conſequently (by *Prop.* 19.) its Centric Place and Diſtance from the Sun be determined, and this be repeated in two other Places of the ſame Planet; three Points of the Planet's Orbit will be had, by which together with one of the Foci (namely the Sun's Place) being given, the Orbit it *ſelf* (by the prec.) is determined. If the Periodic Time be pretty exactly known, the ſame may be done by two Centric Places and Diſtances from the *Sun* being known, which may be gather'd by *Prop.* 17. from twice two Obſervations of a Planet *obſerved* in the ſame two points whatever they be: And by the ſame pains the Place of the Nodes and the Inclination of the Orbit to the Ecliptic are determined by the 19th and 20th Propoſitions.

PROPOSITION XXXI. LEMMA.

TO *find the conjugate Diameters of an Ellipſe, which paſſes thro' five given Points.* [Fig. 29.]

Let the five given Points be *N*, *M*, *L*, *K*, *H*, that the Perimeter of the Ellipſe paſſes thro'; a Diameter as *FE* of that Ellipſe is to be found, together with its Conjugate *TT*, in magnitude and poſition. Let right lines be ſuppoſed to be drawn connecting the given Points, ſome two of which are either parallel, or there are no two ſuch. This is eaſily ſeen; for ſince the Points *N*, *M*, *L*, *K*, *H* are given, the right lines that connect them all manner of ways are given, and the angles contained between thoſe right lines

If

herefore it be found that two angles placed al- nately, (for inſtance KMH, MHN) are e- il, it may be concluded that the two right lines K, NH, in which the ſaid angles are alter- ely placed, are parallel.

ASE I. Suppoſe firſt, that there are two of m, as MK, NH parallel to one another. Let m be biſected in B and A, and let BA con- ting them, be produced, till it meet the Ellip- n F and E; and it will be (*by the Elements of ics*) a Diameter of the Ellipſe, and (by 26. .) given by poſition; becauſe paſſing thro' given points B and A. Draw thro' L, the e LX, parallel to EF, meeting the Ellipſe in the right line MK in D, and NH in C. And the right lines XK, LM connected, meet the it line HN (if need be produced) in the nts P and G. By *Prop.* 17. *B.* 3. *Elem. Conic of llonius*, the rectangle XDL, is to the rectangle K as ZFq to ZY^{q}. And $XCL : NCH ::$ $^{q} : ZYq$. An therefore $XDL : MDK ::$ $L : NCH$. But (as ſhall be ſhown hereafter,) $L. MDK :: XCL. GCP$. And therefore *Prop.* 9. *Elem.* 5.) the rectangle NCH is e- to the rectangle GCP. But the rectangle H is given, becauſe its ſides NC, CH are given; the whole line NH is given at firſt as well in nitude as poſition, alſo the right line LC by tion (by 28. *Dat.*) wherefore (by 25. *Dat.*) point C will be given: So likewiſe will the t line GC, ſince its extremes G and C are n: Therefore (by 57. *Dat.*) CP is given in nitude: But it is alſo by poſition; therefore point P itſelf is given. And the point K is n; therefore the right line KP connecting e points is alſo given by poſition, and conſe- ntly (by 25. *Dat.*) the point X in the Ellipſe, re the lines LC, KD given by poſition meet)

is

is alſo given. Join the right lines *NX*, *LH* meeting the diameter *EF* (if need be produced) in *R* and *S*: And then (by the above cited *Prop.* 17. *B.* 3. *Elem. of Conic.*) both *NCH* will be to *XCL*, and *NAH* to *EAF*, as *ZYq* to *ZFq*; from whence *NCH. XCL*::*NAH. EAF.* And by what is below demonſtrated, *NCH. XCL*::*NAH. RAS*; therefore (by *Prop.* 9. *Elem.* 5.) the rectangle *RAS* is equal to the rectangle *EAF*. But *RAS* is given (ſince the three points *A*, *R*, and *S* are given, *A* as was ſhewn above, *R* and *S* by the 25 of the *Data*, namely the concourſe of the right line *EF* given by poſition with the right lines *XN*, *HL* given by poſition;) therefore the rectangle *EAF* will be given. After the like manner the rectangle *EBF* may be found; and the points *A* and *B* are given, therefore ſo is *EF* in magnitude; as we ſhall ſhow hereafter. But its poſition was found before; wherefore the diameter *EF* is given in Poſition and Magnitude. Moreover TY^q is to EF^q as *NCH* to *XCL*, as was ſhewn above; that is, (by 1 *Dat.*) in a given ratio, becauſe the rectangles *NCH*, *XCL* are given; wherefore the ſubduplicate ratio of the former, namely the ratio of *TY* to *EF* (by 24 *Dat.*) is alſo given: But *EF* is given, as was ſhewn above; therefore by 2 *Dat.* *TY* will be given in magnitude. But its poſition is given by 28 *Dat.* becauſe paſſing thro' the point *Z*, which is in the middle of the right line *EF* given in poſition and magnitude, parallel to the right line *NH* given in poſition. Therefore the right lines *EF*, *TY* are given in Poſition and Magnitude, namely the conjugate diameters of the Ellipſe, paſſing thro' the five given points *N*, *M*, *L*, *K*, *H*.

CASE II. But now let us ſuppoſe that the right lines *MH*, *NK*, [*fig*: 30.] are not parallel;

lel, nor any other two: Therefore theſe lines *MH*, *NK* (at leaſt produced) will interſect one another; for inſtance in the point *L*, which will be given by 25 *Dat.* draw thro' *L* the right line *LV* parallel to *MH*, meeting the right line *NK* in *Q*, which (by 28 and 25 *Dat.*) will alſo be given. And the ratio of the rectangle *NQK* to the rectangle *LQV* will be given; namely the ſame (by *Prop.* 17. *B.* 3. of the *Elem. of Con.*) with that of the given rectangle *NIK* to the given *MIH*. But the rectangle *NQK* is given, ſince its ſides *NQ* and *KQ* are given; and therefore the rectangle *LQV* is alſo given: From whence, ſince *LQ* is given by 57 *Dat.* the right line *QV* is given in magnitude; but alſo in poſition, becauſe a parallel thro' the given point *Q* to the right line *MH* given by poſition; therefore the point *V* itſelf by 27 *Dat.* ſince the magnitude and poſition of the right line *QV* and the other extremity *Q* are given. Thus therefore we come back to the former Caſe; for there are given the five points *N*, *M*, *L*, *V*, *H*, or *M*, *L*, *K*, *V*, *H*, thro' which the Ellipſe may paſs, and yet ſuch as the two right lines *LV*, *MH* may be parallel. By what has been ſaid therefore the conjugate diameters of that Ellipſe may be found. *Q. E. F.*

Let there be two right lines GP, MK, *parallel to one another, on which let the right line XCDL* [Fig. 31.] *fall, and from any points of it* X *and* L *draw any how the right lines* LMG, XPK *meeting the parallels in* M, G, P *and* K: *I ſay, the rectangle* XDL, *is to* MDK *as* XCL *to the rectangle* GCP.

The ratio of the rectangle *XDL* to the rectangle *MDK* (by *Prop.* 23. *Elem.* 6.) is compounded of the ratio of *XD* to *DK*, and of the ratio of *LD* to *DM*; and the ratio of the rectangle *XCL* to the rectangle *GCP* of the ratios of *XC* to *CP* and *LC* to *CG*. But becauſe of the parallels MDK, GCP, XD is to $DK :: XC.\ CP$; and $LD.\ DM :: LC.\ CG$. Since therefore the compounding ratios are equal, the compounded ratios will alſo be equal; that is, XDL to MDK, as XCL to GCP.

If both the rectangles EAF, EBF *are given, and the points* A *and* B *are given; the points* E *and* F *are alſo given, ſuppoſing* EABF *a rightline.*

For ſuppoſe the rectangle BAC to be equal to the rectangle EAF, and the rectangle ABD equal to the rectangle EBF: And therefore, ſince (by *Hyp.*) EAF and BA are given, AC (by 57 *Dat.*) is alſo given. In like manner, ſince EBF and AB are given, AD will be given. By reaſon of the former equality (by 16. *Elem.* 6.) $FA.\ AB :: AC.\ AE$, and by converſion of ratios, $FA.\ BF :: AC.\ CE$. Again, by reaſon of the latter equality $EB.\ BD :: AB.\ BF$, [*Fig.* 32.] and by compounding $ED : BD :: FA.\ BF$. From whence (by *Prop.* 11. *El.* 5.) $AC.\ CE :: ED.\ BD$: And therefore the rectangle contained under AC and BD is equal to the rectangle under CE, ED. But the rectangle under AC and BD is given, (for both right lines are given) therefore the rectangle under CE, ED is alſo given. And becauſe this given rectangle is applied to the given rightline CD, (compoſed of the parts CA and AD which are given) and wants of a ſquare Figure, CE (by 58 *Dat.*) will be given; and conſequently AE (by 4 *Dat.*) and becauſe the rectangle EAF was given at firſt, the right line AF will alſo be given; that is, the points E and F will be given.

PROPOSITION XXXII. LEMMA.

TO find the Foci and Vertices of an Ellipſe that is to paſs thro' five given Points, and conſequently to deſcribe the Ellipſe paſſing thro' the five given Points.

Pappus in *Book* 8. *Math. Coll.* propoſes and conſtructs this Prob. and the prec. for the uſe of Architects: Namely, having a portion of the ſuperficies of a right Cylinder given, that has no part preſerved intire in the circumferences of the Baſes, to find the thickneſs of the Cylinder. Theſe the Aſtronomers afterwards converted to their uſe. By the preced. *Lemma* find any two conjugate Diameters of the Ellipſe, that is to paſs thro' the five given Points, and let them be *A B, C D* biſecting one another in the Center *E*. Produce *B A* to *H* [*Fig.* 33.] ſo as that the rectangle *E A H* may be equal to the ſquare of *D E*; by which means (by 58 *Dat.*) *E H* is given. Let a perpendicular *K L*, from *K* the middle of *EH*, meet with *M A G*, a right line parallel to *D C*, thro' the point *A* in the point *L*, which conſequently is given on the account of the right lines *M G, K L* given by poſition. On the Center *L* thro' *E* or *H*, draw a Circle interſecting the right line *M A G* in *M* and *G*, whoſe diameter *MG* conſequently is given as well in poſition (becauſe 'tis parallel thro' the given point *A* to the right line *C D* given by poſition,) as in magnitude, (ſince it is double to the right line connecting the given points *E* and *L*) the right lines *G E, M E* connecting them, are alſo given as well in poſition as magnitude. But their poſition is the ſame with the poſition of the Axes ſought: For ſince the rectangle *E A H* (by 35. *Elem.* 3.) is equal to the rectangle *M A G*, and by conſtruction *EAH* equal to the ſquare of *D E*; *E D*^{q} will be equal to *M A G*, or (by 17 *Prop. Elem.* 6.) *E D* will be a mean proportional between *MA* and *AG*; and conſequently (by *Prop.* 88. *B.* 3. *of Greg. St. Vincent.*) the Axes of the Ellipſe deſcribed upon the conjugate Diameters *A B, C D* are in the right lines *E G, E M.* And the magnitude of the Axes is thus expeditiouſly determined. Thro' *A*, let fall perpendiculars *A Q*, *A N* to *EG. E M*, which therefore are given. Take in the right line *E G*, *E O* a mean proportional between *E G*, *E Q*; in like manner, in *E M*, *E R* a mean proportional between *E M*, *E N*; the right lines *E O*, *E R* (given by 24 *Dat.*) will be the Semi-axes of the Ellipſe (by *Prop.* 37. *B.* 1. *Conic.*) If in *G E* produced you take *E P* equal to *E O*, and in *M E* produced, *E T* equal to *E R*, the Axes *O P*, *R T* of the Ellipſe will be given in poſition and magnitude, together with with the points *O*, *R*, *P*, *T*, which are the principal Vertices of the ſame. Moreover, if the greater Axe *P O* be ſo divided in *S* and *F*, that the rectangle under *P S* and *S O*, as alſo the rectangle under *P F* and *F O* be equal to the *Square made upon RE*, *S* and *F* will be the Foci of the Ellipſe (by *Prop.* 52. *B.* 3. *Elem. Conic.*) But theſe Points thus found (by 58 *Dat.*) are given: and if the Axes and Foci of the Ellipſe are given, the Ellipſe may be deſcribed by *Prop.* 54. *B.* 1. *Conics of Apollonius.*

There is no need of annexing here the form of a Trigonometrical Calculation ſuited to this Problem: For it is well known to Geometers, that whatever is found out by the help of *Euclid*'s *Data*, may alſo eaſily be found by a Calculation, proceeding exactly after the ſame manner, and generally in a Trigonometrical manner, as it happens here: Indeed the inveſtigation of any Problem by the help of *Euclid*'s *Data*, is an univerſal method of determining it, and very uſual with the Ancients.

PROPOSITION XXXIII.

Five Centric places of a Planet and the Diſtances from the Sun being given, to find its Orbit.

In the preceding Propoſitions, while we were determining the Orbits of the Planets, we ſuppoſed nothing, but what was agreeable to Obſervations exactly made as well as to the Phyſical Cauſes of their Motions demonſtrated in *Book* 1: Namely, that the Orbit of any Planet is an Ellipſe, one of whoſe Foci is the Sun at reſt, and that a Planet deſcribes it after ſuch a manner, as that its Area's or Sectors contained between right lines drawn to the Sun, and the Arcs of the Ellipſe are proportinal to the Times wherein the ſaid Arcs are deſcribed by the Planet. For tho' we have ſuppoſed in *Prop.* 14. with *Ward*, for a little while, that a Planet deſcribes its Orbit ſo, as that its angular motion at the Focus of the Ellipſe in which the Sun is not; yet this was only done, becauſe in the Orbit of the Earth, which is not very excentric, an approximation is made to the equal deſcription of Areas by a radius drawn to the Sun, as is not to be diſregarded, as was ſhown in *Prop.* 7. And we corrected the Orbit of the Earth defined according to that Hypotheſis, as much as we pleaſed in the following 15th Propoſition. But in this Propoſition we ſhall not ſo much as ſuppoſe that the Sun is placed in the Focus of each Orbit; but ſhall deſcribe the Orbit by the help of five given Centric Places, and as many Diſtances from the Sun, independently upon it: And by this means the truth of the preceding Prop. will appear clearer, ſince the Sun will be found to be placed in the common Focus of the Orbits thus deſcribed.

The Centric places of the Planet being given, the right lines, wherein it is ſeen from the Sun, are alſo given by poſition: Let them be *S N*, *S M*, *S L*, *S K* and *S H*; [*Fig.* 34.] and the correſponding Diſtances from the Sun being given, the points *N*,*M*,*L*,*K* and *H* (by 27 *Dat.*) are given. Thro' theſe points (by the help of the preceding Lemma) deſcribe an Ellipſe, and it will be the Orbit of the Planet ſought. And by the ſame Lemma the Vertices will be found, as alſo the Foci, Poſition of the greater axe and the other things that are required in the Orbit of the Planet. *Q. E. F.*

PROPOSITION XXXIV.

TO *determine the Centric Place and Diſtance from the Sun, of any Planet propoſed, at any given Time.*

The plane of the Planets Orbit, the poſition of the Line of the Apſides and the Species of the Planet's Orbit being determined, (by the preced. *Prop.*) that is, the Orbit it ſelf being deſcribed in its proper Situation about the Sun, ſuppoſe *ALP* [*Fig.* 35.] and the Moment being known (by *Prop.* 25.) when the Planet was ſome time or other in the Apſis alſo the Periodic Time of the Planet (by *Prop.* 23.) the Time ſince it was *laſt in* the Apſis, (*v. g. A*) will be known, or the ratio of the ſaid Time to the whole Periodic Time will be known; that is, by the Law of Motion of the Planets, demonſtrated in *B.* 1. ſuppoſing the Place of the Planet *L*, the ratio of the area *ALS* to the whole Ellipſe *ALPA* is known. Find therefore the Angle *ASL* by *Prop.* 3, 4, 6, or 7. Therefore the right line *SL* is given by Poſition, ſince it is inclined in a given angle to *PA* the Line of the Apſides given in poſition: As alſo in magnitude, by the ſame Propoſitions, (ſince this is given in the parts of the Aphelion diſtance of the Sun *SA*, which is defined above in the parts of the Mean diſtances of the Earth from the Sun, which is looked upon as the common meaſure of theſe lines;) that is, the Place of the Planet propoſed ſeen from the Sun, and its Diſtance from the ſame.

And *vice verſa* the Time may be found when a Planet ſhall be in a given Heliocentric Place; by finding the mean Anomaly (by the preced. *Prop.*) from the coequated Anomaly given, to the former of which the Time is proportional.

PRO-

PROPOSITION XXXV.

AT a given Time to find the Geocentric Place of a Planet, as to Longitude and Latitude, and the Diſtance of the Planet from the Earth.

Find (by *Prop.* 18.) the Heliocentric Place of the Earth in its proper Orbit, at the Time given, and (by the preceding) the Heliocentric Place alſo of the Planet propoſed in its Orbit, with the reſpective Diſtances from the Sun.

Let *S* (in *Fig.* 36.) repreſent the Sun, *T* the place of the Earth in its Orbit, *L* the place of the Planet in its Orbit. Let the right line *SN* be the common interſection of the Planes of theſe Orbits, which is extended thro' the Sun, (becauſe the Sun is in both Planes) and whoſe poſition is given (by *Prop.* 19.) Beſides, the place of the Earth *T* being given, the right line *ST* and the angle *TSN* in the Plane of the Ecliptic, are given; and the Place *L* of the Planet being given, the right line *SL* and the angle *LSN*, in the plane of the Orbit of the Planet are given. Draw thro' *L* and *S*, and through *L* and *T*, the Planes of the Circles of Latitude perpendicular to the plane of the Ecliptic, whoſe common interſection the right line *LE* (by *Prop.* 19. *Elem.* 11.) is perpendicular to the plane of the Ecliptic. Draw in the plane of the Planet's Orbit the right line *LN*, perpendicular to *SN*, and the line *EN* (by the converſe of *Prop.* 11. *Elem.* 11.) will alſo be perpendicular to the ſame *SN*.

In the Triangle *SLN* rightangled at *N*, *SL* and *NSL* are given; from whence the Side *LN* will be given. In the Triangle *LEN* rightangled at *E*, there are given the Side *LN* lately found, and the angle *LNE*, which (by *Def.* 6. *Elem.* 11.) is equal to the Inclination of the Orbit of the Planet to the Ecliptic, found by

*Prop.*20. *LE* and *EN* are therefore given. Beſides in the Triangle *LES*, rightangled at *E*, the Sides *LE* and *LS* are given; therefore the Side *SE*, the *Curtate Diſtance of the Planet from the Sun* is given. In the Triangle *ESN*, beſides the rightangle at *N*, there are given *ES*, and *EN* found before; therefore the angle *ESN* will be given, the Heliocentric Longitude of the Planet computed in the Ecliptic from the Node *N*: But the Place of the Node in the Ecliptic is given; wherefore the Heliocentric Place of the Planet in the Ecliptic will alſo be known. Again, in the Triangle *TSE*, *TS* is given at firſt, and *SE* was found juſt now; the angle *TSE* is alſo given (equal to the ſum or difference of the given angles *TSN*, *ESN*, becauſe the three right lines *ST*, *SE*, SN, are in the Plane of the Ecliptic, which is alſo called *the Angle of Commutation*: And therefore all the reſt is given, namely, the angle *TES*, call'd, *The Parallax of the Orbit*, and *STE the Elongation of the Planet from the Sun*; that is, the poſition of the right line *TE*, or the Geocentric Place of the Planet in the Ecliptic, and *TE* the Magnitude of the ſame. Laſtly, in the Triangle *TEL*, rightangled at *E*, the Sides *TE* and *EL* are given; therefore *TL* the Diſtance of the Planet from the Earth, and the angle *LTE* the Geocentric Latitude of the Planet will be known.

SECTION IV.

Of the greateſt Elongation of the Planets from the Sun, their Direction, Station, and Retrogradation.

PROPOSITION XXXVI.

TO determine the greateſt Elongation from the Sun of a given Planet, ſeen from a ſuperior one alſo given, and its Time.

Let *S* be the Sun, *TR* the Orbit of a more remote Planet, [*Fig.* 37.] *VP* of a nearer. Let the radius *SV* of the nearer Planet be drawn any how, and perpendicular to it *VT*, meeting the more diſtant Orbit in *T*, which conſequently (by 16. *El.* 3.) will touch the Circular Orbit *VP* in *V*. The inferior Planet will be ſeen by the ſuperior in *T*, to be at its greateſt Elongation from the Sun in *V*: For a right line drawn from any other point (as *v*) of the Orbit *PV* to *T*, will contain a leſſer Angle with *TS*. Therefore in the Triangle *TVS*, rightangled at *V*, the ratio of the Sides *ST*, *SV* (namely the Diſtances of the given Planets from the Sun) is given; therefore the other angles *TSV*, *STV* are known, the latter of which is the greateſt Elongation of the Planet from the Sun, the other *TSV* is the angle of Commutation or Heliocentric Diſtance of the Planets. Whenever therefore the angle of Commutation, found juſt now, happens, it at the ſame time happens that the inferior Planet, ſeen from the ſuperior, is at its greateſt Elongation from the Sun.

We have hitherto ſuppoſed the Orbit of both Planets to be Circular; but if due regard be had to their Elliptic Figure, the Problem is ſo much after the ſame manner. For the *SVT* is given, that the right line *VT* [*Fig* touching the Orbit, makes with the radi inclined in an aſſumed Angle to the Line Apſides; beſides *SV* is given in magnitude, ſi Inclination to the Diſtance of the Aphelion ready given in magnitude is known; and be the angles *AS a*, *a SV* are known, alſo the *VST*, (very nearly equal to that, which be were both Orbits circular;) the angle will alſo be known, and conſequently *S* be known in magnitude; from whence wh mains is done as before. In *this caſe the* gr Elongation changes according to the differe ſtance of the point *V* (where it is celebr from the Aphelion of its proper Orbit: Fo greater (*cæteris paribus*) in *a* the Aphelion inferior, leſs in *π* the Perihelion, a mean mean Longitude; it is alſo various, the p of the inferior remaining the ſame, accord the ſuperior is ſituated in *T* or *t*.

PROPOSITION XXXVII.

ANY *Primary Planet ſeen from any other P appears in ſome part of its Orbit, Station another Direct, and laſtly in another Retrograde*

Let *S* be the Sun, [*Fig*. 39.] about whic Planets revolve from *D* thro' *C* towards *B*, th bit of the nearer *PT*; of the more diſtant the arcs deſcribed by them in equal ſpac time *AR*, *PB*. Draw *SR* any how m the nearer Orbit in *P*. Between *SR* and *SM* be a Geometric Mean. Then (by *Pr B*. 1.) the Velocity of the more diſtant Pla to the Velocity of the nearer, as *SP* to But the Spaces deſcribed by Bodies in Moti equal Times are as their Velocities; and therefore *AR* is to *PB*, as *SP* to *SM*; wherefore *AR* is leſs than *PB*. And it is plain that it is poſſible for two parallel right lines *HADb*, *GRCg* to be drawn thro' *A* and *R*, ſo as that the arc of the nearer Orbit intercepted by them, is equal to the arc *PB*. For parallel right lines may be ſo drawn, that as much as *CD* (equal to *PB*) is greater than *AR*, ſo much it may be more oblique, (and the arc of the interior Orbit is given, which is as oblique to the parallels thro' *A* and *R* as you pleaſe,) that the arc *CD* compenſating the greater length by its greater obliquity, may touch both of the parallel right lines in their extremities. Which being ſuppoſed, if the Planets be found in ſuch a ſituation in regard of one another as *A* and *D* are here; that is, if the angle of Commutation at the Sun be equal to the angle *ASD*, (which, it is evident, it will be twice neceſſarily between the two next Conjunctions of the Planets propoſed ſeen from the Sun,) the one ſeen from the other all that ſpace of time, wou'd be refer'd to the ſame Fix'd Stars; that is, it would appear Stationary. For the ſuperior at *A* is ſeen from the inferior at *D*, in the right line *DAH*, and while the ſuperior is going to *R*, the inferior is going to *C*, (ſince by ſuppoſition *DC* is to *AR* as the Velocity of the Planet, which is nearer to the Sun, is to the Velocity of the more diſtant:) That therefore ſeen from this, in the right line *CR* will be referred to the ſame place among the Fix'd Stars, to which it was before referred, ſince the diſtance between the Parallels *DA*, *CR* vaniſhes in compariſon with the diſtance of the Fix'd Stars. And, for the ſame reaſon, the inferior Planet deſcribing the arc *DC* viewed from the ſuperior, while it deſcribes the arc *AR* in the ſame time, will alſo appear Stationary. In like manner, if the Planet next the Sun be on the other ſide of the point *P*, (in which being viewed from the Sun, it ſeems in conjunction with the more diſtant Planet; that is, in which the Sun and ſuperior Planet viewed from the inferior, are ſeen in oppoſition, and where the inferior ſeen from the ſuperior appears in conjunction with the Sun,) and as far diſtant from it, namely where the viſual rays of any ſenſible time are again parallel to one another, for inſtance, at *cd*, the one ſeen from the other will again appear Stationary.

I ſay further, that before the Planet, which is nearer to the Sun, arrives at the beforemention'd ſituation in reſpect of the ſuperior, that is, the ſuperior being in *R*, before the inferior arrive at *D*; the one ſeen from the other will appear to move *in conſequentia*. For let the arc *DF* be taken equal to *PB* or *DC*, and join the right lines *FA*, *DR*, and produce them both ways in *KAFL* and *IRDL*; they will not be parallel, but diverge towards the parts beyond *RA* (in reſpect of the Sun, but on this ſide they will converge and concur, for inſtance, in the point *L*, from whence they will again diverge. For ſince (by ſuppoſition) *DA*, *CR* are parallel, and *DF* equal to *CD*, but more oblique to *RD* than *CD* is to *RC*; 'tis evident that *DR*, *FA* diverge on the other ſide of *RA*, and converge on this ſide of *RA*; *viz.* either behind *DF*, if it be in that part of the nearer Orbit that is turned towards *R* (as in the caſe of this Figure) or between *RA* and *DF*, if in that part which is turned from it. If now the inferior being in *F*, the ſuperior be in *A*, the latter viewed from the former will be ſeen in *K*: But the inferior being arrived at *D*, the ſuperior is got to *R* (by ſuppoſition) and is ſeen from it at *I*.

But to come from the ſituation of *K* to the ſituation of *I* is to move *in conſequentia*, or to be Direct; for any of the Planets ſeen from the Sun, moves *in conſequentia* from *A* to *R*, or from *D* to *C*. Nay, even the inferior ſeen from the ſuperior at the ſame time ſeems Direct: For the inferior in *F* will be ſeen from the ſuperior *A* at *k*; and afterwards the inferior at *D* will be ſeen from the ſuperior in *R* at *i*: But to change place from *k* to *i*, is alſo to tend *in conſequentia*; for the Sun ſeen from any Planet ſeems to move *in conſequentia*; namely from *S* towards ♌. In like manner, after the Planet, which is nearer to the Sun, has paſſed the ſecond Station at *d*, it becomes direct again, and continues ſo till the inferior arrive again to a ſituation in reſpect of the ſuperior, (as is *D* in regard of *A*;) which it will do after it has compleated an intire revolution from the former place *D*, and ſo much of another beſides, as is requiſite for it to recover the ſaid ſituation again, in regard of the more diſtant Planet, that had in the mean time gone forwards.

I ſay thirdly, that the one will appear Retrograde to the other, after that the Planet that is nearer to the Sun has got beyond the above deſcribed boundary *DC* of the firſt Station, and has not arrived at the ſecond *cd*. Let *CE* be taken equal to *PB*, or *DC*; draw the right lines *CA*, *ER*, and let them be produced both ways into *OCAYo*, and *NERYn*, which are not parallel, but diverge towards the parts *CE*, and converge towards *AR*, and being produced meet, for inſtance in *Y*, becauſe by ſuppoſition *DA*, *CR* are parallel, and *CE* is equal to *DC*, but leſs oblique towards *RE* than *CD* is towards *RC*. If now while the ſuperior is in *A*, the inferior be in *C*, this will be ſeen from that in *O*, and that *vice verſa* will be ſeen from this in *o*: But when the ſuperior is got to *R*, and the inferior is found at *E*, this will be ſeen from that at *N*, but that will be ſeen from this at *n*. But 'tis evident, that to ſeem to move from O to *N*, or from *o* to *n*, is to move contrary to the order of the Signs; ſince from *S* to *Q*, or from *A* to *R* is according to the order of the Signs; namely that wherein the Primary ſeen from the Sun, or *vice verſa*, the Sun ſeen from the Primary ſeems to tend. And this apparent Retrogradation of the Superior ſeen from the Inferior, or of the Inferior from the Superior, continues from the celebration of the firſt ſtation at *D C*, to the ſecond at *c d*: But by reaſon of the motion of the more diſtant Planet, this ſecond *ſtation* is removed a little *in conſequentia*, or beyond the place *c d* here expreſſed. 'Tis therefore evident, that any Primary ſeen from another Primary in ſome part of its Orbit is Direct, and in ſome Retrograde; and laſtly, in ſome Stationary. *Q.E.D.*

COROLLARY. 1.

'Tis evident from the demonſtration of this Propoſition, that a ſuperior Planet ſeen from an inferior appears at the ſame time Direct, Stationary, and Retrograde, that the inferior ſeen from that ſuperior does.

COROLLARY 2.

The more unequal the ratio between the Orbits of two Planets is, the nearer its place in the Orbit of the inferior, in which the inferior is found while the one ſeen from the other appears Stationary, is to that point of it, from which a right line drawn to the more diſtant Planet, touches the Orbit of the inferior; that is, where the inferior ſeen from the ſuperior (by *Prop.* 26.) appears in its greateſt Elongation from the Sun; and the angle greater wherein the Sun and ſuperior Planet, ſeen from the inferior, ſeem to be diſtant from one another. For the greater the ratio of *S R* is to *S P*, the greater will the ſubduplicate of the latter, namely the ratio of *R S* be to *M S*; that is, (by conſtruction) the ratio of *C D* to *R A*. And therefore the line *C D* muſt be more oblique to *R C* or *A D*, that *D A*, *R C* may become parallel; but it is evident that *C D* is ſo much the more oblique as it is nearer to the point of the inferior Orbit, from whence a right line drawn to *R* touches it.

From hence it is, that to make Saturn Stationary there is need of ſo ſmall a receſſion of the Earth from the point of its Orbit, where a right line drawn to Saturn touches it, towards the point where the Earth comes between the Sun and Saturn; to make Jupiter Stationary, a greater receſſion from the point of contact of the Orbit of the Earth, with a right line touching it drawn to Jupiter; to make Mars the greateſt. Therefore Saturn is Stationary, when ſeen from the Earth it is diſtant a little more than a Quadrant from the Sun; that is, between the Quartile and Trine Aſpect; Jupiter will appear Stationary almoſt in the Trine of the Sun; Mars beyond the Trine.

Hence it is alſo that the arc of the Orbit of Mercury intercepted between the point where Mercury is ſeen from the Earth at its greateſt Elongation from the Sun, and the point where its next Station is ſeen, ſubtends a leſs angle at the Sun than the arc of the Orbit of Venus contains between its greateſt Elongation from the Sun and next Station: And therefore ſince it ſubtends both a leſs angle at the Sun, and belongs to a leſs Circle, and beſides is preſented more Obliquely to the Earth, obſerving it, and is more diſtant from it; 'tis evident that the Stations of Mercury

are

are much nearer to its greateſt Elongations from the Sun than thoſe of Venus.

COROLLARY 3.

And from hence again it follows, that in reſpect of the ſame inferior, in two ſuperior Planets compar'd together; the farther off the Sun the ſuperior Planet is, the longer its Retrogradation continues; for (by the preced. *Corol.*) it begins ſooner and ends later. But the angle of Retrogradation is leſs, becauſe the Orbit of the given inferior Planet ſeen from the more diſtant ſuperior Planet appears under a leſs angle, than if it were ſeen from a nearer: From this *Prop.* 37. 'tis evident, that the angle of Retrogradation (contain'd between two right lines from the ſuperior Planet oppoſite to the Sun, to the places of the inferior Orbit, where the one appears Stationary to the other) is almoſt equal to the angle wherein the Orbit of the inferior is ſeen from the ſuperior, ſince the above mention'd points of the Stations are not far from the points of contact.

Hence it is, that the Retrogradation of Saturn ſeen from the Earth is longer, but made along a leſs arc, than that of Jupiter; and that of Jupiter in like manner to that of Mars.

COROLLARY 4.

But in reſpect of the ſame ſuperior, in two inferior Planets compared together; the farther off the ſuperior Planet is from the Sun, the longer is its Retrogradation and along a greater arc. The former is evident from the preced. *Corol.* ſince the Retrogradation of two Planets is mutual by *Corol.* 1. The latter is evident from hence, that a Planet that is more diſtant from the Sun, while it appears Retrograde, deſcribes a longer arc ſlower, and therefore ſpends more time.

Hence it is, that Venus ſeen from the Earth, is both longer Retrograde, and along a greater arc than Mercury. *COROL-*

COROLLARY 5.

Hence it alſo follows, that when two ſuperior Planets are ſeen from an inferior in Conjunction, if that which is farther off from the Sun be Direct, that which is nearer the Sun will alſo be Direct; if that which is nearer the Sun be Retrograde, that which is farther off the Sun will alſo be Retrograde. Let *S* repreſent the Sun in [*Fig.* 40.] *IT* the Orbit of an inferior Planet; *PZ*, *RV* the Orbits of two ſuperior Planets. Draw *ST* any how interſecting the Orbits of the ſuperior in *S* and *R*. Let *B* be the Place in the Orbit of the inferior, where the inferior Planet is when the more diſtant one in *R* is ſeen from thence to be Stationary; and let *A* be the Place in the ſame, where the inferior is found when the next in *P* ſeen from the inferior appears to be Stationary. Join the right line *AP*, *AS*, *BR*, *BS*. From *Corol.* 2. it is evident that the Place *A* is nearer to the point *I* than *B*, and that the angle *PAS* is greater than *RBS*. When the Planet in *R* appears Direct from the inferior, the Place of the inferior ought to be farther diſtant from *I* than the Place *B*, for inſtance in *C*. Suppoſe now that the nearer of the ſuperior Planets ſeen from the inferior is in conjunction with the more diſtant one, and it will be therefore in the point *E*, where the right line *CR* interſects the Orbit *PZ*. By *Prop.* 21. *Elem.* 1. the angle *ECS* is leſs than the angle *RBS*; therefore it is much leſs than *PAS*. Therefore a Planet ſeen in *E* from *C* is Direct; becauſe to make it Retrograde, 'tis requiſite that *ECS* be greater than *PAS*, which is requiſite to make it Stationary. But the inferior being between *B* and *A*, the more diſtant at *R* will apappear Retrograde, and that which is nearer to it in conjunction, ſtill Direct; becauſe the Planet that takes the view is not as yet arriv'd to the limit *A*.

But if the nearer of the ſuperior Planets appears Retrograde at *P* from the inferior, the inferior will be between *A* and *I*, for inſtance in *D*. If now the more diſtant Planet appear in conjunction with it, that will be in the right line *DS* produced to the Orbit *RV*, namely in *F*: But the angle *FDS* is greater (by *Prop.* 21 *El.* 1.) than *PAS*, which again is greater (by what has been ſhown) than *RBS*. And therefore the more diſtant Planet ſeen in *F* from *D*, appears Retrograde, ſince to make it appear Direct, a leſſer angle at the inferior than *RBS* is requiſite.

And hence it is that when Saturn is ſeen from the Earth to be Direct, Jupiter and *Mars can't* be in conjunction with him unleſs they alſo be *Direct*; or Mars can't be in conjunction with Jupiter Direct, unleſs when he himſelf is Direct: But either Jupiter or Mars Direct may be in conjunction with Saturn Retrograde, and Mars Direct may be in conjunction with Jupiter Retrograde. But Jupiter and Saturn can't be in conjunction with Mars Retrograde, unleſs they themſelves be Retrograde; or Saturn with Jupiter Retrograde, unleſs Retrograde.

LEMMA.

GIven in the quadrilateral figure CGDK [Fig. 41.] *inſcribed in a Circle,* (*one of whoſe ſides* CK *is a diameter of the Circle,*) *the ſide* CG *and the diagonal* CD, *together with the ratio between the oppoſite ſide* KD *and the other diagonal* GK, (*the ſame that is between the giveu right lines* N *and* M;) *to find the angle* GCD *contained between the given ſide and the given diagonal.*

Becauſe the ſum of the Squares of the right lines *CG* and *KG* is equal to the ſum of the Squares of the right lines *CD* and *KD*, (for each of the ſums are equal to the Square of *CK*;) the difference of the

Squares of the right lines *G K* and *K D* will be equal to the difference of the Squares of the right lines *C D* and *CG*, and conſequently given. Moreover, becauſe *G K* is to *D K* as *M* is to *N*, the Square of *G K* will be to the difference of the Squares of *G K* and *D K*, as the Square upon the right line *M* is to the difference of the Squares upon *M* and *N*; being made out of thoſe in the ſame ratio with them that are ſimilarly made out of theſe, which are proportional to thoſe: Therefore the Square upon *GK* (by *Dat.* 2.) will be given; the Square alſo of *CK*, made up of the given Square of *GK* and the Square of *CG*, is alſo given: Conſequently the right lines themſelves *CK* and *GK* are given, the right line *DK* is alſo given, becauſe it has a given ratio to *GK*. But it is a known Theorem among Geometers, demonſtrated by *Ptolemy*, *B.* 1. *of the great Conſtruction*, *Chap.* 9. that in a quadrilateral *CGDK* inſcribed in a Circle, the rectangle contained under *CD* and *GK* is equal to the rectangle under *CG* and *DK*, together with the rectangle under *CK* and *GD*. And therefore the rectangle under *CD* and *GK*, leſſen'd by the rectangle under *CG* and *KD* is equal to the rectangle under *C K* and *G D*. But the rectangled Parallelograms that are contained under the given right lines are given, and the difference of the given rectangles is given; and conſequently the rectangle under *CK* and *DG* is given, and *CK* is given; therefore *GD* is given. And the three ſides in the Triangle *GCD* being given, the angle *G C D* will be found. *Q. E. F.*

PROPOSITION XXXVIII.

TO *find the Angle of Commutation, wherein two given Planets ſeen from the Sun are diſtant, and the angle wherein the Sun and either of the Planets ſeen from*

from the other are diſtant, when one of the given Planets ſeen from the other appears Stationary.

Let *S* be the Sun [Fig. 42.] *RV*, *PT* the Orbits of the given Planets; and let *CD*, *RA* be the Arcs deſcribed in the ſame time by the Planets, while they are ſeen from the other Stationary, which are given by what has been ſaid at *Prop.* 37. Join *SR*, *SC*, and *RC*, *AD*, which (by the preced.) will be parallel. Thro' *C* draw the lines *CG* parallel to *RA*, alſo thro' *G* the right line *GK* parallel to *RS*, and thro' *D* the right line *DK* parallel to *CS*, which may meet in *K*, join the right line *CK*. The Triangle *GKD* will be equiangular to the Triangle *RSC*, (ſince each of the ſides of that, is parallel to each of this,) and conſequently ſimilar. Becauſe the Time of the Station of the Planet is very ſmall, if compared with the periodic Time even of the ſwifteſt, the line *CD* does not ſenſibly differ from a right line, as alſo *RA* or *CG*, much leſs. The former therefore does not differ from *HC* a right line produced touching the Orbit *PT* in *C*, and the latter produced will be the right line *CL* perpendicular to *SR*, as *AR* is to the ſame. Moreover, a Circle made upon the diameter *CK* paſſes thro' *D* and *G*, becauſe the angles *CGK*, *CDK* are right; being by conſtruction equal to the right ones *SLC*, *SCH*. *CGDK* therefore is a Quadrilateral inſcribed in a Circle, one of whoſe ſides *CK* is the diameter of the Circle, and in which are given the ſide *CG* and the diagonal *CD*; as alſo the ratio between the oppoſite ſide *DK* and the other diagonal *GK*, namely, the ſame that there is between the Semidiameters *CS*, *RS* of the given Orbits. And therefore (by the Lemma premiſed) the angle *GCD* will be found, to which the vertical angle *HCL* is equal; and to this again the angle *RSC* is equal, which is the Angle of Commutation, whereby the two given Planets ſeen from the Sun are diſtant, when the one ſeen from the other appears Stationary. Moreover, in the Triangle *RSC* the two ſides *SR*, *SC*, and the angle *RSC* contained by them being given, the angles *SRC*, *SCR* will be found; the former of which is that angle whereby the inferior is diſtant from the Sun, while it appears Stationary from the ſuperior, the latter is the angle whereby the ſuperior is diſtant from the Sun, while it appears Stationary from the inferior.

This Method of defining the Points of the Stations fit for Calculation and more natural, and as it were depending upon the motion of the Spectator about the Sun, I have demonſtrated; paſſing over the Geometrical Conſtruction of this Problem laid down by *Ptolemy in B.* 12. *of his Almageſt.* accomodated to the Hypotheſis of the Earth's being at reſt, which *Apollonius Pergæus* formerly gave, and which *Copernicus* notwithſtanding ſays, that it no leſs agrees with his Principles of the Motion of the Earth, in *B.* 5. *Ch.* 35. *Revolut.* and which you may ſee in the abovemention'd Authors.

If the plane of two Orbits be not the ſame, the one may be reduc'd to the other, as letting fall from all the points of the Orbit, which is to be reduced, perpendiculars to that other, by which means the circular Orbit becomes Elliptic, the the Elliptic is chang'd into another Ellipſe, or perhaps into a Circle. But a new Orbit is more eaſily drawn (by *Prop.* 33.) thro' five Points, two of which are the ſame with two of the Orbit, to whoſe Plane the other is reduc'd, namely, the common Sections of that with this; the three others are found, by ſeeking the Curtated Diſtances from the Sun, as was done in *Prop.* 35. And if the other Orbit, or both after the reduction become Elliptic, the Problem is conſtructed in them

after

mean Time pitched upon according to the reckoning of the place, for which the Tables are made, you have marked by the help of the preceding Propoſitions, the Heliocentric Longitude of the ſaid Planet, of the Aphelion and aſcending Node of the ſame; the firſt whereof, namely the Diſtance of the Planet from the Equinoctial, conſiſts of two heterogeneous parts; namely the diſtance of the Aphelion of the Planet from the Equinox, expreſſed by the angle, and the diſtance of the Planet from the Aphelion, expreſſed by the area of its Orbit.

In the ſecond place, the Mean Motions from the Equinox of the Planet it ſelf, *of the Aphelion* and Node, for ſingle Years, Tens, *Hundreds, &c.* alſo for Months, Days, Hours, and parts of Hours, are diſpoſed into Tables.

In the third place, a Table is ſubjoined, exhibiting the true Anomaly of the Planet to the ſeveral degrees of the mean Anomaly of the ſame. This Table may be made either by *Prop.* 3. or by Approximation by the 4th, 6th, or 7th Propoſitions, according to the nature of the Approximation which a Perſon has a mind to uſe.

In the fourth place, you have a Table exhibiting the Diſtance of the Planet from the Sun, (found by the ſame Propoſitions, together with *Prop.* 22.) in parts, whereof the mean Diſtance from the Earth contains 100000, for the ſeveral degrees of the mean Anomaly. Theſe two Tables are advantageouſly reduced into one, by ſetting next to each degree of the mean Anomaly, the agreeable true Anomaly and correſponding Diſtance. To theſe Diſtances, for eaſe of Calculation, are annex'd the correſponding Logarithms; or even the Logarithms of the Diſtances alone are diſpoſed into Tables, ſince they are ſufficient for Calculation.

In the fifth place, is put a Table of the Centric Latitude, or Inclination of a Planet to the Plane of the Ecliptic for the ſeveral degrees of the Diſtance from the neareſt Node, which Diſtance from the Node is commonly called by Aſtronomers, *The Argument of the Latitude.*

In the ſixth place, you have a Table of *Reduction*, whereby the Place of a Planet in its proper Orbit is reduced to a Place in the Ecliptic, ſhewing how much more or leſs forward it is in the latter than in the former.

In the ſeventh place, you have a Table ſhewing how much the interval of a Planet from the Sun in its proper Orbit is to be curtated, to reduce it to the plane of the Ecliptic, by letting fall from the Planet a perpendicular upon it. And ſince theſe three (namely the Inclination, Reduction and Curtation) are fitted to the ſeveral degrees of the Argument of Latitude, the three preceding Tables will commodiouſly come into one; as was ſaid before concerning the third and fourth Tables. Theſe three things are found by common Trigonometry, having the Inclination of the Orbit of the Planet to the plane of the Ecliptic, which is done by *Prop.* 20.

In the eighth place, is advantageouſly placed the Angle of Commutation (found by *Prop.* 38.) making in that Planet an apparent Station from the Earth. And becauſe neither the Orbit of the Planet nor of the Earth is circular, neither can the ſame Angle of Commutation ſerve in every caſe, nor the ſame in the two next Stations; the Angles of Commutation producing both the firſt and ſecond Station of the Planet, are diſpoſed into a Table, for four or eight of the principal Anomalies.

In every one of the Inferiors may commodiouſly be added a Table, (made by *Prop.* 36,) coalexing with the former, of the Angles of Commutation in the greateſt Elongation of the Planet from the Sun, as well towards the Eaſt as towards the Weſt, fitted to the ſame *Anomalies.* To theſe, the Depreſſion of the Sun below the Horizon in the Moments of Heliacal Riſing or Setting is alſo added. And theſe are the common Tables uſually made by Aſtronomers for the ready finding out the Places of the Planets.

PROPOSITION XL.

TO calculate the Heliocentric and Geocentric Place of any Planet, in regard of Longitude and Latitude at any time propoſed, by the help of the above deſcribed Tables, and to define its Paſſions.

Firſt of all, let the Time propoſed be equated, as was ſhewn in *Prop.* 17; let the Place of the Earth ſeen from the Sun, and its Diſtance from the Sun be found for that mean Time, as in the 18th. And ſince by the help of the firſt Table at the aſſum'd Epocha there are given the diſtances of the Planet propoſed, and of its Aphelion and aſcending Node from the Equinox; and by the ſecond, their Mean Motions made between the ſaid Epocha and the Time propoſed, being equated, as was ſaid; the mean Diſtance of the Planet from the Equinox, and the Diſtances of the Aphelion and the Nodes from the ſame will be given. The difference of the two former is the Planets mean Anomaly; to which, the correſpondent true one or angular Heliocentric Diſtance from its Aphelion may be taken, out of the third Table; and out of the fourth, the Diſtance of the ſame from the Sun. The Diſtance of a Planet from the Node, or the Argument of Latitude will likewiſe be given; to which, the correſponding Inclination, or Centric Latitude may be

en, out of the fifth Table; and out of the
h, the Reduction, whereby it is reduced to the
iptic; and out of the ſeventh, the Curtation,
ich being ſubtracted from the Diſtance of the
net from the Sun in its own Orbit, firſt taken
of the fourth Table, leaves the Curtate Di-
ice of the Planet from the Sun.

fter this manner may be had the following
igs in the Diagram of *Prop.* 35; *viz.* the right
ST [*Fig.* 43.] in magnitude, namely the Di-
ice of the Earth from the Sun; the ſame in
ition, namely the Heliocentric Place of the
th; the right line SE in magnitude, *viz.* the
rtate Diſtance of the Planet from the Sun; and
ſame in poſition, namely the Place of the
net reduced to the Ecliptic: And conſequently
angle TSE is given, contained between right
s given by poſition, which is alſo the angle of
mmutation; therefore the angle STE the E-
gation of the Planet from the Sun ſeen from
Earth will be found. But the Place of the
n ſeen from the Earth is given; and therefore
Geocentric place of the Planet in the Eclip-
, and TE the Diſtance of the Planet from the
th are given.

Again, becauſe LE is perpendicular to the
ne of the Ecliptic $TSNE$, the Triangles
L, TEL are rightangled at E; and therefore
Tangents of the angles ETL, ESL are in
ſame ratio with ES, ET: But ESL is the
liocentric Latitude of the Planet firſt taken
t of its proper Table, and the ratio of ES to
is given; for the right lines themſelves are
ven, (but the ratio of theſe is the ſame with the
io of the Sines of the oppoſite angles STE of
ongation, and TSE of Commutation;) where-
e ETL the Geocentric Latitude of the Planet
known.

[illegible] it may [illegible] whether
by [illegible] in their general Elongation
And by the Place of the [illegible]
and the known Declination of the
[illegible] according to the Table it may
[illegible] by [illegible] whether the P
or [illegible] Heliacally, or less and
of the Sun. [illegible] by the
[illegible] the Phasis of the
[illegible] the Phases
as of Mercury. It is great time
[illegible] But Jupiter and [illegible] always
as we [illegible] in [illegible]

In the Prorogation we [illegible]
[illegible] of all this, [illegible] whether this
subtracted from that, [illegible]
that in the different [illegible] of Declina
tions Northern and Southern. Elon
[illegible] and Western, [illegible]
how [illegible] of [illegible]
lar [illegible] is to be found by
For we do not here [illegible]

Nodes of the Orbits of the Planets to be at reſt; as *Street* of all the Planets, *Flamſtead* at leaſt of the Apſides of the Earth.

For every Planet's Radixes are to be ſettled, not of the Longitude of a Planet, and of its Aphelion and Node, as before, but of the mean Anomaly of the Planet: And the Motion of the Anomaly for Years and ſums of Years, Months, Days, (or rather for the particular days of the ſeveral Months current,) Hours, *&c.*

To theſe are added the following things, which always continue the ſame, determining the ſituation of the Orbit of the Planet, and exhibiting the intire Theory expreſſed by numbers; namely 1. The Diſtance computed in the Ecliptic of the Aphelion from ſome one of the Fix'd Stars (for inſtance, the firſt of Aries, or the difference of Longitude of the ſaid Fix'd Star and Aphelion of the Planet, which is called *the Longitude of the Aphelion from a Fix'd Star.* 2. The Longitude of the aſcending Node of a Planet from the ſame Fix'd Star. 3. The Inclination of the Orbit of a Planet to the plane of the Ecliptic. 4. The Magnitudes of the greater Axe of the Orbit of a Planet and of its Excentricity, expreſſed in parts, whereof the mean diſtance from the Earth contains 100000. For from theſe, by the reſpective Propoſitions, the Longitude of the Planet from the firſt Star of Aries, the Inclination or Heliocentric Latitude Northern or Southern, and the Curtate diſtance of a Planet from the Sun are found, for any mean Anomaly of the Planet. Out of theſe artificially annexed to the mean Anomaly, to which they belong, are made Tables; though inſtead of the Curtate Diſtance itſelf, the Logarithm alone will ſuffice for Calculation. For ſince (by *Prop.* 43. *B.* 1.) the mutual ſituation of a Fix'd Star and the Orbits of the Planets remains the ſame, or the Lines of the Nodes and Apſides are at reſt, (at leaſt they are taken to be ſo by thoſe Authors,) to the ſame point of the ſame Orbit, that is, to the ſame mean Anomaly, theſe three continue the ſame and unvaried in the ſame Planet. Theſe being taken out of the Tables, the reſt is done, as above in the Propoſition. But the Longitude of the firſt Star of Aries at a given Time is found by the Tables of the Fix'd Stars, of which ſee *Prop.* 68. *B.* 2. What remains about the Paſſions of the Planets is to be done after the ſame manner as above.

Such Tables as are more ready for uſe than thoſe mention'd above, are called *Ephemerides of the Celeſtial Motions*, being calculated, from the perpetual Tables deſcribed above, for a certain number of Years, and they ſhew the Places of the ſeveral Planets as to Longitude and Latitude, their Paſſions, and mutual Aſpects, as well when they are viewed from the center of the Sun, as from the center of the Earth, for every day; to which alſo are added the Places of the Moon computed for every day, its Phaſes, the Eclipſes of the Sun and Moon, and other things found in the following Book.

PROPOSITION XLI.

IN the Primary Planets compared together, the Squares of the Periodic Times are as the Cubes of the greater Axes of the Elliptic Orbits, which they deſcribe about the Sun.

Let *S* [*Fig.* 44.] repreſent the Sun; *MFTN* the Orbit of the Earth, whoſe Apſides are *M* and *N*, *AEPB* the Orbit of any other Planet, whoſe Apſides are *A* and *P*, which are ſuppoſed to be ſo delineated in the Scheme, as that the Inclination of the lines *MN*, *AP* may be the ſame with that found by what has been ſaid about the Lines of the Apſides of the ſaid Planets in the Heavens;

and the ratio of the Cubes of them, the ſame with that of the Squares of the Periodic Times of the ſame Planets. I ſay, that the Orbit of the Earth remaining, another Planet can't move in any other Orbit beſides that juſt now deſcribed *A E P B*. For if it can, let the different Orbit of the ſaid Planet be *α ε π β*; that is, greater or leſs than the Orbit *A E P B*. Let *F* denote the point in the Orbit of the Earth, wherein the Earth is when it is in conjunction with any other Planet Heliocentrially. The line *S F* drawn (and if need be, produced) will ſhew in the Orbit *A E P B*, the point *B*, wherein the other Planet would then be found if it moved in this Orbit; and in the Orbit *α ε π β*, which it is ſuppoſed to deſcribe, the point *β*, in which it is really found. The Earth removing from *F* and the other Planet from *β*, after a certain time, let the former be found in *T*, the latter in *ε*: *S ε* joined will meet the Orbit *A E P B* in the point *E*, in which the Planet would have been found, if it had deſcribed the Orbit *A E P B*, ſince the Periodic Time is ſufficiently known and certain. The right lines *E T*, *ε T* being drawn, the angle *S T ε* will be the Elongation of the Planet from the Sun at the ſaid time: But this Elongation is not agreeable to obſervation, for 'tis the Elongation *S T E* (drawn from the ſuppoſition, that the Planet deſcribes the Orbit *A E P B*) that does. And therefore ſuppoſing *M F T N* to be the Orbit of the Earth, that other Planet does not deſcribe the Orbit *α ε π β* different from the Orbit *A E P B*. But the Cubes of the greater Axes of the Orbits *A E P B*, *M F T N* are as the Squares of the Periodic Times of the Planets moved in thoſe Orbits. And the ſame is collected of the other Planets compared with the Earth, for the like reaſons: And therefore *ex æquo*, in any two Primary Planets compared together, the Squares of the Periodic Times are as the Cubes of the greater Axes of the Orbits, wherein they are carried.

And generally the agreement of the Places *by* calculation with thoſe obſerved, argues the true Situation and true Species of the Orbit of any Planet, as well as the true Magnitude. For the Situations of the Earth and the other Planets are ſo variouſly intermingled, that any notable errors committed in the ſettling the Orbit and Theory of any Planet would ſometimes manifeſtly diſcover themſelves, and become very ſenſible: But on the contrary, the Places of the Planets drawn from Tables depending upon the *ſaid* Theories, ſtrangely agree with their Places by *obſervation.* The very ſmall difference from obſervation in the mean while is owing to other cauſes; namely the action of the Planets upon one another, the effects whereof in diſturbing the motions of the Planets are afterwards to be conſidered, and their Orbits rectified from thence.

PROPOSITION XLII.

THE Geocentric Place of a Planet at a given Time being given, to find by the Tables the Place of the ſame ſeen from a given Place or Habitation upon the ſurface of the Earth as that time.

If the Habitation given be not in the ſame Terreſtrial Meridian with the Place to which the Tables are fitted, let the given Time be reduced to the reckoning of the Place of the Tables, by the Tables deſcribed *Sect.*9.*B.*2. To that Time find (*by Prop.*36. *B.*2.) its Altitude, ſeen from the center of the Earth, above the horizon of the Habitation given, and the Azimuth. Let the Parallax of the Sun, that correſponds with the Altitude thus found, be taken out of the Table made by *Prop.* 47. *B.* 2; from the Parallax of one Diſtance and of one Altitude, or known by ſome Prop. of the 2d Book, or by one of the following: And (by *Prop.* 48. *B.* 2. the Parallax of the Planet in the ſame Altitude above the Horizon, will be to the Parallax of the Sun taken above, as the Diſtance of the Sun from the Earth, to the Diſtance of the Planet from the ſame: But theſe Diſtances in the Place of the Planet to be computed by the preced. were known; wherefore the Parallax of the Planet correſponding to that Altitude is found. This therefore is to be ſubtracted from the Geocentric Altitude found above, that it may become the Altitude once corrected in the given Habitation. To the Altitude thus corrected add the correſponding Refraction, from the Table of Refraction made by *Prop. 66. B.* 2. of the uſe of which we have ſpoken in *Prop.* 68. of the ſame; and you will have the apparent Altitude of the Planet in the given Habitation, the Azimuth firſt found remaining unchanged. From this apparent Altitude and Azimuth, you may find by *Prop.* 26, and 27. *B.* 2. the Planet's place, which will be the apparent one in the given Habitation. The Time propoſed reckoned in the Meridian to which the Tables are fitted, and afterwards reduced to the Meridian of the given Habitation, ſince it is a mean, (ſuch as it ought to be when Tables are uſed) 'tis reduced to apparent Time, by *Prop.* 17. Therefore the apparent Place of the Planet, at the given Time now made the apparent, is found.

SECTION VI.

Of the Magnitude of the Orbits of the Planets.

IN what precedes, we have conſidered the Planets themſelves, as ſo many points moving in their Orbits; that is, we have abſtracted from their Magnitude. And in this caſe it is the ſame, as to the Phænomena, what Magnitude the Orbit of any of them is ſuppoſed to be of, provided all the reſt have that ratio and *poſition to this*, as they have in nature. Therefore *we have every* where ſuppoſed the common meaſure of all the Orbits and their parts to be the greater Axe of the Orbit, that the Earth (the Habitation of the Spectator) deſcribes about the Sun, or half of it, the mean Diſtance of the Earth from the Sun, without being concerned about the Magnitude of this apparent Diſtance. But now when the Magnitude of the other Planets comes to be compared with the Magnitude of the Earth, the aforeſaid Diſtances muſt be compared with the Diameter of the Earth; which may be done by the help of the Parallax being known, as was ſhewn in *Book* 2. And tho' the Parallax of the Planet might be found by one of the methods of finding the Parallax delivered in *Book* 2, if you could obſerve it as accurately as you pleaſe; yet notwithſtanding, becauſe there are ſeveral things that are too obvious which are an hindrance to it, I ſhall propoſe two methods by which this obſervation, ſo ſlippery as it is, may be made without any danger of a notable error. In the firſt it ſhall be done by two obſervations made together in two Places of the Earth; in the ſecond, by two made in the ſame Place.

PROPOSITION XLIII.

BY the Conjunction of two Planets, obſerved in two given places of the Earth, to find the Parallax of both.

Let the true Places of the two Planets obſerved in *Corporal* Conjuction (that is, where the one ſeems to touch or cover the other) be *L* and *E*. Imagine the Circle *A B* [*Fig.* 45.] to be drawn thro' the vertices *A* and *B* of the Places on the Earth. Draw the vertical Circles *A E M*, *A L O*, *B E P*, *B L N*. Let the Planets be obſerved from one of the given Places (namely that whoſe vertex is *A*) in *O* and *M*. And ſince the Places are given where the obſervations are made, (by *Prop.* 32. *B* 2.) the difference of the Meridians will be given; and ſo the names of the Hours, whereby any the ſame inſtant of time is expreſſed in the ſaid two Places: And therefore the ſame moment of Time may be pitched upon to make the obſervation in the given Places. Therefore at the ſame inſtant that the Planets in *O* and *M* are obſerved from the Place whoſe vertex is *A*, let them be obſerved from the Place whoſe whoſe vertex is *B*, for inſtance, in *N* and *P*; Draw the great Circles *L E*, *O M*, *N P*. Obſerve alſo (either by a Micrometer, or what is inſtead of it) Threads ſtretched various ways in the Focus of the Teleſcope, or by the help of an Icoſcope throwing the Image upon a Plane, or by any other method to be left to the induſtry of the Artiſt) the magnitudes of the Arcs *NP*, *O M*, (which in a corporal Conjunction, by reaſon of their ſmallneſs, will be right lines) and of the Angles *P N B*, *N P B*, *A O M*, and *A M O*. Now, by the Theory of theſe Planets, there is given the ratio of their Diſtances from the Center of the Earth, from whence, and their Altitudes (obſerved ſecondly, the reſolution whereof will give the value of the Root, or ſine of the Arc LN; from whence all the reſt that is ſought will become known. But if ſuch Places upon the Earth be choſen, as have the Planets paſſing the common Azimuth of the Places, and that inſtant of Time wherein one of them is in the ſaid common Azimuth; this conſtruction will become that particular one in *Prop.* 87. *Opt. Prom.* of *James Gregory.*

LEMMA.

THE difference of two Quantities and the ratio between them being given, to find the Quantities themſelves.

Let a quantity be taken, that is to the known difference, as the the greater term of the given ratio, is to the difference of the terms; this quantity will be the greater of the quantities ſought: For in four proportional quantities, the products of the two former are in the ſame ratio with the ſimilar products of the two latter.If the greater be leſſen'd by the given difference, what remains will be the leſs.

PROPOSITION XLIV.

BY two Obſervations of the ſame Corporal Conjunction of two Planets, made in the ſame Place, to determine the Parallax of both Planets.

Let the apparent motion of the two Planets reduced to the Ecliptic, made mutually by one another (or to one another) in a given time, be obſerved near the Corporal Conjunction: And by the Theory of both Planets, known by the preceding, let their motion from one another mutually be determined, computed in the Ecliptic, viewed from the Center of the Earth, made in the ſame time; which may be done accurately enough, if the time be not too great. From theſe arcs therefore given, namely, the true and apparent motion of the Planets from one another, made in the ſame time, the Parallax of both is to be determined.

Let *HO* [*Fig.* 46.] repreſent the Horizon; *Z* the Zenith, and *EC* the Ecliptic: And let *SV* be the arc of the Ecliptic, wherein the Planets (viewed from the Center of the Earth) move mutually from one another in a given time, *known* from the Theory; and *AB* the arc of the Ecliptic, whereby they ſeem to move from one another in the ſame time, known by Obſervation; therefore their difference is given. Thro' *A* and *B* imagine Circles of Latitude *AD*, *BF drawn*, meeting the Verticals *ZH*, ZO, drawn thro' *S* and *V*, in *D* and *F*; and *SA* is the Parallax of the Longitude of the nearer Planet from the more remote, or the exceſs of the greater Parallax of Longitude above the leſs in this ſituation; and *VB* the exceſs of the greater Parallax above the leſs in the other. The ratio of the arc *AS* to the arc *BV* is compounded of the ratios of *AS* to *SD*, *SD* to *VF*, and *VF* to *BV*; but the ratio of *AS* to *SD* is known, becauſe in the rectilineal Triangle *ASD*, rightangled at *A*, the Triangle *ASD* is given, found by *Prop.* 36. *B.* 2; the ratio of *SD* to *VF* is found, by *Prop.* 47. *B.* 2; and the ratio of *VF* to *BV* is found as before, in the ratio of *AS* to *SD*; wherefore the ratio of *AS* to *BV* compounded of them is known: But the difference of the arcs *AS* and *BV* is alſo known, namely the ſame with the given difference of the arcs *AB* and *SV*; and therefore (by Lem. preced.) the arcs themſelves *AS* and *BV* will be given. And therefore (by Dat. 2.) *SD* and *VF* will be given; that is, the exceſs of the greater Parallax above the leſs in a given Altitude: For the Altitude *HD* or *OF* may be obſerved exact enough for that purpoſe. And from thence the exceſs of the greater Horizontal Parallax above the leſs (by *Prop.* 47. and 63. *B.* 2.) will alſo be given: But by *Prop.* 35.) the ratio between the Diſtances of the given Planets from the Earth at the time of Obſervation is given, (for the Diſtance of both from the Earth is expreſſed in the parts of the mean Diſtance of the Earth from the Sun;) and therefore the ratio of the Horizontal Parallaxes of the ſame Planets is alſo given, becauſe (by *Prop.* 48. *B.* 2.) the reciprocal of the former: And therefore (by Lem. preced.) the Horizontal Parallaxes of the Planets themſelves are alſo given.

And from hence the Parallaxes of all the Planets are determined. For the Parallax of one of the Planets being given, the Parallax of all of them (by *Prop.* 49. *B.* 2.) are given; becauſe (by the preceding Propoſition) at a given time the ratio between their Diſtances from the center of the Earth is given.

SCHOLIUM.

Tho' any two Planets in any Conjunction (after the manner ſhewn in the preced. Propoſition) may be uſed to determine the Parallax of both, nay any Planet (Mars, for inſtance, in his Achronical ſituation, and neareſt to the Earth) in conjunction with a Fix'd Star, whoſe Parallax is nothing, may ſuffice to determine its Parallax, (at that time the greateſt,) one of the inferior Planets paſſing over the diſc of the Sun, is fitteſt for that purpoſe; eſpecially Venus, whoſe Parallax is very ſenſible, becauſe ſhe is then nearer; and the Theory, ſo far as it is neceſſary for this purpoſe, is accurately enough known.

PRO-

PROPOSITION XLV.

TO *determine the greater Axes of the Orbits of the Planets, Excentricities,* &c. *in known meaſures.*

For any time (by one of the two preceding, or any other method ſhewn in *B.* 2.) find the Parallax of any Planet, correſponding with any known apparent Altitude; and from thence the ratio of the Diſtance of the Planet from the center of the Earth to the Semidiameter of the Earth will be given; namely the ſame (by *Prop.* 46. *B.*2.) with that which is between the ſine of the apparent Diſtance from the vertex and the ſine of the Parallax. But (by *Prop.* 35.) *the ratio* between the Diſtance of the Planet from *the center* of the Earth and the Diſtance of the ſame from the Sun at that known time; and (by the preced. Prop.) for the ſame Time the ratio of the diſtance of the Planet from the Sun to the greater Axe of the Orbit, and its Excentricity, *&c.* is given: And therefore, *ex æquo*, and by (*Dat.*2.) the ratio of thoſe right lines to the Diameter of the Earth will be given, that is, all theſe Diameters of the Orbits will be given, expreſſed by Diameters of the Earth. And ſince, by the *Schol.* of *Prop.* 17. *B.* 2. the Diameter of the Earth is found in known and familiar meaſures, the aforeſaid Diameters of the Orbits will be given in the ſame.

By this means the Diſtances of the Sun and Primary Planets from the Sun at any time propoſed, expreſſed by Semidiameters of the Earth or any other known meaſures, become known. For thoſe Diſtances at that time expreſſed in the parts of the mean Diſtance of the Earth from the Sun are known, (by *Prop.* 34. and 35,) and (by this) the ſaid mean Diſtance of the Earth from the Sun is known in known meaſures.

SECTION VII.

Of the Magnitude and Denſity of the Sun and Primary Planets.

PROPOSITION XLVI.

To define the Magnitude of the Sun and Primary Planets.

'Tis very proper in this place, having laid down the Theory of the Planets, to treat alſo of their Magnitude and Denſity, upon which conjunctly the Quantity of Matter in each depends; eſpecially ſince ſome things in their Motion and the dimenſion of their Orbits come to be rectified by this means.

Let ſome one of them be *ABC*, [*Fig.* 47.] whoſe Center is *P*, and let O be the Eye of the Obſerver. Join the right line O *P*, along which imagine a plane drawn cutting the ſpherical Body of the Sun or Planet in its greateſt Circle *BAC*. From O draw right lines O *A*, O *B* touching it, and join the right line *P A*. The angle *A* O *B* is the ſame with the apparent Diameter of the ſaid Body at O, and is biſected by the right line O*P*. Let this be obſerved by the methods well known to Aſtronomers, by which means, in the Triangle *A* O *P*, all the angles will be known: For (by *Prop.* 16. *El.* 3.) *A* is a right angle, and *A* O *P* is half the obſerv'd apparent Diameter; hence the ratio of the ſides O *P* and *A P* is known: But (by *Prop.* 45.) the ratio between the Semidiameter of the Earth, and O*P* the Diſtance of the Planet is given; therefore *ex æquo*, the ratio between the Semidiameters and the Earth and Planet is given; therefore the triplicate of this is alſo given, namely the ratio between the Earth and the Planet. But the Magnitude of the Earth is given by *Schol. Prop.* 17. *B.* 2; and therefore the Magnitude of the Planet (by 2 *Dat.*) is given. *Q. E. F.*

If thro' *T* and *P* the centers of the Earth and Planet [*Fig.* 48.] a Plane be drawn cutting them both, the common ſections of this plane with thoſe Bodies will be great Circles in them. But if you draw the Tangents *T A*, *PD*, and join *TP*, *T D*, *A P*; theſe two laſt do almoſt coincide with arcs deſcribed upon the Centers *T* and *P* and at the diſtance *T P*; and conſequently they are in the ſame ratio with the angles *T P D*, *P T A* that they ſubtend, the latter whereof is the apparent Semidiameter of the Planet, the former its Horizontal Parallax: And therefore reciprocally the Diameter of the Planet is to the Diameter of the Earth, as the apparent Semidiameter of the Planet to its Horizontal Parallax. *But the three* latter terms of this Proportion are given; and therefore the firſt, namely the Diameter of the Planet.

SCHOLIUM.

By the aforeſaid method, the Magnitudes of the Sun and all the Planets, excepting the Earth, may be accurately compar'd together. For their Diameters are in a ratio compounded of the ratio of the Diſtances, and of the ratio of the Sines of the ſemi-angles under which they appear, that is, almoſt of the apparent Diameters: But the ratio of the Diſtances found by *Prop.* 35. is the ſame with the true by *Prop.* 41; and the ratio of the apparent Diameters is certain by obſervation. But the compariſon of the Magnitudes of thoſe Bodies with the Magnitude of the Earth depends upon the Parallax of ſome Planet. If this be not accurately given, the ſaid compariſon can't be made accurately. We have already given the beſt methods of finding it in their proper places: But the laſt, which you have in *Prop.* 44. ſeems moſt fit for uſe, if it be applied to Venus ſeen in the diſc of the Sun; which has not been yet done, nor can be done till the year 1761, (which will be the ſecond time of Venus being to be ſeen by us in the Sun.) If in the mean while the Diameter of the Earth ſeen from the Sun be made a mean betwee theDiameters of the five others ſeen from the ſame, (which 'tis very probable is not far from the truth, and beſides agrees very nearly with the Parallax of the Sun determined by an obſervation of Mercury ſeen in the Sun; and other obſervations made other ways,) the Magnitudes of all the Planets and Sun will come out near enough to their true Magnitudes, and ſuch as are to be taken for the true, 'till ſomething more certain be ſettled about the Parallax of the Sun by obſervations.

PROPOSITION XLVII. LEMMA.

TO find the ratio of the Diſtance of a given Secondary from its Primary, to the Diſtance of the given Primary from the Sun.

Let *S* repreſent the Sun; [*Fig.* 49.] *T* the Earth; *P* any Planet, whoſe Satellite is *A*. At *T* obſerve its greateſt Elongation from *P*, namely the angle *ATP*. Therefore in the Triangle *PAT* right-angled at *A*, the angle *T* being given, the ratio of the ſide *AP* to *PT* will be given: And by *Prop.* 35. the ratio of *PT* to *TS* is given; and the ratio of *TS* to the Diſtance of any other Planet from the Sun is alſo known; and therefore the ratio between the Diſtance of the given Satellites from its Primary and the Diſtance of any Primary from the Sun will not be unknown.

But if the Satellite propoſed be our Moon, the Problem can be ſolved no otherwiſe than by finding the Diſtance of the Sun from the Earth, (by one of the methods above, or ſome like it) and the Diſtance of the Moon from the ſame (by the ſame, or ſome other already delivered or hereafter to be delivered in *Book* 4th.)

PROPOSITION XLVIII.

TO find the ratio of the Quantity of Matter in the Sun to the Quantity of Matter in a given Primary, about which a Satellite revolves.

Let *S* be the Sun; [*Fig.* 50.] P the Primary Planet given, about which its attendant *A* revolves; and let *V* be any other of the Primaries taken at pleaſure: Make *P B* equal to *S V*, and imagine any Body placed at *B*; and (by *Prop.* 49. *B.* 1.) the Quantity of Matter in *S* to the Quantity of Matter in *P*, will be as the accelerative gravity of the Body *V* towards *S to the accelerative* gravity of the Body *B* towards P. And the ratio of the accelerative gravity of the Body *V* towards *S* to the accelerative gravity of the Body *B* towards *P*, is compounded of the ratio of the accelerative gravity of the Body *V* towards *S*, to the accelerative gravity of the Body *A* towards *P*, and of the ratio of the accelerative gravity of the Body *A* towards *P*, to the accelerative gravity of the Body *B* towards the ſame *P*. But (by *Prop.* 26. *B.* 1.) the accelerative gravity of the Body *V* towards *S*, is to the accelerative gravity of the Body *A* towards *P* in a ratio compounded of the ratio of the right line *S V* to the right line *P A*, and of the duplicate ratio of the periodic Time of the Satellites *A* about *P*, to the periodic Time of the Primary *V* about the Sun; and the accelerative gravity of the Body *A* towards the Planet *P* is to the acceleravity gravity of the Body *B* towards the ſame, in the duplicate ratio of the right line *P B* (or its equal *S V*) to the right line *P A*. And therefore the accelerative gravity of the Body *V* towards *S* is to the accelerative gravity of the Body *B* towards *P*; that is (as was ſhewn above) the Quantity of Matter in *S* is to the Quantity of Matter in *P*, in a ratio compounded of the three following; namely, the ratio of the right line *SV* to the right line *PA*, the duplicate ratio of the periodic Time of the Satellite *A* about the Planet *P*, to the periodic Time of the Primary *V* about the Sun, and the duplicate ratio of the right line *SV* to the right line *PA*. But the firſt and laſt compounding ratio make the triplicate ratio of the right line *SV* to *PA*: From whence the Quantity of Matter in *S* is to the Quantity of Matter in *P* in a ratio compounded of the triplicate ratio of *SV* to *PA*, and the duplicate ratio of the periodic Time of the Satellite *A* about *P* to the periodic Time of the Primary *V* about the Sun. But both the compounding ratios are given; namely the ratio of *SV* to *PA*, by the *Lemma* premiſed, and the periodic Times themſelves by obſervation: Therefore the ratio compounded of them will not be unknown; namely the ratio of the Quantity of Matter in the Sun to the Quantity of Matter in the Primary *P*.

Hence the ratio of the Quantity of Matter in the three Planets that have Satellites may be found; *viz*. by finding the ratio of the Quantity of Matter in every one of them to the Quantity of Matter in the Sun.

SCHOLIUM.

After the ſame manner you may find the ratio of the accelerative Gravity in the ſuperficies of the Sun to the accelerative Gravity in the ſuperficies of any Planet that has a Satellite. For the ratio of Gravity in the ſurface of the Sun, to the Gravity in *V*. *viz*. the duplicate of the ratio of the Diſtance *SV*, to the Semidiameter of the Sun, known from what goes before, and the ratio of Gravity in *V* towards the Sun to the Gravity in *A* towards the Planet, found as above, are had; as alſo the ratio of Gravity in *A* to the Gravity in the ſuperficies of the Planet *P*, the duplicate of the known ratio that the Semidiameter of the Planet has to the Diſtance of the Satellite from the Primary. Now it is evident, that the ratio compounded of theſe three known ratios, is the ſame with the ratio of Gravity in the ſuperficies of the Sun to the Gravity in the ſurface of the Planet *P*. And from hence again, *ex æquo*, the ratio of Gravity in the ſuperficies of one Planet to the Gravity in the ſuperficies of the other Planet, if both have Satellites, will be determined.

PROPOSITION XLIX.

TO *find the ratio of the Denſity of the Sun to the Denſity of any Primary Planet, about which a Satellite revolves.*

If there be two Bodies *A* and *B* of the ſame magnitude, 'tis evident the Quantity of Matter in *A* is to the Quantity of Matter in *B* as the denſity of *A* is to the denſity of *B*: And if they be of the ſame denſity, the quantity of matter in *A* is to the quantity of matter in *B*, as the magnitude of the Body *A* to the magnitude of the Body *B*: And therefore, tho' neither the magnitudes nor denſities are equal, the ratio of the quantity of matter in *A* to the quantity of matter in *B*, is compounded of the direct ratio of the denſities and the direct ratio of the magnitudes of them. Wherefore the ratio of the denſity of the Body *A* to the denſity of the Body *B*, is compounded of the ratio of the quantity of matter in *A* to the quantity of matter in *B*, and the ratio of the magnitude of the Body *B* to the magnitude of the Body *A*; that is, (in the preſent caſe) the Denſity of the Sun is to the Denſity of a Planet in a ratio compounded of the ratio of the Quantity of matter in the Sun to the Quantity of matter in the ſaid Planet, and the ratio of the Magnitude of a

Planet to the Magnitude of the Sun. But (by prec. *Prop.*) the former of the compounding ratios is given, if the Planet have a Satellite, and (by *Prop.* 46.) the latter is given; therefore the ratio compounded of them is given, namely of the Denſity of the Sun to the Denſity of the Planet.

COROLLARY.

From hence, *ex æquo*, the ratio of the Denſities of three of the Primary Planets is had, namely Saturn, Jupiter and the Earth. The Denſities of the three others may be analogically derived from hence. No doubt the Divine Being, the Wiſe Creator of the World, has placed the Planets at different diſtances from the Sun, that each of them may enjoy a greater or leſs degree of the Sun's heat, according to the degree of its Denſity: For denſer Matter requires a greater heat to perform its natural operations withal. But without doubt, ſome reſpect is had to the Magnitude of the Planet, ſince of like Bodies the leſſer, *cæteris paribus*, are heated ſtronger and more intimately; becauſe in reſpect of their Bulk, they have a greater ſuperficies, and conſequently receive more rays.

PROPOSITION L.

TO *correct the greater Axes of the Orbits of the Planets found before.*

In Propoſition 24, the greater Axes of the Orbits are taken in the ſubſeſquiplicate ratio of the periodic Time; that is, the Maſſes of the Planets are abſtracted from, as ought to be done before they are determined, and the Planets are looked upon as points revolving in Ellipſes about the center of the Sun at reſt and placed in the Focus. But becauſe the actions of the Sun and Planet are mutual, and a Planet is made by them to deſcribe an Ellipſe (by *Prop.* 58. *B* 1.) whoſe

Focus is the common center of Gravity of its ſelf and the Sun, ſimilar to that which it would deſcribe (by *Prop.* 51. *B.* 1.) attracted by the ſame Forces, about the Sun at reſt; and (by *Prop.* 54. *B.* 1.) the greater Axe of the Ellipſe that any Planet deſcribes about the Sun, revolving at the ſame time about the common center of Gravity, is ſhown to be to the greater Axe of the Ellipſe, that the ſame Planet would deſcribe about the Sun at reſt in the ſame periodic Time, in the ſubtriplicate ratio of the ſum of the Maſſes of the Sun and Planet to the Maſs of the Sun; therefore the greater Axe of the Orbit of each Planet (found by *Prop.* 24.) is to be increaſed in the ſaid ſubtriplicate ratio of the ſums of the *Maſſes of the* Sun and Planet to the Maſs of the Sun, that it may be correct. But (by *Prop.* 48.) the ratio between the Maſſes of the Sun and Planets are given, and therefore the ratio is given, wherein the greater Axes (found by *Prop.* 24.) of the Orbits are to be increaſed, that they may be correct.

PROPOSITION LI.

TO eſtimate the Errors of the motion of the Planets about the Sun, ariſing from their Action upon one another.

If *S* repreſent the Sun, [*Fig.* 51.] *ABCE* and *DL* the Orbits of two of the Planets; 'tis evident that while the nearer to the Sun is in *A*, and the more remote in *D*, (namely when the Planets are tending towards an Heliocentric Conjunction,) the nearer to the Sun will be a little accelerated, and the more diſtant retarded by the Attraction upon one another; and when they are arrived after the Conjunction to ſuch a ſituation as *C* and *D* is, that the more diſtant one is accelerated, and the nearer is retarded; and that near the Heliocentric Conjunction the Diſtance of the more remote

Planet from the Sun is leſſened, of the nearer is increaſed. And ſince the Maſſes of the Sun and both the Planets (by *Prop.* 48.) are given, and the mutual Diſtances from the ſuperiors are alſo given, and that (by the preced. *Prop.*) correctly; the ratio of the Attraction of each towards the Sun to the Attraction towards the other will be given, and conſequently the ratio of the Error ariſing from the mutual Attraction to the effect of that Force, whereby each is retained in its proper Orbit about the Sun.

But if all the Error is to be thrown upon the Planet that is more remote from the Sun, the Orbit of the nearer remaining, this will be done almoſt (by *Prop.* 58. *B.* 1.) by placing the Focus of the exterior Orbit in the common Center of Gravity of the Sun and of the interior.

Theſe Errors are moſt ſenſible in Jupiter and Saturn, becauſe of the vaſt Maſſes of thoſe Planets.

SCHOLIUM.

There are alſo ſome inequalities ariſing from the mutual Action of the Primary and their Satellites: For (by *Prop.* 63. *B.* 1.) 'tis not the Center of the Primary, but the Center of Gravity of the Primary and the Satellites, that deſcribes the Elliptic Orbit about the Sun. But becauſe theſe things depend upon the Quantities of Matter in theſe Secondaries, the method of determining which properly belongs to the following Book, we ſhall at preſent ſay nothing of them.

SECTION VIII.

Of the Figure of the Sun and Planets.

PROPOSITION LII.

TO *determine the Figure of the Earth; that is, the ratio that its Axe has to the Diameters that are perpendicular to them.*

In the firſt Book, at *Prop.* 31, we ſhewed in general, from the proper cauſes, that the Sun and Planets are of a Figure depreſſed towards the Poles, but elevated towards the Circle, that *lies* exactly between the Poles, approaching that of an oblate Spheroid. Now we come to determine this Figure by obſervations. And firſt the Figure of the Earth is to be defined.

Let the Figure *APEp*, [*Fig.* 52.] whoſe Center is C, repreſent the Earth; *Pp* its Axis, *AE* any Diameter of the Equator, perpendicular to the Axe. Let the lengths of Pendulums oſcillating in a Cycloid, in a given ſpace of time, in the Places *A* and *P*; and *AC*, *PC* will be *reciprocally* as the ſaid lengths of the Pendulums, and therefore *in* a known ratio. Let the ſame be done in any intermediate Place *B*, whoſe Latitude (*viz.* the angle *ACB*) is known, (by *Prop.* 18. *B.* 2;) and *AC*, *BC* will be reciprocally as the lengths of the Pendulums oſcillating in an equal ſpace of Time in thoſe Places *A* and *B*, and therefore in a given ratio.

For imagine a Canal filled with a Fluid and bent from *A* to *C*, and from thence to *P*. From the reſt of the Fluid 'tis evident that the Fluid in the Leg of the Canal *AC* is in equilibrio with the Fluid in the Leg *PC*, the centrifugal Force ariſing from the motion about the Earth's Axis, and the greater diſtance from the Earth's Center raiſing and retaining the Fluid in the Leg *CA*, at a greater height than in the Leg *CP*. Wherefore any portion of the Fluid in *CA*, is in equilibrio (*or* equally heavy) with the like ſimilarly poſited portion of the Fluid in the Leg *CP*, (which conſequently is alſo true of any homogeneous Bodies, tho' they be not Fluid;) and the upper points *A* and *P* are ſimilarly poſited in the Legs: And therefore homogeneous Bodies placed in *A* and *P*, which are as *AC*, *PC* are equiponderant towards the Center of the Earth. But the Gravity of a Body placed in *A*, which is as *PC*, is to the Gravity of another homogeneous Body ſtanding there, that is, as *AC*, as *PC* to *AC*, (namely the weights of homogeneous Bodies placed near one another, are as the Bodies themſelves;) and therefore the Gravities of homogeneous equal Bodies placed in *A* and *P* are as *PC* and *AC*; that is, reciprocally as the diſtances from the Center. Juſt after the ſame manner the Gravity of the Body in *B* may be ſhewn to be to the Gravity of an equal and homogeneous Body in *P*, as *CP* to *CB*; for the Fluid will continue at reſt in the Canal *BCP*, as in the former *ACP*: Wherefore *ex equo*, the abſolute Gravities of equal and homogeneous Bodies placed any where on the Surface of the Earth are reciprocally as the Diſtances from the Center. But the accelerative Gravity of a Body is as its abſolute Gravity applied to its Maſs. And therefore the accelerative Gravities of Bodies placed on the Surface of the Earth are as the diſtances of the Places from the Center reciprocally. Moreover, becauſe the oſcillation of a pendulous Body placed at *A*, and moved in a Cycloid, is ſuppoſed (by obſervation) to be performed in the ſame ſpace of time with the oſcillation of another at *P*; and (by *Prop.* 25. *Part* 2. of *Hugenius's Horologium Oſcillatorium*) the time of a free deſcent along a given axe of a Cycloid has a given ratio to the time of Oſcillation in that Cycloid; and (by *Prop.* 6. and 7. *Part* 3. of the ſaid Book) the axis of the Cycloid, which a heavy Pendulum deſcribes, has a given ratio to the length of the String ſuſpending the weight; and conſequently the time of a free deſcent along the given axis of the Cycloid, has alſo a given ratio to the time of a free deſcent along the length of the ſtring, namely, the ſubduplicate of the former: The time of a free deſcent along the length of a Pendulum placed at *A*, will be equal to the time of a free deſcent along the length of a Pendulum placed at *P*. But the ſpaces at *A* and *P*, run in equal times by a falling Body, are as the accelerative Gravities in the ſaid Places *A* and *P*, *viz.* the effects are as their cauſes; and the ſaid accelerative Gravities have already been ſhewn to be reciprocally as the diſtances from the Center; and therefore the length of a Pendulum oſcillating at *A*, is to the length of a Pendulum oſcillating at *P* in an equal ſpace of time as *CP* to *CA*. And after the ſame manner may it be ſhewn, that the length of a Pendulum at *B* is to the length of a Pendulum at *P* oſcillating in an equal ſpace of time as CP to CB: Wherefore, *ex æquo*, univerſally the lengths of Pendulums oſcillating in equal Times are reciprocally as the diſtances of the Places from the Center of the Earth. Since therefore the ratio of *AC* to any *BC* and the inclination of the ſaid *BC* to *AC*, and (among the reſt) the ratio of *AC* to *PC*, are found; the nature of the Figure *APEp* is defined, namely, of the Section of the Earth with the plane of the Meridian, by whoſe rotation about the Axis *Pp*, the Figure of the Earth is generated. *Q. E. F.*

If the Figure of the Earth were to be determined after the manner above deſcribed, there would be no need of meaſuring the length of a Pendulum oſcillating at the Pole *P*, in a given ſpace of time: For by obſerving ſeveral ſuch lengths in different Latitudes, the length of a Pendulum oſcillating at the Pole in the ſaid ſpace of time, may ſafely be concluded upon.

Again, in ſolving this Problem, there will be no need of fitting a Pendulum after ſuch a manner in different places, as to make it perform its oſcillations in the ſame ſpace of time: It will be ſufficient to carry into the other places a Pendulum fitted to one place, ſo as that its ſeveral ſingle oſcillations may anſwer a given ſpace of time, and mark how much a Clock, that has the ſaid Pendulum for its Ballance, is ſwifter or ſlower during one apparent revolution of a Fix'd Star to the ſame place, according as the places, into which it is carried, are nearer the Equator or Pole, than that place is to which the Movement is adjuſted, and from that acceleration or retardation it may be found by calculation, what length is neceſſary to make the ſingle oſcillations to be performed in the ſame ſpace of time, as they are in the place to which the Pendulum was firſt adjuſted.

COROLLARY.

Hence it follows, (the ſame things remaining) if a Circle be deſcribed upon the interval *CA* or *CE* [*Fig.* 53.] and the line *CB* drawn any how be produced to *D*, that *DB* is as the increment of the accelerative Gravity in the receſs from the Equator towards either Pole, or as the increment above the length of the Pendulum oſcillating at the Equator in the given time, agreeable to the place *B*. For it has been ſhewn above, that if *AC* repreſent the gravity or length of a Pendulum at the Pole, *CP* will repreſent the increment of the gravity or length of the ſaid Pendulum oſcillating in an equal time at the Equator; and therefore (producing *Pp* to *F* and *G*) *PF* will repreſent the increment of the gravity or of the length of the ſaid Pendulum at the Pole *P*. In like manner, if *CB* repreſent the gravity or length of the Pendulum at *A*, *AC* (that is, *DC*) will expreſs the gravity or length of the Pendulum at *B*; and conſequently *DB* the exceſs or increment of the gravity or length of the Pendulum at B, above the gravity or length of the Pendulum oſcillating in the ſame time at A.

If $APEp$, the ſection of the Earth with the plane of the Meridian be an Ellipſe, the increments of the gravity or of the length of the Pendulum in the different places of the Surface of the Earth, above the gravities or length of the Pendulum at the Equator, will be very nearly in the duplicate ratio of the right Sines of the Latitudes of thoſe Places. Thro' D draw DM parallel to the Axe Pp, meeting the Ellipſe in L, and the right line AE in the point M, from whence let fall the perpendicular MK upon CD; take DH equal to PF, wherefore CH is equal to CP, and join LH: Becauſe APE is an Ellipſe, (by *Prop.* 31. *B.* 1. *El. Conic.*) CP^q is to ML^q as the rectangle under AC and CE is to the rectangle under AM and ME: And, by the ſame, in the Circle AFE, CF^q is to MD^q as the rectangle under AC and CE to the rectangle under AM and ME; and therefore CF^q will be to MD^q as CP^q to ML^q. Wherefore CF is to MD as CP to ML; and therefore (by *Prop.* 19. *El.* 5.) PF is to LD as CP to LM; that is, HD is to LD as CH to ML; and therefore (by *Prop.* 2. *El.* 6.) LH is parallel to CM; and conſequently the Triangle LDH is ſimilar to the Triangle MDC, and ſimilarly poſited. But from the right angle M of the Triangle CMD the right line MK is drawn perpendicular to the Hypothenuſe DC, which alſo is very nearly done in the Triangle HLD, the particle LB of the curve of the Ellipſe being almoſt a right line and perpendicular to CD, becauſe the Ellipſe $APEp$ does not differ much from the Circle $AFEG$: Wherefore CD is ſimilarly divided in K, as HD is divided in B; that is, HD is to DB as CD to DK, or PF to BD as CD to DK.

Again, becauſe of the rightangled Triangle *CMD* divided by the right line *MK* perpendicular to *CD*, (by *Prop.* 8. *Elem.* 6.) *CD*, *DM*, *DK* will be proportional, or the ratio of *CD* to *DK* the duplicate of the ratio of *CD* to *DM*; that is, of the ratio of *CF* to *MD*. But it has already been ſhewn, that *PF* is to *BD* as *CD* to *DK*; and therefore *PF* is to *BD* in the duplicate ratio of *CF* to *MD*: But *CF* is the whole Sine, and *MD* the right Sine of the angle *ACB*, namely of the Latitude of the place *B*; and *PF*, *BD*, are the increments of the gravity or length of the Pendulum oſcillating in the given time, agreeable to the Places *P* and *B*, above the gravity or length of the Pendulum at the Equator: And therefore ſuppoſing *APEp* an Ellipſe, the increment of the gravity or length of the Pendulum at the Pole, above the gravity or length of the Pendulum at the Equator, is to the like increment in any other Place *B* of the ſurface of the Earth, very nearly in the duplicate ratio of the Radius to the right Sine of the Latitude of the ſaid place *B*. And the ſame may be ſhewn in any other place on the ſurface of the Earth: And therefore, *ex æquo*, if *APEp* be an Ellipſe, the increments of the gravity or of the length of the Pendulum in different places of the ſurface of the Earth, above the gravity or length of a Pendulum oſcillating the like Time at the Equator, in the duplicate ratio of the right Sines of the Latitude of thoſe places verynearly.

SCHOLIUM.

All theſe Premiſes are true, ſuppoſing the Earth to conſiſt of uniform and equally denſe matter. For we ſuppoſed the Fluid, wherewith the Canals *ACP*, *BCP* are filled, to be uniform and homogeneous. But if the matter whereof the Earth conſiſts, be denſer at the Center than towards the ſurface, the above mentioned increments of the di-

diſtances from the Center going from the Poles towards the Equator, and the proportional increments of Gravity and of the length of the Pendulum oſcillating in the given Time going from the Equator towards the Poles, will be greater than the Calculus upon the former ſuppoſition makes them; and the Axis of the Earth will be leſs in reſpect of the Diameters perpendicular to the ſame. For if this matter towards the Center, by which the Earth is denſer there, be imagined to be removed, or rather be looked upon ſeparately, that the remaining matter may be every where equally denſe; from what precedes 'tis evident, that the accelerative Gravity in different places of the ſurface of this equally denſe Earth, is reciprocally proportional to the diſtance from the Center. But the ſaid accelerative Gravity is ſtill increaſed, by reaſon of that redundant matter about the Center; and that (as is ſhewn in *Book* 1.) nearly in the duplicate reciprocal ratio of the Diſtance from the Center; Since therefore in the caſe of the uniformly denſe Earth its ſuperficies is elevated towards the Equator and depreſſed towards the Poles; and the Gravity at the Equator leſs than at the Pole in the ratio of the diſtance of the Pole from the Center to the ſemidiameter of the Equator; and further the gravity at the Equator to the aforeſaid redundant matter about the Center ſhould be leſs than at the Pole, in a duplicate ratio of the diſtance of the Pole from the Center to the ſemidiameter of the Equator, which is leſs than the former ſimple ratio, ſince that ratio is a ratio of leſſer inequality; and therefore from both cauſes together, or in the caſe of the Earth's being denſer toward the Center, the Gravity at the Center compounded of the two former, is leſs than the Gravity at the Pole in a ratio leſs than that is which the diſtance of the Pole from the Center has to the ſemidiameter of the Equator. And therefore from this leſſer Gravity at the Equator in reſpect of that which holds at the Poles, the Earth will be more elevated at the Equator, than what it would be from what goes before. And the ſame holds in the length of the Pendulum oſcillating in a given ſpace of time; becauſe it is ſhewn above to be proportional to the accelerative Gravity.

PROPOSITION LIII.

TO *determine the ratio that the Axis of the Sun, or of a given Planet, has to the Diameters perpendicular to it.*

Find by *Prop.* 26. *B.* 1. the ratio of the centrifugal Force in the Circle that is in the middle between the Poles of the Sun or of the given Planet to the centrifugal Force in the Equator of the Earth: But the ratio between the centrifugal Force in the Equator of the Earth to the Gravity on the Surface of the Earth is given, *viz.* the ſame with the ratio found above, that the exceſs of the Diameter of the Earth along the Equator has to the Axis of the ſame; and by *Schol. Prop.* 48. the ratio of the Gravity on the ſurface of the Earth to the Gravity on the ſurface of the Sun or of he given Planet is given: And therefore the ratio compounded of theſe will be given, namely he ratio that the centrifugal Force in the middle Circle, between the Poles of the Sun or of the iven Planet, has to the Gravity on the ſurface f the ſame. And this is the ſame with that which e exceſs of the Diameter of the Equator of the n or of the given Planet above the Axe of the me to the ſaid Axe, ſince (by preced. *Prop.*) that ceſs ariſes only from that centrifugal Force, d conſequently is proportional to it: There-

fore

fore (by *Dat.* 6.) the ratio between the Axes of the Sun or of a given Planet and its Diameter perpendicular to the Axe will be given.

SECTION IX.

Of the Diſtances of the Fix'd Stars.

PROPOSITION LIV.

TO *determine the Diſtance of a Fix'd Star by Obſervation.*

Having determined the Diſtance and *Magnitude* of the Sun and Primary Planets, what remains is only to diſcourſe a little of the Diſtance of the Fix'd Stars. And tho' the Diſtance of the Fix'd Stars be ſo immenſely great in compariſon of the Diameter of the Earth, that the later intirely vaniſhes and becomes inſenſible; and therefore all the ways laid down in this and the preceding Book for finding the Parallax of any Phænomenon, and conſequently the Diſtance, are uſeleſs in this matter: Yet the Diameter of the Orbit that the Earth deſcribes in the ſpace of a Year about the Sun, may ſeem big enough to be compared even with the Diſtance of the Fix'd Stars; and ſince we, who are carried on the Earth, poſſeſs ſometimes this, ſometimes that extremity of the Diameter of the ſaid Orbit, there ought to ariſe a very ſenſible Parallax of the Fix'd Stars from ſo ſenſible a removal of the Obſerver. The Obſervation may be made very commodiouſly after the following or ſome ſuch like method.

Let *S* [*Fig.* 54.] repreſent the Sun, *AMB* the *Orbis Magnus.* Chuſe two Stars *C* and *F* near one another, ſituated not far from the Ecliptic. Obſerve their Diſtance, when one, as C, is in oppoſition to the Sun, *viz.* the angle *FAC*. Obſerve their Diſtance again, when C is in conjunction with the Sun, namely the angle *FBC*. Therefore in the Triangle *FAB* all the angles being given, the ratio of the ſides is given, namely of *FA* the Diſtance of the Fix'd Star from *A* to *AB* the Diameter of the *Orbis Magnus* known by *Prop.* 45; therefore *AF* is determined. After the like manner the Diſtance of a Fix'd Star may be found, if the ſecond obſervation of the apparent Diſtance of the Fix'd Star be made, when C is not in conjunction with the Sun, (which perhaps would be difficult, by reaſon of the Sun's brightneſs,) but in any other known ſituation of the Earth, as when it is in *M*: For the right line *AM*, and its ratio to the known one *AB*, is known by the above cited *Prop*. 45.

Having once determined the Diſtance of a Fix'd Star, its Magnitude will be defined by the method laid down in *Prop.* 46, provided the apparent Diameter of the Fix'd Star be ſenſible.

SCHOLIUM.

Becauſe the Obſervations deſcribed above, or others like them neceſſary to determine the Diſtance of the Fix'd Stars, are ſo ſlippery and require ſo much nicety, that Artiſts dare not promiſe they ſhall obtain it by thoſe means; yet they think they have done ſomething very conſiderable, if they can ſhow by obſervation that there is a Parallax of the Fix'd Stars in reſpect to the *Orbis Magnus*: For by this means they could put the Motion of the Earth out of all doubt, which all acknowledge to be a matter worth their while to do. But this may be done pretty well by the following method.

The other things remaining, let the two Fix'd Stars *F* and C (*fig.* 55.) be ſuppoſed not to be near theEcliptic. Let a Plane drawn thro' them and the Sun cut the Orbit of the Earth in *A* and *B*, which Points will be given exactly enough for the purpoſe, by what has been ſaid in the preced. Book. Let the angles *CAF*, *CBF* (or the apparent Diſtances of the Fix'd Stars *F* and *C*, ſeen from the Earth, when ſhe was in *A* and *B*,) be obſerved, which will be remarkably enough different from one another, if *AB* be ſenſible in regard of *AC*, and the Stars C and *F* very unequally diſtant from the Sun, which ſeems probable to hold in Stars of a different Magnitude. Perhaps if inſtead of the two Stars C and *F*, two, ſuch as C and D, be obſerved, D will be ſeen from *A to be on this ſide* of C, from *B* on that ſide, (which may happen *in* ſo great a multitude of Fix'd Stars) or it may be, theſe two Stars will appear to coincide at one time of the Year, and to be ſenſibly diſtant from one another at another time of the Year: *In* which caſe the Annual motion of the Earth about the Sun will be evident by the bare uſe of a Teleſcope, without any Inſtrument neceſſary to take the diſtance of the Stars,.

The above mentioned method of obſerving, beſides that it is capable of receiving all the improvement, that any Inſtrnments have or will receive, is hardly any ways chargable, ſince there needs only a Teleſcope with a Micrometer, there is no need of erecting an Obſervtaory nor of rectifying a Perpendicular, all which are very uncertain, ſince it is not abſolutely certain that either Walls tho' the firmeſt, or Rocks or Mountains themſelves always retain the ſame ſituation. To this head we may refer perhaps what that celebrated Aſtronomer Mr. *Caſſini* has diſcovered about the Fix'd Stars; namely that the Firſt of Aries ſometimes appears ſplit into two equal ones, and diſtant by the length of theDiameter of either from one another,

ther, which he has alſo obſerved of the preceding Head of Gemini ; and ſome others, as ſome of the Pleiades ; and the middle in Orion's Belt, appear to be three, nay, four ſometimes.

PROPOSITION LV.

TO *obſerve the Parallax of the* Orbis Magnus, *by the acceſs and receſs of a Fix'd Star to or from the Pole of the Equator made in different Seaſons of the Year.* [Fig. 56.]

Let *S* repreſent the Sun ; *HME* the Orbit of the Earth, whoſe points *E* and *H* are the places of the Earth in the Solſtices. Further let *F* be a Fix'd Star near the Pole of the World, that is, let *AB* the Axis of the Earth *ATB* contain with a a right line connecting the Earth and the Fix'd Star an angle *BHF* or *BEF* pretty ſmall ; and let this Star be not far from a Solſticial Colure, and therefore in conjunction almoſt with the Sun in the other Solſtice, or let the Plane *EFH* be perpendicular to *HME* the plane of the Ecliptic. 'Tis evident that the angle *BHF* the diſtance of the Fix'd Star from the Pole of the World, when the Star is in oppoſition to the Sun, is leſs than the angle *BEF* the diſtance of the ſame from the Pole, when the Fix'd Star is in conjunction with the Sun. And converſely, if the angle *BHF* be found leſs by obſervation than *BEF*, this inequality muſt be thrown upon the Parallax of the Annual Orbit. And the Diſtance of the Fix'd Star is gained after this manner: The difference of the angles *BEF*, *BHF* is equal to the angle *F* ; and in the Triangle *FEH* the other angle *FEH* is given, namely the Latitude of the Fix'd Star at the Time, when the Earth is at *E* : From whence the ratio between *HE* the Diameter of the *Orbis Magnus*, and *HF* the Diſtance of the Fix'd Star will be known.

SCHOLIUM.

This method of finding by Obſervation the Parallax of the *Orbis Magnus*, that moſt induſtrious Obſerver Mr. *John Flamſteed* makes uſe of, and ſo anſwers the objection of ſuch, as with *Ricciolus*, deny the motion of the Earth about the Sun, becauſe it is confirmed by the obſervation of a Parallax of the *Orbis Magnus*. For he found the Diſtance of the Pole Star from the North Pole to be greater about the Summer Solſtice than about the Winter one, by about 40 or 45 Seconds, and that by a continued ſeries of Obſervations for ſeven Years, allowing for the correction of the Inſtrument, and the change of the Star's place on the account of the receſſion of the *Equinoctial points*: as Mr. *Flamſteed* learnedly diſcourſes in a Letter to Dr. *Wallis*, dated *Decemb*. 20, 1698. publiſhed in 3 Vol. of *Wallis's Mathemat. Works*. But this method of determining the Parallax of a Fix'd Star, ſuppoſes the Axis of the Earth *A B* [*Fig*. 57.] to be exactly parallel to itſelf, when the Earth is in the points *E* and *H* of its Orbit, when the Obſervations are made. And tho' that ſmall Nutation of the Axis which ariſes from the diminution of the Inclination of the Ecliptic and Equator at the Solſtices, and the increaſe of it at the Equinoxes (by *Corol*. 20. *Prop*. 66. *B*. 1. *of* Newton's *Math. Princip. Nat. Philoſ.*) does not affect or hinder this obſervation, as Mr. *Flamſteed* obſerves in the ſaid Epiſtle; yet another Nutation ariſing from another cauſe may produce all this diverſity in the diſtance of the Pole-Star from the Pole: Namely, if the denſity of the Southern Hemiſphere of the Earth be a little greater than that of the Northern, (either becauſe of the leſſer Heat there and greater Cold here, or becauſe of the inequalities of the Continents of the Earth placed about the Poles, or ſome other cauſe unknown to us,) for then ſince in the Winter Solſtice the South Pole *A* is inclined toward to the Sun, and at the ſame time is nearer to it than the North Pole *B*, when in the time of the Summer Solſtice the latter is inclined towards the Sun, *H A* will be more inclined to the plane of the Ecliptic in the Winter time than *E A* in the Summer time; on which account *B H F* will be leſs than *B E F*, tho' the right lines *H F*, *E F* ſhould be parallel. Since therefore all that can be made out of Mr. *Flamſteed*'s obſervation is, that the angle *B H F* is leſs than the angle *B E F*; and this may ariſe from a double cauſe, namely from the concourſe of the right lines *H F*, *E F* towards *F*, if the Axis of the Earth in one obſervation be parallel to the ſame in the other (which is ſuppoſed to be the caſe by Mr. *Flamſtead*, and expreſſed by the *Fig.* 56.) or from the concourſe of the lines right *H A*, *E A* towards *G* the quite contrary way to *F*, right lines *H F*, *E F* being ſuppoſed to be parallel, as in the Figure 57; from this obſervation the Parallax of the Fix'd Stars is not evinced, becauſe the obſervation may ſtand intire though the right lines *E F*, *H F* remain parallel; that is, tho' the Parallax of the *Orbis Magnus* be nothing.

Nay, this obſervation does not immediately prove even the Annual Motion of the Earth: For tho' the Earth be in the middle (revolving about its Axe, as in the *Semi-Tychonic* Syſtem, [*Fig.* 58.] and ſo make the apparent Diurnal Revolution of the Celeſtial Bodies;) the Sun in the Southern Signs may then attract the Southern Hemiſphere of the Earth turned to it, being nearer and perhaps denſer, ſo as to make the angle *BHF*, the diſtance of the Fix'd Star from the Pole, leſs than *BEF*, when the Sun in the Northern Signs being more diſtant, attracts the Northern Hemiſphere leſs, which alſo perhaps is leſs denſe.

The ſame likewiſe may be ſaid of the different

Diſtance of a Fix'd Star from the Zenith, which is the way Dr. *Hook* formerly ſought after the Parallax of the Fix'd Stars. For if the direction of the Axis of the Earth be changed, the direction of any other right line in the Earth (whoſe poſition to the Axe is given) will likewiſe be changed: And the motion of the Earth about the Sun, can't be certainly concluded, from the various inclination of a right line connecting the Fix'd Star and the Earth to that right line in the Earth. It were therefore to be wiſhed, that our exact Obſervers would for the future manage their obſervations for proving the Annual Motion of the Earth after this manner, according to the prec. *Prop.* or its *Schol.* For ſo at laſt the Parallax *of the Fix'd* Stars *in regard of the Orbis Magnus* will with any certainty be found (if it be ſenſible,) ſince the change *of thoſe* angles can't ariſe from any other cauſe, but the change of the Earth's place or of the ſituation of the Fix'd Stars among themſelves; but the identity of the ſituation of the Fix'd Stars in regard of one another is taken as a principle by all ſuch as endeavor to find their Parallax. In the mean while the reaſons laid down in the following Propoſition incline me to believe with *Copernicus*, that the Diameter of the *Orbis Magnus* in regard of the Fix'd Stars (even the neareſt) is inſenſible.

PROPOSITION LVI.

THE *Diſtance of the Sun from the Earth in reſpect of the Diſtance of a Fix'd Star (even the neareſt) from the Earth; vaniſhes almoſt and is inſenſible.*

Since the Sun and Fix'd Stars are the only great Bodies of the Univerſe that have any native Light, they are juſtly eſteemed by Philoſophers to be of the ſame kind, and deſigned for the ſame uſes; and 'tis the effect of a Man's temper, that ſets a greater value upon his own things than he ought, that makes him judge the Sun to be the biggeſt of them all. Let them therefore be ſuppoſed equal, ſince that the one exceeds the other has no ground or foundation to ſtand upon.

Let *T* be the Earth, [*Fig.* 59.] *A E B* the Sun whoſe Center is *S*; *CGD* a Fix'd Star equal to the Sun, whoſe Center is *F*. Since the angle *BTS*, that the ſemidiameter of the Sun ſeen from the Earth appears under, exceeds 16 Minutes, then will *B S* or *C F* equal to it, be almoſt the two hundredth part of *ST*; or 200 times *C F* will be equal to *TS*. In the Triangle *FST* rightangled at *S*, the Sine of the angle *S F T* is to the ſide *S T* as the radius to the ſide *CT*, equal to *FT*: From whence (by *Prop.* 11. *Elem.* 5.) the Sine of *SFT* is to *ST* as the Sine of the angle *CTF* to *CF*. And ſince, in theſe ſmall angles, the angles themſelves are as their Sines, the angle *SFT* will be to the angle *CTF* as the right line *ST* to *C F*; and therefore 200 *CTF* is equal to *SFT*, or 100 *CTD* to *TFS*. But that moſt acute Obſerver *Chriſtian Hugens* (in *his Syſtem of Saturn*) affirms, that he, aſſiſted by a Teleſcope, (by the help of which the diameter of an object was magnified an hundred times) could not obſerve any magnitude in the Diameters of the Fix'd Stars tho' the brighteſt, but that they appeared like a point; that is, the angle *DTC* made an hundred times bigger, was ſtill inſenſible; or an hundred times *CTD* (that is, *S F T*) could not be taken notice of by Senſe. Therefore the Semidiameter of the *Orbis Magnus S T* ſeen from the neareſt Fix'd Star (ſuch as we are to ſuppoſe the brighteſt to be) would appear like the ſmalleſt Point; that is, *TS* in reſpect of *S F* or *TF* vaniſhes and is inſenſible.

This ſmallneſs of the apparent Diameters of the Fix'd Stars is the cauſe of their Twinkling: For the ſmalleſt opake corpuſcles, which are always moving in the Atmoſphere, intirely cover any Fix'd Star, and then immediately uncover it, that is, make it Twinkle.

SCHOLIUM.

Becauſe *ST* compared with *SF* is inſenſible, 'tis no matter in what point of the *Orbis Magnus* the Fix'd Stars be obſerved; for the whole *Orbis Magnus* is but as a point in reſpect of the Diſtance of the Fix'd Stars: And ſince Saturn the outmoſt of the Planets, is not above ten times farther off from the Sun than the Earth, there will be immenſe Spaces between this outmoſt Planet and the neareſt Fix'd Stars to the Sun, into which the very excentric Orbits of the Comets may run, of which we have treated in *Prop.* 35. *B.* 1. And 'tis on the account of this immenſe Diſtance, that the Fix'd Stars have no ſenſible effects upon our Solar Syſtem, and don't diſturb the Planets contain'd in it.

Since therefore the Diſtance of the Fix'd Stars can't be determined by obſervations made in different places of the Earth's Orbit, it remains that we ſhow how to meaſure it by ſome Optical Methods: Two of which we ſhall propoſe, one by Mr. *James Gregory* publiſh'd in the year 1668, the other by Mr. *Hugens* in his *Coſmo-Theoria.*

PROPOSITION LVII. LEMMA.

THE illuminations of the ſame Sphere, in different diſtances from the lucid Body, are reciprocally as the Squares of the diſtances.

Tho' this Propoſition be an eaſy conſequence of *Prop.* 48. *B.* 1. yet I ſhall take the pains to demonſtrate it here. Let *A* [*Fig.*60.] be any point of a lucid Body taken at pleaſure; and let *BC* be the diameter of the Sphere that is illuminated, directly oppoſed to the Rays, at a ſmaller diſtance from the lucid Body; *βγ* the ſame diameter at a greater diſtance; that is, let AE, a ray reaching to the Center E be perpendicular to BC, and $A\varepsilon$ to $\beta\gamma$: Join AB, AC, and being produced let them meet $\beta\gamma$, alſo produced, in D and F. On the Center A, at the diſtance AD or AF, deſcribe an arc of a Circle DF, which let $A\beta$, $A\varepsilon$ and $A\gamma$ produced, meet in H, G and K; and let this Figure be imagined to revolve about the axe AG. 'Tis evident that all the Rays falling upon the Sphere, whoſe diameter is BC, upon the removal of it, will be ſpread upon the Sphere whoſe diameter is DF: And therefore the illumination of the Sphere, whoſe diameter is BC, is to the illumination of the Sphere, whoſe diameter is $\beta\gamma$, as all the rays, contained within the Sector of the Sphere generated by the rotation of ADG, to all the rays contained within the Sector of a Sphere generated by the rotation of AHG; that is, as the Surface of a Sphere generated by the rotation of the arc DG, to the Surface of a Sphere generated by the rotation of the arc GH; that is, (by *Prop.* 49. *B.* 1. *of Archimed. of the Sph. and Cylind.*) as the Circle whoſe radius is DG, to a Circle whoſe radius is HG, or as the Square of DG to the Square of HG. But in ſmall angles, ſuch as we ſuppoſe DAG and HAG to be, DG is to HG very nearly as DE to $\beta\varepsilon$: And therefore the illumination of a Sphere, whoſe Center is in E, is to the illumination of the ſame or an equal Sphere, whoſe Center is in ε, in the duplicate ratio of $D\varepsilon$ to $\beta\varepsilon$, or $D\varepsilon$ to BE, or $A\varepsilon$ to AE; that is, reciprocally as the Square of the diſtances. And ſince the ſame, for the ſame reaſons, is true of any other point of a lucid Body, as well as of A, what was propoſed becomes evident.

And after the like manner, the illuminations of the ſame Sphere made by equal lucid Bodies at unequal diſtances, are reciprocally as the Squares of the diſtances from thoſe lucid Bodies.

PROPOSITION LVIII. LEMMA.

ALL the Rays of the Sun falling upon one Planet are to all the Rays of the Sun falling upon another Planet, as the Squares of the Chords of the Horizontal Parallaxes of the Sun ſeen from thoſe Planets. [Fig.61.]

Let *S* repreſent any radiating point of the Sun; and the Spheres *AB*, *CD*, whoſe centers are *T* and *P*, any two Planets. Join *ST*, *SP*, let a plane drawn thro' them cut the Spheres in the Circles *AB*, *CD*; and from *S* draw Tangents to the ſaid Circles. Upon the Center *S*, at any diſtance, draw an arc of a Circle meeting the right lines *SA*, *ST*, *SB*, *SC*, *SP* and *SD* in *F*, *E*, *L*, *H*, *G* and *K*; join the right lines *EF*, *GH*. Let the Figure *AFSET* be imagined to *revolve about its* Axis *ST*, and *CHSGP* about the Axis SP. All the Rays flowing from the point *S* that fall upon the Planet *T* (*viz.* all within the Cone *ASB*) will be to all the Rays falling upon the Planet P (namely all in the Cone *CSD*) as the Superficies of a Sphere deſcribed by the rotation of the arc *EF*, to the Superficies of a Sphere deſcribed by the rotation of the arc *GH*; that is, (by *Prop.* 49. *B.* I. *of Archimedes of the Sphær. and Cyl.*) as a Circle deſcribed upon the radius *EF* to a Circle deſcribed upon the radius *GH*, or as the Squares of the right lines *EF*, *GH*: But *EF* and *GH* are Chords of the angles *AST*, *CSP* to the radius *SE*, and the ſaid angles are the horizontal Parallaxes of the Sun *S*, ſeen from the Planets *T* and *P*; and therefore all the Rays flowing from the point *S* and falling upon the Planet *T*, are to all falling upon *P*, as the Square of the Chord of the Sun's Parallax ſeen from *T*, to the Square of the Chord of the Sun's Parallax ſeen from *P*. And ſince the ſame is in like manner true of Rays flowing from any other point of the Sun, the Propoſition is evident.

Inſtead of the Chords the Arcs themſelves, or the Angles proportional to them, and ſubtended by them, may be taken, as in the preced. Prop.

SCHOLIUM.

After the like manner, all the rays of a lucid and every way radiating Sphere, are to all its rays that fall upon a given Sphere and illuminate it, in the duplicate ratio of theDiameter (or Chord of the Semicircle) to the Chord of the horizontal Parallax of the lucid Body, ſeen from the illuminated Body; or as the Square of the Diameter to the Square of the Chord of the ſaid horizontal Parallax. And conſequently (by taking the halves of the antecedents,) the half of all the Rays of the lucid Sphere, or all the Rays flowing from a radiating Hemiſphere (ſuppoſing the other Hemiſphere opake) are to all its Rays falling upon the Sphere to be illluminated, to which it appears full, as the half of the Square of the diameter (that is, the Square inſcribed in a Circle) to the Square of the Chord of the Parallax of the lucid Body, ſeen from the Sphere to be illuminated.

PROPOSITION LIX. LEMMA.

THE ratio of the illumination of the Earth by the Sun to its illumination by a Planet ſhining at the full, is compounded of the duplicate ratio of the Chords of the Parallaxes of the Sun ſeen from the Earth and that Planet, and the ratio of a Square inſcribed in a Circle to the Square of the Chord of the horizontal Parallax of the ſaid Planet ſeen from the Earth.

The Rays, with which the Planet illuminates the Earth, are ſuch as flowing originally from the Sun are reflected back again upon the Earth. We ſuppoſe therefore that each Planet reflects every way all the Rays of the Sun that fall upon it; that is (for greater eaſe) we ſuppoſe a Planet to emit Rays every way from that Hemiſphere of it that is turned towards the Sun, *viz.* ſo many as fall upon it from the Sun. 'Tis evident that the illumination of the Earth by the Sun is to the illumination of the ſame by a Planet ſhining in the full, as all the Rays of the Sun falling upon the Earth to all the Rays flowing from the Planet and falling upon it: But the ratio of the Sun's Rays falling upon the Earth to the Planet's Rays falling upon the ſame, is compounded of the ratio of the Sun's Rays falling upon the Earth to the Sun's Rays falling upon the Planet, (that is, all the Rays flowing from the Planet's Hemiſphere,) and the ratio of all the Rays flowing from the Planet's Hemiſphere to all *thoſe of them* that fall upon the Earth. And the former of theſe compounding ratios, or the ratio of the Sun's Rays falling upon the Earth to the Sun's Rays falling upon the Planet, is equal (by the preced. *Prop.*) to the duplicate ratio of the horizontal Parallax of the Sun ſeen from the Earth to the horizontal Parallax of the Sun ſeen from that Planet; but the latter, namely the ratio of all the Rays flowing from the Planet's Hemiſphere illuminating the Earth to all of them that fall upon the Earth and inlighten it, is equal (by the preced. *Schol.*) to the ratio that a Square inſcribed in a Circle has to the Square of the Chord of the horizontal Parallax of the ſaid Planet ſeen from the Earth. And therefore the ratio of the illumination of the Earth by the Sun to the illumination of the ſame by a Planet ſhining in the full, is compounded of the duplicate ratio of the Chords of the horizontal Parallaxes of the Sun ſeen from the Earth and that Planet, and of the ratio of the Square inſcribed in a Circle to the Square of the Chord of the horizontal Parallax of the ſaid Planet ſeen from the Earth. *Q. E. D.*

PROPOSITION LX.

TO *determine the Diſtance of a Fix'd Star.*

Chuſe a Fix'd Star of nearly the ſame brightneſs with ſome Planet, when it is in the Full; in which caſe the Earth is equally illuminated by theſe two Stars. But the mutual Diſtances of the Sun, Earth, and Planet at the time of the abovementioned Obſervation, and the Magnitudes of theſe three Bodies (by *Prop.* 46.) being known, the horizontal Parallaxes of the Sun and Planet ſeen from the Earth (by *Prop.* 48. *B.* 2.) become known, together with the angle the ſaid Planet appears under from the Sun, whoſe half is the horizontal Parallax of the Sun ſeen from that Planet. And conſequently (by the preced. *Prop.*) the ratio of the illumination of the Earth by the Sun to the illumination of the ſame by the ſaid Planet becomes known; that is, the ratio between the illumination of the Earth by the Sun and by the Fix'd Star: But (by *Prop.* 57.) the illumination of the Earth by the Sun is to the illumination of the ſame by a Fix'd Star (equal to the Sun and equally lucid) in the duplicate ratio of the Diſtance of the Fix'd Star from the Earth to the Diſtance of the Sun from the ſame; therefore the ratio between the Diſtances of the Fix'd Star and the Sun from the Earth is known; and conſequently the Diſtance it ſelf of a Fix'd Star from the Earth is known.

SCHOLIUM.

For the ſame purpoſe you may commodiouſly take Jupiter in its Achronical ſituation, and Sirius, which (if any other) may be looked upon as equal to the Sun. But ſince Jupiter in the ſaid ſituation exceeds Sirius in brightneſs, and beſides the Rays of the Sun falling upon Jupiter are not reflected towards the Earth without being weakened, but are moſt of them ſtifled near the ſurface of Jupiter, the Diſtance of Sirius determined after this manner will be leſs than the true Diſtance.

PROPOSITION LXI.

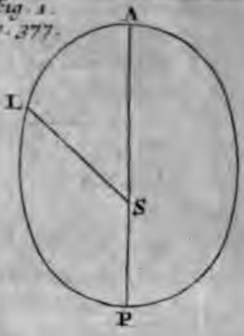
TO meaſure the Diſtance of a Fix'd Star another way.

Tho' the diameter of the brighteſt Fix'd Star can't by any Optical Inſtrument yet invented, be made ſo big as to become ſenſible, and ſuch as may be compared with the Sun's Diameter; however, the Sun's Diameter may be leſſen'd ſo much, as that its Light thus leſſen'd may (in appearance) be no greater than the light of *Sirius* twinkling in the Night; which will be done if its Rays be let into a Tube thro' a hole ſmall enough; or if its image be leſſened as much by means of a Lens, or partly by one partly by the other method: For the diameter of the image of the Sun, tho' inſenſible (if looked for inconſiderately) becomes known by calculation founded upon Optics and Dioptrics, having the Diameter of the hole, the Lens and ſituation of the Eye given. But the Diameter of an image of the Sun, as bright as Sirius to an Eye given by poſition, is equal to the Diameter of Sirius; therefore the apparent Diameter of Sirius, before inſenſible, becomes known by calculation: But the Sun and Sirius are ſuppoſed equal; therefore their Diameters are equal: And therefore the Diſtance of Sirius will be to the Diſtance of the Sun, as the Sine of the apparent Diameter of the Sun to the Sine of the apparent Diameter of Sirius, or (by reaſon of the ſmallneſs of theſe angles) as the apparent Diameter of the Sun to the apparent Diameter of Sirius; therefore the Diſtance of Sirius will be known. *Q. E. I.*

The End of the Firſt Volume.

Plate 1. Book 3.

Fig. 1.
p. 377.



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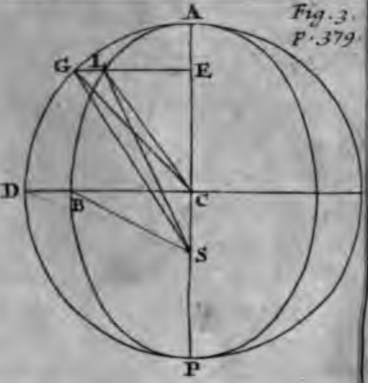
p. 379.



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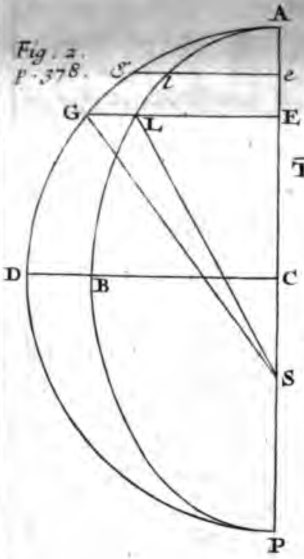

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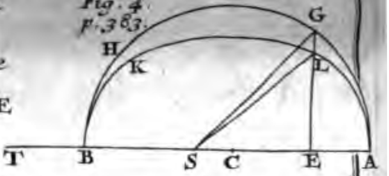

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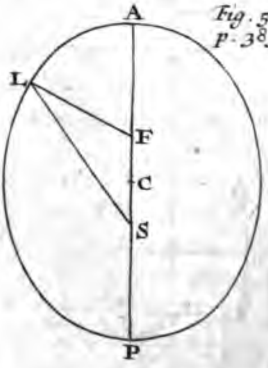

Plate 2. Book 3.

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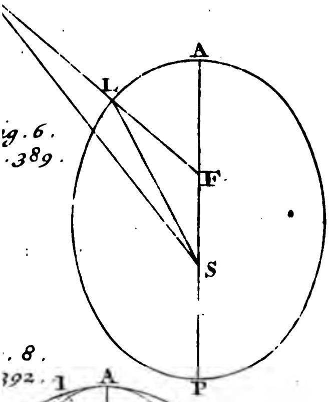

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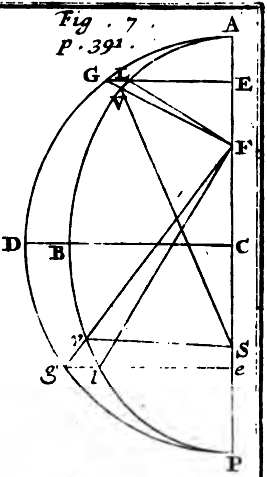

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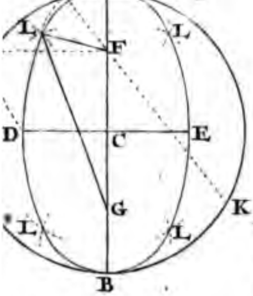

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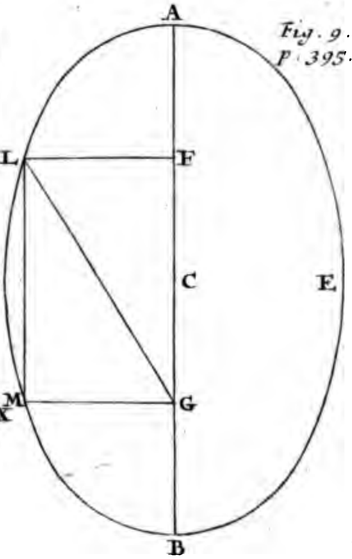

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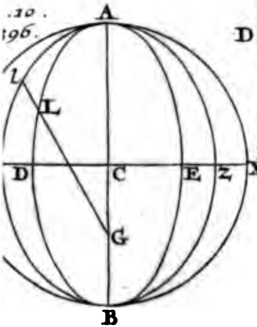

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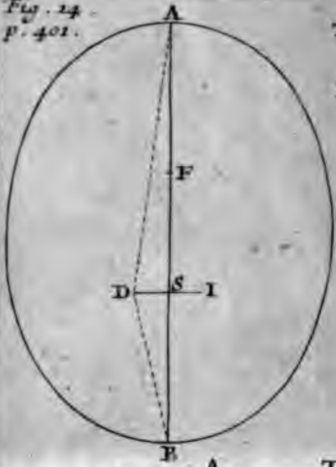

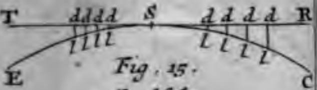

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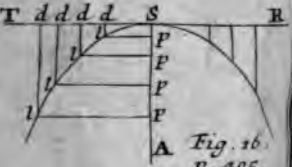

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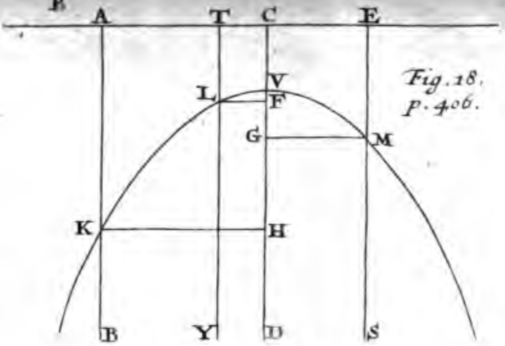

Fig. 18.
p. 406.

Fig. 17.
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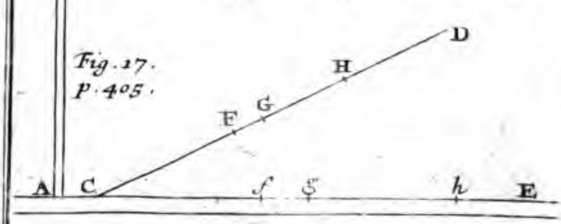

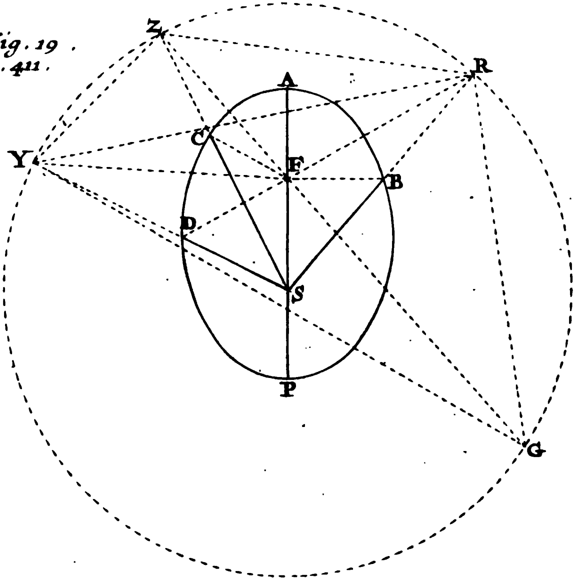

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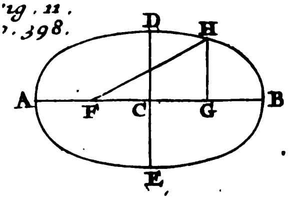

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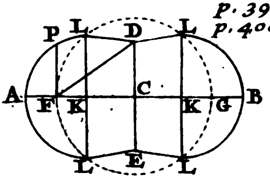

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p. 400.

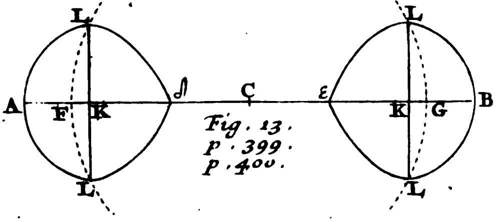

Fig. 13.
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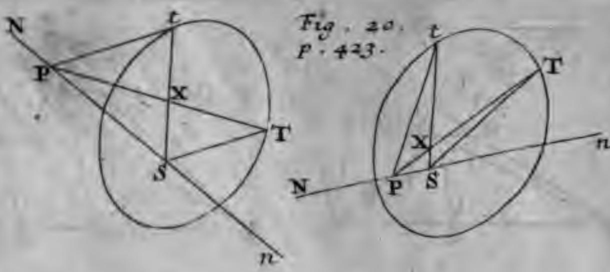
p. 400.

Plate 5. Book 3.

Fig. 20.
p. 423.



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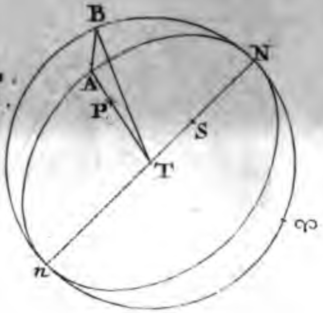

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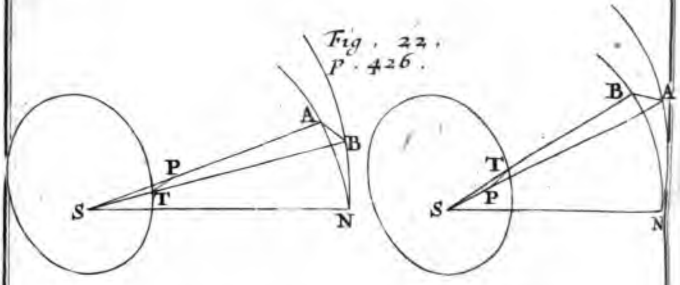

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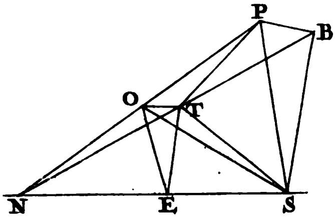

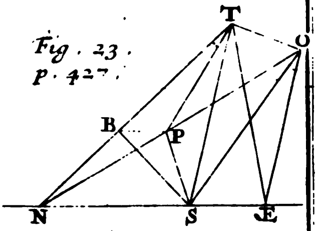

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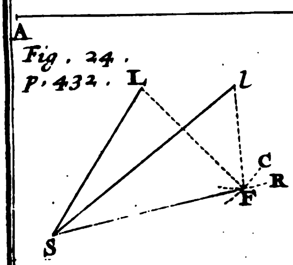

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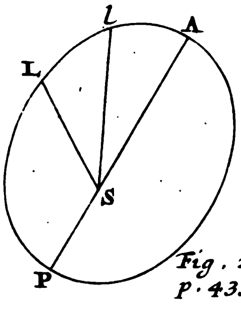

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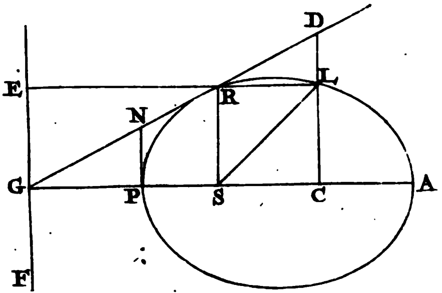

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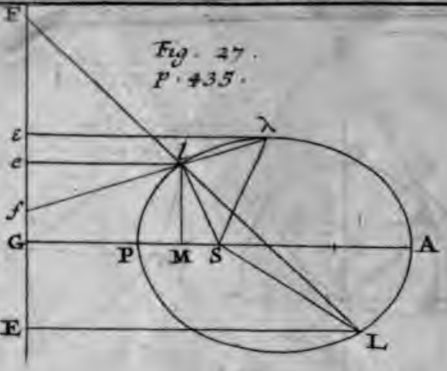
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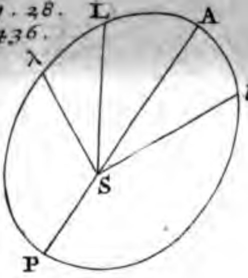

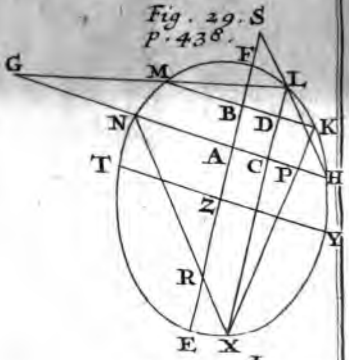
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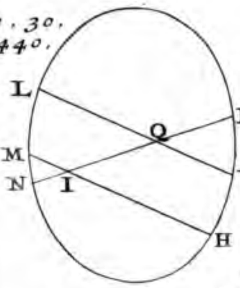

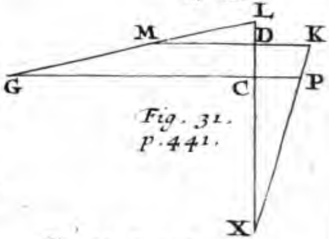
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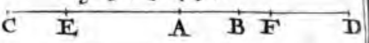
Fig. 32. p. 442.


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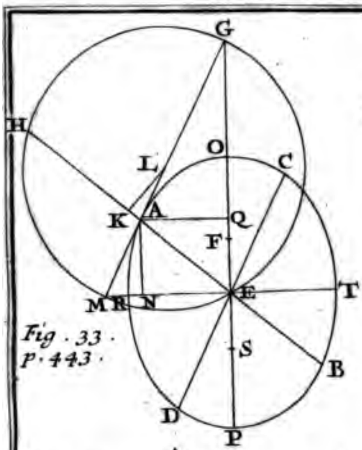

Fig . 33 .
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Fig . 34 .
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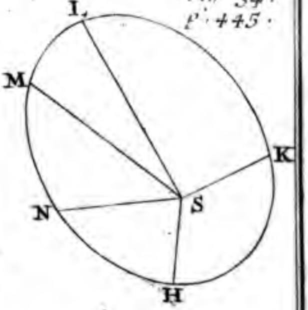

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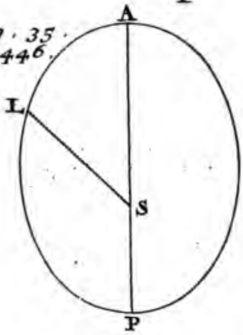

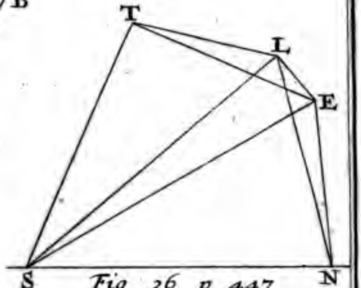

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Fig . 37 .
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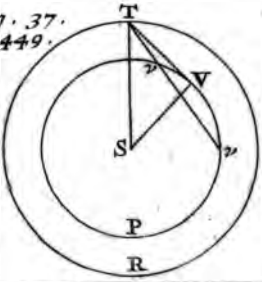

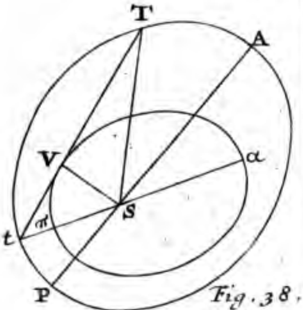

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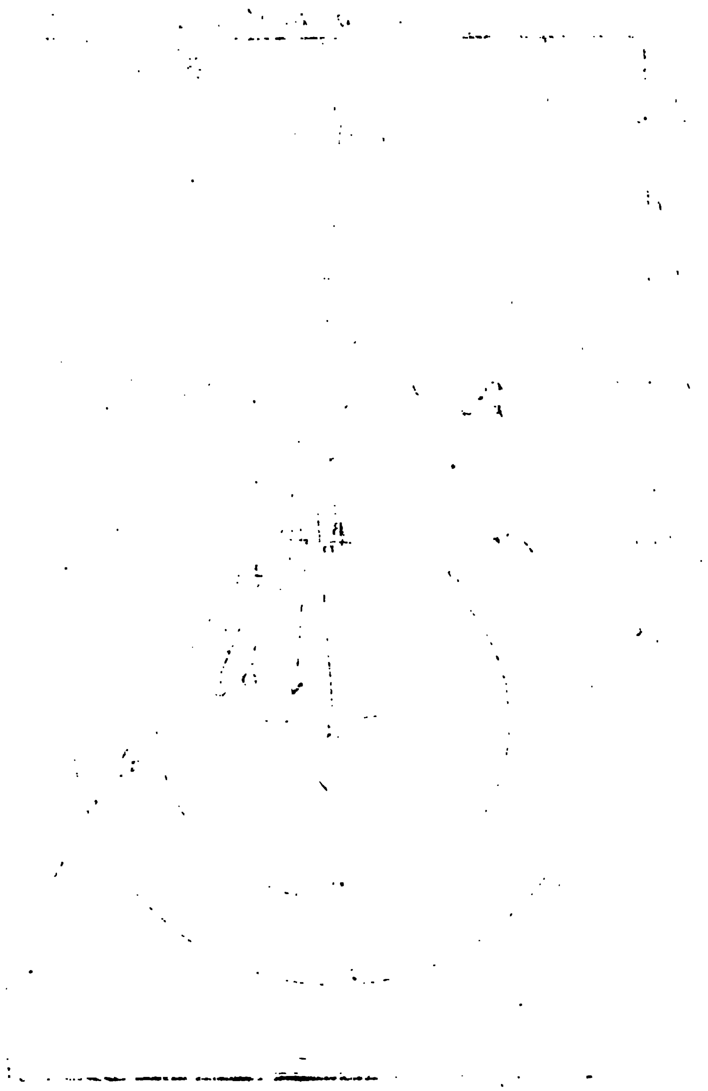

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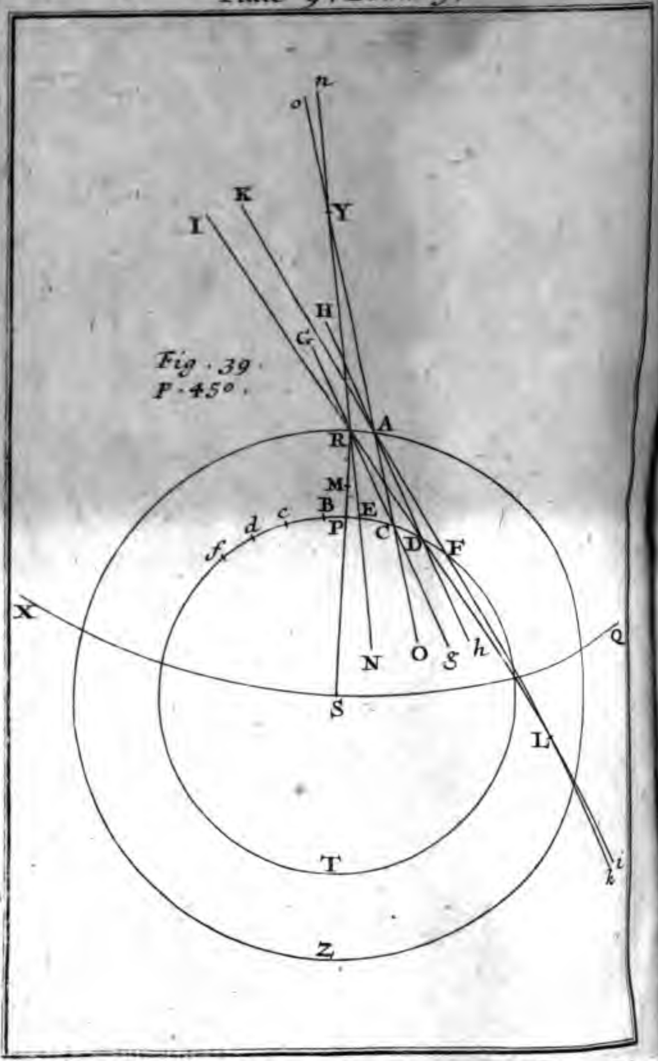

Fig. 39.
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Plate 10, Book 3.

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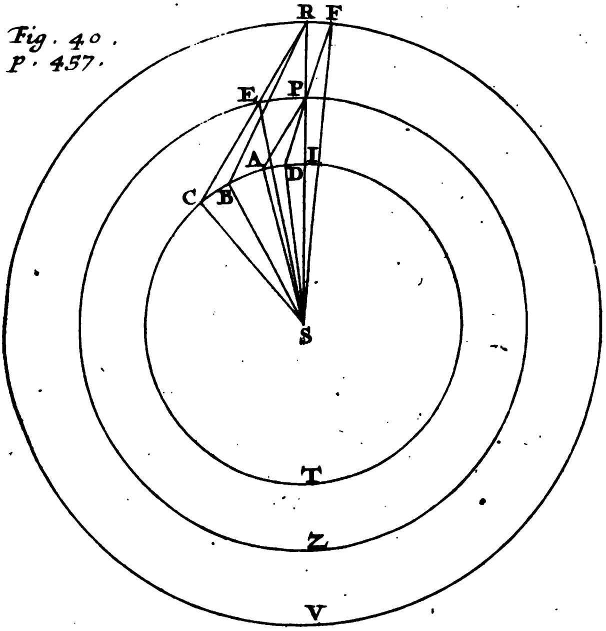

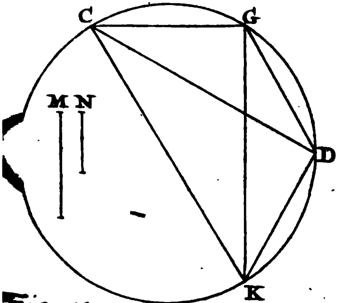

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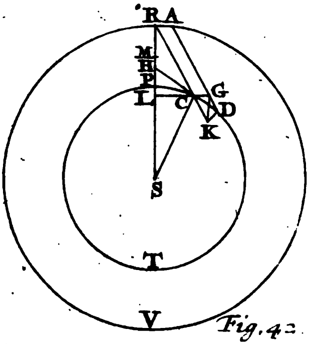

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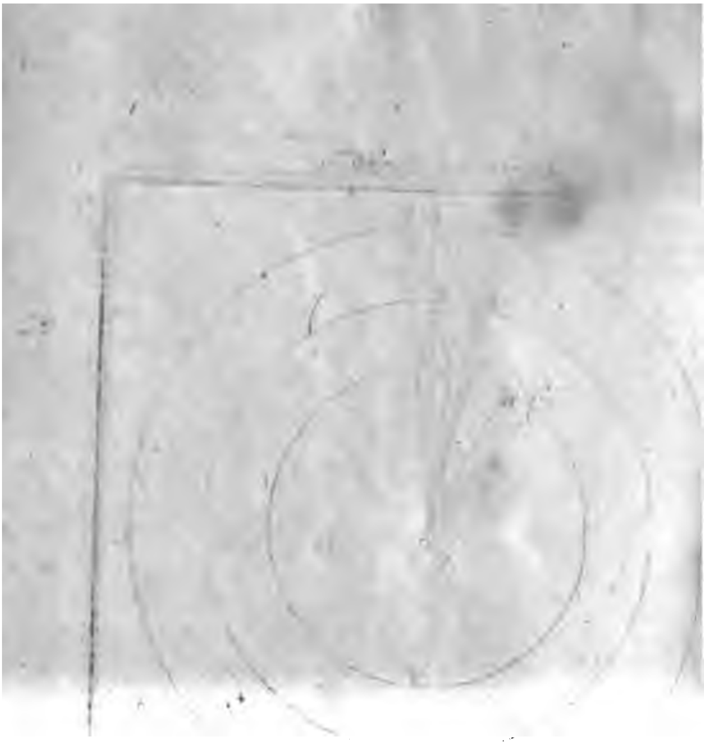

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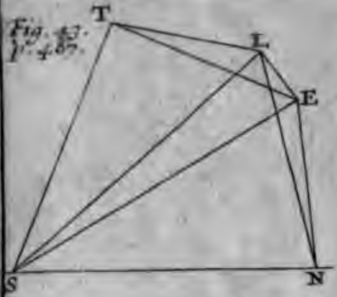
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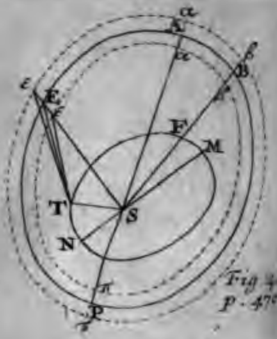

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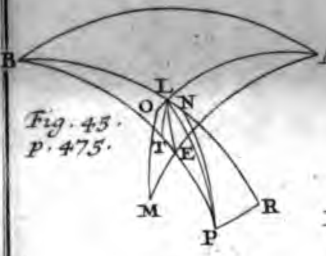
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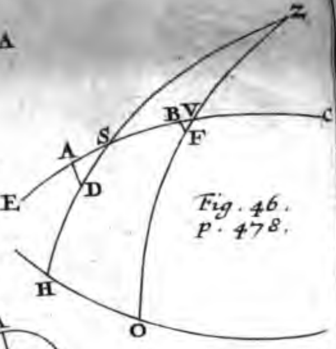

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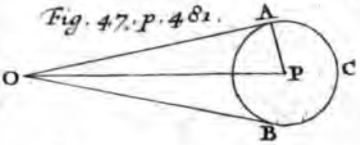
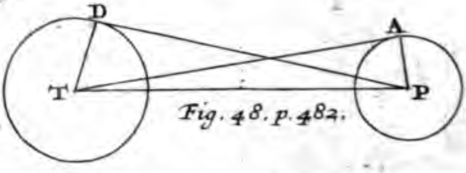
Fig. 47. p. 481.




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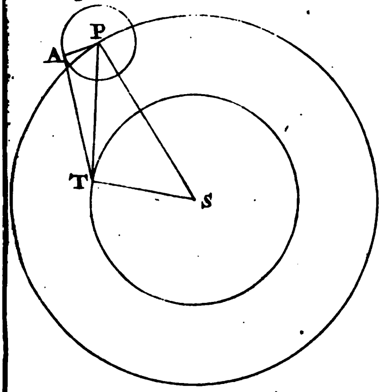

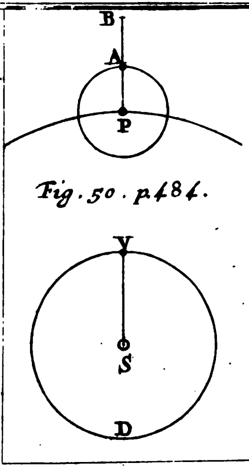

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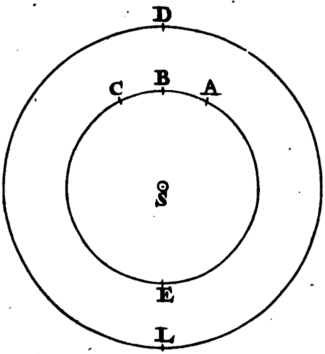

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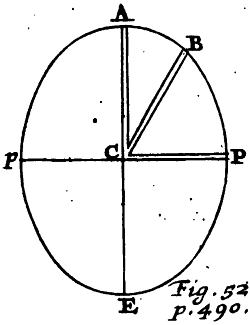

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Fig. 53.
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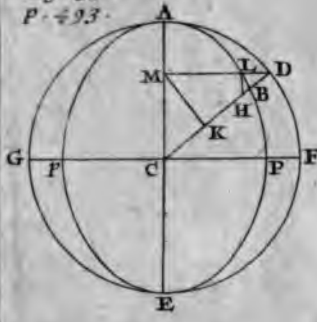

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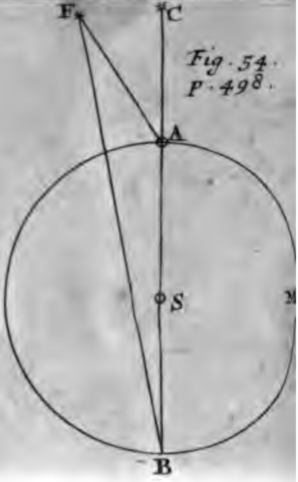

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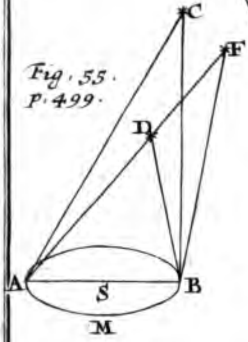

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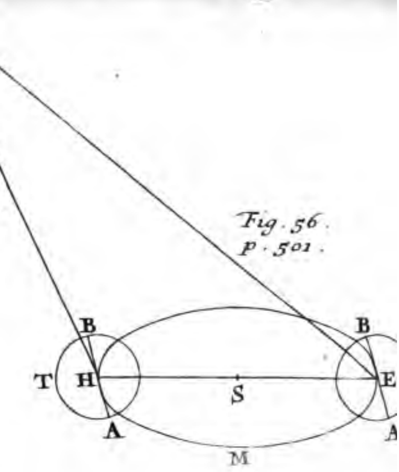

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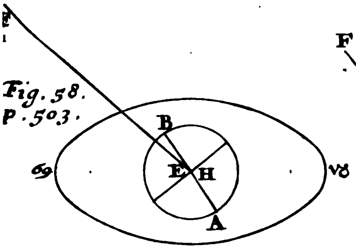
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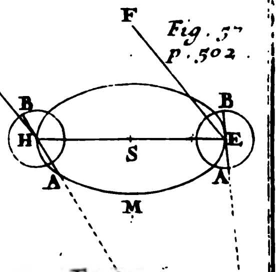
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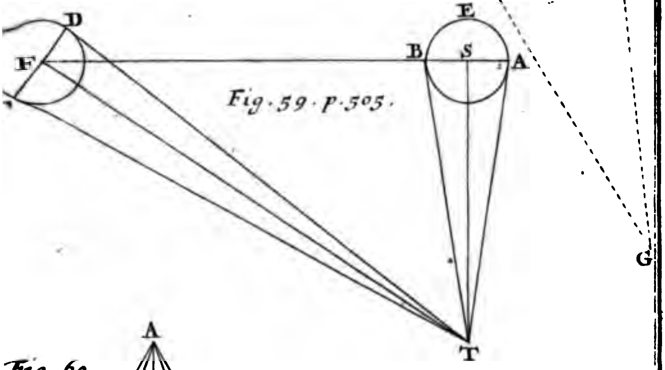
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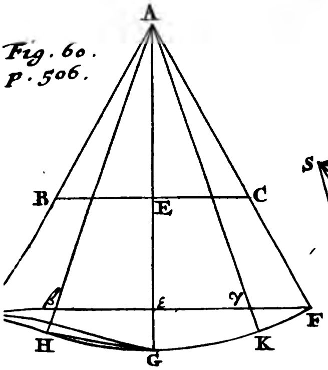
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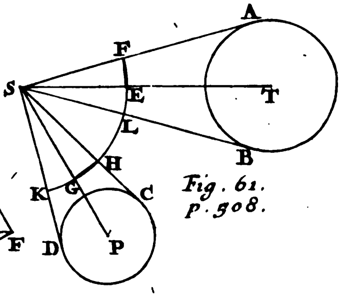
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